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# STATIC AND DYNAMIC SIGNAL DETECTION METHODS FOR ROAD TRAFFIC NOISE ENVIRONMENT BASED ON FUZZY OBSERVATION

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ABSTRACT: In the actual sound environment, a specific signal shows various types of probability distribution forms apart from a standard Gaussian distribution due to the diversified causes of fluctuation. Furthermore, the actually observed data often contain some fuzziness due to the existence of confidence interval in measuring instruments, permissible error in experimental data and a level quantized error in digital observation. In this study, a new estimation theory for a specific signal, based on the observed data containing the fuzziness and the effects of non-Gaussian property is proposed from the static and dynamic viewpoints. More specifically, by applying fuzzy probability to a probability expression with an infinite expansion series form, a static method to estimate the probability density function of the specific signal based on the fuzzy observation data is first proposed. Next, a dynamical method of estimating only a specific signal state embedded in the additional noise (i.e., background noise) with random fluctuation of non-Gaussian type is theoretically derived especially in a flexible form applicable to those fuzzy observation data. Finally, the validity and the effectiveness of the proposed method are confirmed experimentally by applying it to the actual road traffic noise data.

## 1. INTRODUCTION

In the actual sound environment, the observation data are usually contaminated by the additional external noise (i.e., background noise) showing various types of probability distribution forms of non-Gaussian distribution. Therefore, in order to estimate several evaluation quantities for the specific signal, like $L_x$, $L_{eq}$ and peak value, based on the observed noisy data, it is necessary to consider not only the lower order statistics like mean and variance but also the higher order ones connected with non-Gaussian properties.

On the other hand, the actual observed data often contain the fuzziness due to the existence of confidence interval in measuring instruments, permissible error in experimental data and a quantized error in a usual digital observation. Many standard signal detection methods proposed previously have not considered positively the fuzziness in the observation data under the restriction of Gaussian type fluctuation, for the simplification of theory [1]-[3]. Though several state estimation methods for a stochastic environmental system with non-Gaussian fluctuations have been proposed in our previous studies [4],[5], the fuzziness contained in the observed data has not been considered. Therefore, there arises a problem on how to extend our previous methods to a flexible form applicable also to the ill-conditionedness with fuzzy observation.

In this paper, a new estimation theory for a specific signal, based on the observed data containing the fuzziness and the effects of non-Gaussian property is proposed from the static and dynamic viewpoints. More specifically, by applying the fuzzy probability [6] to a probability expression with an infinite expansion series form, a static method to estimate the system parameters for an acoustic environment based on the fuzzy observation data and to predict the output response probability distribution for an arbitrary input signal is first proposed. Next, a dynamical state estimation method of only a specific signal embedded in the additional noise (i.e., background noise) with random fluctuation of non-Gaussian type is theoretically derived in a flexible form applicable to the fuzzy observation data. Finally, the validity and the effectiveness of the proposed method are confirmed experimentally by applying it to the actual road traffic noise data.

## 2. STATIC SIGNAL DETECTION METHOD BASED ON FUZZY OBSERVATION

Let us focus on the input x and the output y of acoustic environmental systems in an intensity variable under the existence of the external noise v. Based on the additive property of acoustic intensity, the following linear system model is considered.

$$y = ax+b+v, \tag{1}$$

where a and b denote two unknown system parameters, and the statistics of v are known. The observation data in an acoustic environment are often measured in the quantized level form on a decibel scale, and sometimes contain the effects of a permissible error of the accuracy in the measurement. By regarding these quantized level measurement and various error factors as some kind of fuzziness in the observation from the functional viewpoint, the following membership function can be introduced.

$$\mu_S(Y) = \exp\{-\alpha(Y-S)^2\}, \quad (Y = 10\log_{10}\frac{y}{10^{-12}}), \tag{2}$$

where S denotes the fuzzy observation data and the parameter $\alpha$ can be generally regarded as unknown one. In this section, a method to estimate the system parameters a and b based on the fuzzy observation S is considered.

By use of the fuzzy probability [6], the probability distribution P(S) of fuzzy data S can be expressed as:

$$P(S) = \frac{1}{K}\int \mu_S(Y)P(Y)dY, \tag{3}$$

where K is a constant satisfying the normalized condition:

$$\int P(S)dS = 1, \quad (\text{or } \sum_i P(S_i)) = 1. \tag{4}$$

Since the output level Y shows an arbitrary fluctuation form of non-Gaussian type, the statistical Hermite expansion expression [7] is adopted as the probability density function of Y:

$$P(Y) = \frac{1}{\sqrt{2\pi\sigma_Y^2}}\exp\{-\frac{(Y-\mu_Y)^2}{2\sigma_Y^2}\}\{1+\sum_{n=3}^{\infty}A_n\frac{1}{\sqrt{n!}}H_n(\frac{Y-\mu_Y}{\sigma_Y})\},$$

$$\mu_Y = <Y>, \quad \sigma_Y^2 = <(Y-\mu_Y)^2>, \quad A_n = <\frac{1}{\sqrt{n!}} H_n(\frac{Y-\mu_Y}{\sigma_Y})>, \tag{5}$$

where < > denotes the averaging operation with respect to the random variables. Hereupon, by use of a relationship between the cumulant statistics on decibel scale and the moment statistics on intensity scale [8], the parameters $\mu_Y$, $\sigma_Y^2$, $A_n$ (n=3,4) in Eq.(5) can be given by

$$\mu_Y = M\{\ln\frac{1}{10^{-12}} + 4\ln<y> - 3\ln<y^2> + \frac{4}{3}\ln<y^3> - \frac{1}{4}\ln<y^4>\},$$

$$\sigma_Y^2 = M^2\{-\frac{26}{3}\ln<y> + \frac{19}{2}\ln<y^2> - \frac{14}{3}\ln<y^3> + \frac{11}{12}\ln<y^4>\},$$

$$A_3 = \frac{M^3}{\sqrt{3!}\sigma_Y^3}\{9\ln<y> - 12\ln<y^2> + 7\ln<y^3> - \frac{3}{2}\ln<y^4>\},$$

$$A_4 = \frac{M^4}{\sqrt{4!}\sigma_Y^4}\{-4\ln<y> + 6\ln<y^2> - 4\ln<y^3> + \ln<y^4>\}, \qquad (M = 10/\ln 10). \tag{6}$$

Furthermore, from Eq.(1), the following relationship is easily derived:

$$<y^r> = \sum_{i+j=r} \frac{r!}{i!j!(r-i-j)!} a^i <x^i> b^j <v^{r-i-j}>. \tag{7}$$

Substituting Eq.(2) and Eqs.(5)-(7) into Eq.(3), and considering the orthonormal condition of Hermite polynomial [7]:

$$\int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma_Y^2}} \exp\{-\frac{(Y-\mu_Y)^2}{2\sigma_Y^2}\} \frac{1}{\sqrt{n!}} H_n(\frac{Y-\mu_Y}{\sigma_Y}) \frac{1}{\sqrt{m!}} H_m(\frac{Y-\mu_Y}{\sigma_Y})\, dY = \delta_{nm}, \tag{8}$$

the probability distribution P(S) is expressed as follows:

$$P(S) = \frac{1}{K}\sum_{n=0}^{\infty} J_n(S; \alpha, a, b, <x^j>, <v^j>), \quad (j=1, 2, ...) \tag{9}$$

with

$$J_0(S;*) = \frac{e^{C_3}}{\sqrt{2C_1\sigma_Y^2}}, \qquad J_1(S;*) = J_2(S;*) = 0,$$

$$J_3(S;*) = \frac{e^{C_3}}{\sqrt{2C_1\sigma_Y^2}} \frac{1}{\sqrt{3!}} \{\frac{3(C_2-\mu_Y)}{2C_1\sigma_Y^3} - 3(\frac{C_2-\mu_Y}{\sigma_Y}) + \frac{(C_2-\mu_Y)^3}{\sigma_Y^3}\}A_3,$$

$$J_4(S;*) = \frac{e^{C_3}}{\sqrt{2C_1\sigma_Y^2}} \{\frac{9}{4\sigma_Y^4}(\frac{1}{2C_1})^2 + \frac{6(C_2-\mu_Y)^2 - 6\sigma_Y^2}{\sigma_Y^4}\frac{1}{2C_1} + \frac{(C_2-\mu_Y)^4 - 6\sigma_Y^2(C_2-\mu_Y)^2 + 3\sigma_Y^4}{\sigma_Y^4}\}A_4, \quad .....,$$

$$C_1 = \frac{2\alpha\sigma_Y^2+1}{2\alpha\sigma_Y^2}, \quad C_2 = \frac{2\alpha\sigma_Y^2 S + \mu_Y}{2\alpha\sigma_Y^2+1}, \quad C_3 = C_1(C_2^2 - \frac{2\alpha\sigma_Y^2 S^2 + \mu_Y^2}{2\alpha\sigma_Y^2+1}). \tag{10}$$

Let us assume that S takes the finite numbers of levels $S_1$, $S_2$, ..., $S_L$, and $S_i$ denotes a fuzzy data about $S_i$ dB around the true state value. Therefore, by selecting artificially several values of $S_i$ with the same numbers as unknown parameters, the simultaneous equations in the same form as in Eq.(9) can be written, as follows:

$$P(S_i) = \frac{1}{K}\sum_{n=0}^{\infty} J_n(S_i; \alpha, a, b, <x^j>, <v^j>), \quad (i=1, 2, 3) \tag{11}$$

As a method of solving Eq.(11), the well-known Newton Raphason method can be adopted.

By use of the estimates on the parameters of acoustic environmental systems, the output probability distribution for the system with an arbitrary input signal in an ideal case without the external noise and fuzziness can be predicted as follows:

$$\hat{Y} = 10\log_{10}\frac{\hat{a}x+\hat{b}}{10^{-12}}, \tag{12}$$

where $\hat{a}$ and $\hat{b}$ denote the estimates of a and b. Based on the time series data of $\hat{Y}$, the cumulative probability distribution connected with an evaluation index $L_x$ can be directly constructed by use of the computer.

## 3. DYNAMIC SIGNAL DETECTION METHOD BASED ON FUZZY OBSERVATION

A sound environmental system exhibiting a non-Gaussian distribution is considered. Let the specific signal intensity at a discrete time k be x(k), and express the dynamical model for the specific signal as:

$$x(k+1) = Fx(k) + Gu(k), \tag{13}$$

where u(k) denotes the random input with known statistics, and x(k) and u(k) are uncorrelated each other. Furthermore, F and G are known system parameters.

The observed data in the actual sound environment inevitably contain some kind of fuzziness due to the quantized error in the digitization of observation data and several error factors in the measurement. Therefore, in addition to an avoidable external noise, the effects of the fuzziness contained in the observed data have to be first considered in order to derive a state estimation method for the specific signal. The observation equation can be formulated by dividing it into two types of operation from a functional viewpoint:

i) The additive property of acoustic intensity, under the existence of external noise:

$$y(k) = x(k) + v(k). \tag{14}$$

We assume that the statistics of the external noise intensity v(k) are known in advance.

ii) The fuzzy observation s(k) obtained from y(k): The fuzziness of s(k) is characterized by the membership function $\mu_{s(k)}(y(k))$, especially by reflecting it to y(k) on an intensity scale, not to the directly observed data.

In order to derive an estimation algorithm for a specific signal x(k), based on the successive observations of fuzzy data s(k), we focus our attention on Bayes' theorem:

$$P(x(k)|S(k)) = \frac{P(x(k),s(k)|S(k-1))}{P(s(k)|S(k-1))}, \tag{15}$$

where S(k)(=(s(1), s(2), ..., s(k))) is a set of observation data up to a time k. After applying the fuzzy probability [6] to the right side of Eq.(15), expanding it in a general form of the statistical orthonormal expansion series, the conditional probability density function P(x(k)|S(k)) can be expressed as:

$$P(x(k)|S(k)) = \frac{\int \mu_{s(k)}(y(k))P(x(k),y(k)|S(k-1))dy(k)}{\int \mu_{s(k)}(y(k))P(y(k)|S(k-1))dy(k)}$$

$$= \frac{\sum_{m=0}^{\infty}\sum_{n=0}^{\infty} A_{mn}P_0(x(k)|S(k-1))\psi_m^{(1)}(x(k))\int \mu_{s(k)}(y(k))P_0(y(k)|S(k-1))\psi_n^{(2)}(y(k))dy(k)}{\sum_{n=0}^{\infty} A_{0n}\int \mu_{s(k)}(y(k))P_0(y(k)|S(k-1))\psi_n^{(2)}(y(k))dy(k)} \quad (16)$$

with

$$A_{mn} = <\psi_m^{(1)}(x(k))\psi_n^{(2)}(y(k))|\ S(k-1)>, \quad (17)$$

The functions $\psi_m^{(1)}(x(k))$ and $\psi_n^{(2)}(y(k))$ are the orthonormal polynomials of degrees m and n with weighting functions $P_0(x(k)|S(k-1))$ and $P_0(y(k)|S(k-1))$, which can be artificially chosen as the probability density functions describing the dominant parts of $P(x(k)|S(k-1))$ and $P(y(k)|S(k-1))$ from the viewpoint of convergence of infinite expansion series. Based on Eq.(16), and using the orthonormal relationship of the function $\psi_m^{(1)}(x(k))$, the recurrence algorithm for estimating an arbitrary Nth order polynomial type function $f_N(x(k))$ of the specific signal can be derived as follows:

$$\hat{f}_N(x(k))=<f_N(x(k))|S(k)>=\frac{\sum_{m=0}^{N}\sum_{n=0}^{\infty} A_{mn}c_{Nm}\int \mu_{s(k)}(y(k))P_0(y(k)|S(k-1))\psi_n^{(2)}(y(k))dy(k)}{\sum_{n=0}^{\infty} A_{0n}\int \mu_{s(k)}(y(k))P_0(y(k)|S(k-1))\psi_n^{(2)}(y(k))dy(k)}, \quad (18)$$

where $c_{Nm}$ is the expansion coefficient determined by the equality:

$$f_N(x(k)) = \sum_{m=0}^{N} c_{Nm}\psi_m^{(1)}(x(k)). \quad (19)$$

In order to make the general theory for estimation algorithm more concrete, the well-known Gaussian distribution is adopted as $P_0(x(k)|\ S(k-1))$ and $P_0(y(k)|\ S(k-1))$, because this probability density function is the most standard one, as follows:

$$P_0(x(k)|S(k-1)) = N(x(k); x^*(k), \Gamma(k)),\quad P_0(y(k)|S(k-1)) = N(y(k); y^*(k), \Omega(k)) \quad (20)$$

with

$$N(x; \mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}}\exp\{-\frac{(x-\mu)^2}{2\sigma^2}\},$$

$$x^*(k)=<x(k)|S(k-1)>,\ \Gamma(k)=<(x(k)-x^*(k))^2|S(k-1)>,\ y^*(k)=<y(k)|S(k-1)>=x^*(k)+<v(k)>,$$
$$\Omega(k) = <(y(k)-y^*(k))^2|S(k-1)> = \Gamma(k) + <(v(k) - <v(k)>)^2>. \quad (21)$$

Then, the orthonormal functions with two weighting probability density functions in Eq.(20) can be given in the following Hermite polynomials [7]:

$$\psi_m^{(1)}(x(k)) = \frac{1}{\sqrt{m!}}H_m(\frac{x(k) - x^*(k)}{\sqrt{\Gamma(k)}}),\quad \psi_n^{(2)}(y(k)) = \frac{1}{\sqrt{n!}}H_n(\frac{y(k) - y^*(k)}{\sqrt{\Omega(k)}}). \quad (22)$$

As the membership function $\mu_{s(k)}(y(k))$, the following function is adopted.

$$\mu_{s(k)}(y(k)) = \exp\{-\ \beta(y(k) - s(k))^2\}, \quad (23)$$

where β (>0) is a parameter. Through the similar calculation process to the derivation in Sect.2, the estimation algorithm of the specific signal can be given by

$$\widehat{f_N}(x(k)) = \frac{\sum_{m=0}^{N}\sum_{n=0}^{\infty} A_{mn} c_{Nm} I_n(s(k))}{\sum_{n=0}^{\infty} A_{0n} I_n(s(k))} \tag{24}$$

with

$$I_0(s(k)) = \frac{e^{K_3}}{\sqrt{2K_1\Omega(k)}}, \quad I_1(s(k)) = \frac{e^{K_3}(K_2-y^*(k))}{\sqrt{2K_1\Omega(k)}},$$

$$I_2(s(k)) = \frac{e^{K_3}}{\sqrt{2K_1\Omega(k)}}\frac{1}{\sqrt{2!}}\{\frac{(K_2-y^*(k))^2}{\Omega(k)} + \frac{1}{2K_1\Omega(k)} -1\},$$

$$I_3(s(k)) = \frac{e^{K_3}}{\sqrt{2K_1\Omega(k)}}\frac{1}{\sqrt{3!}}\{\frac{3(K_2-y^*(k))}{2K_1(\Omega(k))^{3/2}} - 3(\frac{K_2-y^*(k)}{\sqrt{\Omega(k)}}) + \frac{(K_2-y^*(k))^3}{(\Omega(k))^{3/2}}\}, \ldots\ldots , \tag{25}$$

where the fuzzy data s(k) are reflected in $K_1$, $K_2$ and $K_3$ as:

$$K_1=\frac{2\beta\Omega(k)+1}{2\Omega(k)}, \quad K_2=\frac{2\beta\Omega(k)s(k)+y^*(k)}{2\beta\Omega(k)+1}, \quad K_3=K_1(K_2{}^2 - \frac{2\beta\Omega(k)s(k)^2+y^*(k)^2}{2\beta\Omega(k)+1}). \tag{26}$$

Finally, by considering Eq.(13), the prediction step which is essential to perform the recurrence estimation can be given by

$$x^*(k+1) = F\langle x(k)|S(k)\rangle + G\langle u(k)\rangle,$$
$$\Gamma(k+1) = F^2\langle (x(k)-\langle x(k)|S(k)\rangle)^2|S(k)\rangle + G^2 \langle (u(k)-\langle u(k)\rangle)^2\rangle, \ldots\ldots . \tag{27}$$

By replacing k with k+1, the recurrence estimation can be achieved.

## 4. APPLICATION TO ROAD TRAFFIC NOISE ENVIRONMENT

In order to examine the practical usefulness of the proposed signal detection method based on the fuzzy observation, the proposed method is applied to the actual sound environmental data. First, the system identification method in Sect. 2 is applied to acoustic data observed indoors and outdoors for a house. Under the actual situation contaminated by a background noise (road traffic noise), the fuzzy data quantized roughly with 5 dB width on the output fluctuation of the acoustic environmental system are sampled. Based on the 500 data, the system parameters are estimated. The 200 sampled data following the data used for the estimation of parameters are adopted for predicting the output response probability distribution form. The mean level of the background noise is daringly set in advance to be equal to the mean value of the system output.

Figure 1 shows the prediction result for the output probability distribution. The observed values contaminated by the background noise and affected by the fuzziness with rough 5dB width are shown in white circles (○), and the experimentally sampled points on the output probability distribution without considering the effects of the background noise and the fuzziness are shown in black circles (●). Furthermore, a solid curve (——) shows the theoretically predicted output probability distribution in the case of considering the expansion coefficients $A_3$ and $A_4$ in Eq.(5) when the system parameters are estimated, and the dotted curve (-----) shows the predicted result when the system parameters are estimated by considering only the first term (i.e., Gaussian distribution) in Eq.(5). By considering the expansion terms $A_3$ and $A_4$ into consideration, the theoretically predicted curve approaches to the experimentally sampled values for the output probability distribution.

Furthermore, the proposed method is compared with the standard method based on the least squares error criterion under the situation without the background noise, after assuming the linear system model on decibel scale. Figure 2 shows the comparison between the theoretically predicted curves and the experimentally sampled points for the output probability distribution. The theoretically predicted curves based on the proposed method show better agreement with the experimentally sampled values than the result using the usual least squares error criterion.

Next, in order to examine the practical usefulness of the proposed digital filter based on the fuzzy observation in Sect.3, the proposed method is applied to the actual sound environmental data. The road traffic noise is adopted as an example of a specific signal with a complex fluctuation form. Applying the proposed estimation method to actually observed data contaminated by background noise and quantized roughly with 2 dB width, the fluctuation wave form of the specific signal is estimated. Figure 3 shows the estimation result of the fluctuation wave form of the specific signal in the case when the proposed method is applied. Hereupon, the horizontal axis shows the discrete time k, of the estimation process, and the

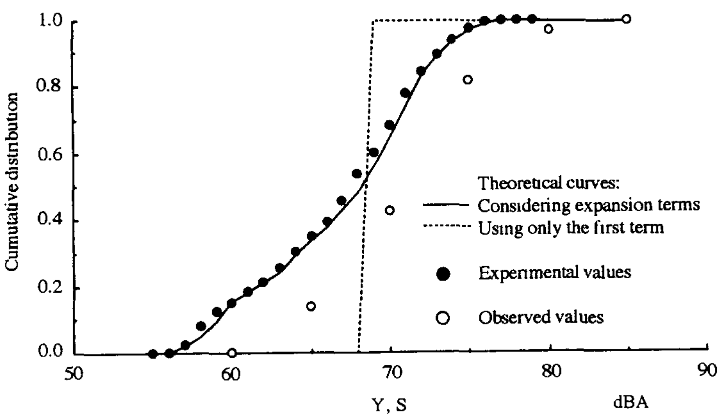


Fig.1 Comparison between theoretically predicted curves and experimentally sampled points on the output probability distribution in the actual case of existing the background noise.

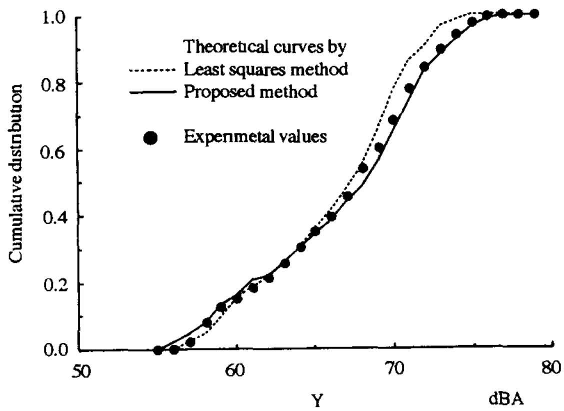


Fig.2 Comparison between theoretically predicted curves on the output probability distribution by use of the proposed method and the least squares method.

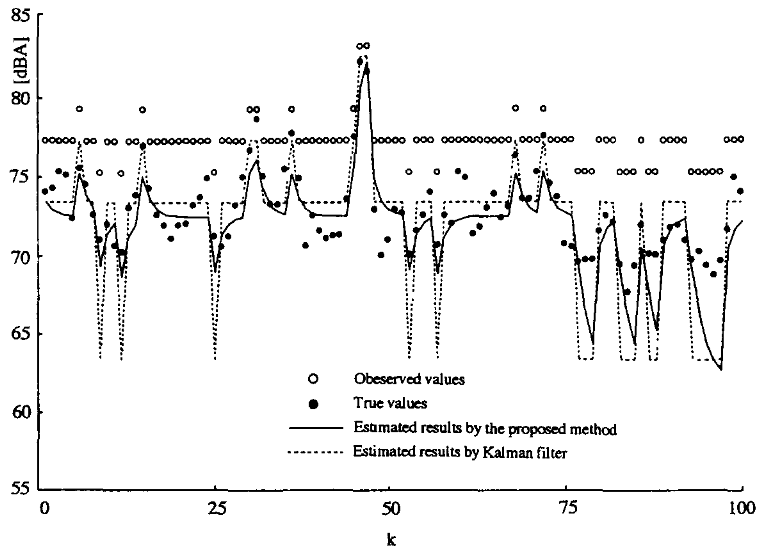


Fig.3 State estimation results for the road traffic noise during a discrete time interval of [1, 100] (sec).

vertical axis expresses the sound level. For comparison, the estimation result calculated using the usual method without considering any membership function is also shown in this figure. Since Kalman's filtering theory is widely used in the field of stochastic system [1, 2], this method is also applied to the fuzzy observation data as a trial. The result estimated by the proposed method considering the membership function shows good agreement with the true values. On the other hand, there are great discrepancies between the estimates based on the standard type dynamical estimation method (i.e., Kalman filter) without consideration of the membership function and the true values, particularly in the estimation of the lower level values of the fluctuation.

## 5. CONCLUSION

In this paper, based on the observed data containing some fuzziness after contamination by background noise, new methods for estimating a specific signal have been proposed especially from two viewpoints of static and dynamic methods. The proposed estimation methods have been realized by introducing the fuzzy probability into the probability distribution of expansion series type. The proposed methods have been applied to the actual estimation problems of the specific sound environment, and these have been experimentally verified that the better results have certainly been obtained than the results employing usual methods without considering any membership function.

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