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THE YO-YO AS A STRUCTURAL VIBRATION EXCITER - FORCE TIME HISTORIES

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ABSTRACT

A yo-yo can be used as a vibration exciter as the device generates an approximately half sine wave force as its translatory motion changes from down to up. This paper reviews the theory of Bürger and its use in prediction of yo-yo forces as well as a series of measured force time histories and associated spectra. Both theory and measurements show that a new design of the toy yo-yo is required to be an efficient vibration shaker.

1. INTRODUCTION

The mechanical action of a yo-yo has several attributes which may give the "toy" a useful function as a vibration exciter for structures which have relatively low natural frequencies such as bridges and towers. The first attribute is that when a yo-yo is released from a structure, the structure is exposed to a step input due to the loss of mass of the yo-yo. The second attribute is that of an almost impulsive force input to the structure when the body of the yo-yo reaches the bottom of its stroke due to the change in linear momentum over a short period of time. The third attribute is associated with the return of the yo-yo body to its starting point if certain conditions on initial angular momentum, damping and string length are satisfied.

The dynamics of the solid body motion of a yo-yo as it descends on its string is treated in references [1] and [2], effects of string elasticity are not considered in these works. In order to understand how a structure will respond to a yo-yo excitation when the string is attached to the structure, measurements of yo-yo string forces were made in the dynamics laboratory at Monash University. The largest amplitude force occurs when the yo-yo is at the bottom of its stroke. This investigation was undertaken for three different yo-yo types, the results from measurements differ from solid body dynamics theory presented in the above references. It is believed the string elasticity plays a major role in the development of the peak force and its period of action and is the cause of the discrepancy.

2. THEORY OF BÜRGER

Theory and equations presented in this section are taken from reference [1]. Shown in Figure 1 are three different types of yo-yos which can be considered for analysis, the one treated by Bürger here is the classical yo-yo as it naturally ascends the string without slip when the toy has reached the bottom of its stroke. Velocity of the yo-yo, v, at a distance x from its starting point is given by

$$v = \left(\frac{2gx}{1+J/mr^2}\right)^{1/2} \tag{1}$$

Where g is 9.81 ms^{-2}, J is the rotary inertia, m is the mass and r is the spool radius of a classical yo-yo. Spool radius, r, varies as the string unwinds; r is the axle radius, ro, when completely unwound and is the outer radius of the string, r_m, when fully wound. The relationship between r, string thickness, d, and string length, l, is given by

$$\pi r^2 = \pi r_o^2 + d\,(l-x) \tag{2}$$

The yo-yo translatory velocity is slowed by the factor, β,

$$\beta = \left(1+J/mr^2\right)^{-1/2} \tag{3}$$

as potential energy is fed into rotational energy and the term J/mr^2 is the actual ratio of rotational to translational kinetic energies. The time taken for the toy to descend is given by an elliptic integral of the second kind. At the bottom of the stroke, the yo-yo turns around 180° and the angular speeds, at this position, is given by

$$w = \left(2\,mgl/J\right)^{1/2} \tag{4}$$

and the period of time τ taken by the yo-yo to complete a 180° turn is

$$\tau = \pi\left(J/2mgl\right)^{1/2} \tag{5}$$

For a typical "long spin" yo-yo Bürger measured the following parameters: $r_o = 3mm$, $r_m = 13mm$, $l = 1000mm$ and $d = 0.5mm$. The time taken for the yo-yo to descend was calculated at 1.3s and the turning time to be less that 0.02s. Thus, the translatory linear velocity changes abruptly from descent to ascent.

3. ESTIMATES OF FORCE AND IMPULSE

Estimates of the force and impulse generated in the string due to the sudden velocity change can be made using the previous equations. Change in linear momentum of the yo-yo from decent to ascent is given by

$$m\Delta v = \Delta\ \text{momentum} = 2m\left(\frac{2\,gl}{1 + J / m{r_o}^2}\right)^{1/2} \tag{6}$$

where Δv is change in velocity and m is the mass of a yo-yo. An average force over the turning period is given by

$$\bar{F} = m\Delta v / \tau = 2m\left(2gl / \left(1 + J / mr_o^2\right)\right)^{1/2} \frac{1}{\pi\left(\dfrac{J}{2\,mgl}\right)^{1/2}} \tag{7}$$

which can be simplified to give

$$\bar{F} = \frac{4\,m^2 glr_o}{\pi}\left\{\frac{1}{Jmr_o^2 + J}\right\}^{1/2} \tag{8}$$

Using Bürger's data the average force is calculated as 4.2N for a 54gm yo-yo, see Table 1 for data.

4. MEASURED DATA

Measurements of the force in a yo-yo string were made by attaching the string to the transducer end of a PCB impact hammer, which has a sensitivity of 99.6N per volt. The string force time histories were captured and displayed on an AND 3525 two channel spectrum analyser. Three different yo-yos were tested, however, the Duncan Imperial yo-yo was investigated for two conditions, the first is "modern" where the string is tied around the shaft to allow for slip and the second is "classical" where the string was glued into the shaft to prevent slip between the shaft and string.

The Yomega brands of yo-yos were considered also, the "Brain" with spring return clutch and the "Raider" with ball bearings (allows for long sleeping time).

Typical time histories of force for the four cases are shown in Figures 2 through 5. Peak forces and duration's of these results are given in Table 2.

The largest amplitude force was provided by the Yomega "Raider" and the smallest amplitude force was obtained from the Duncan Imperial (modern). Durations of the positive force period were about the same for all the yo-yos investigated. The Yomega "Raider" force time history, Figure 5, is similar to that of a normal or Gaussian probability distribution.

The force calculated from the theory of Bürger was 4.2N for a 54gm yo-yo and this result differs considerably from the force data in Table 2 by a factor of 10. Two possible phenomena may be the cause of the discrepancy, the first is string elasticity and the second is slip between the string and spindle of the "modern" yo-yo. Both concepts were investigated, the second phenomena was investigated by converting a "modern" yo-yo to a "classical" yo-yo by drilling a hole in the spindle and gluing the string in the hole. This action doubled the force amplitude for the Duncan Imperial yo-yo however the force duration was the same.

As the yo-yo descends and reaches the end of the string the yo-yo obtains its maximum velocity. Although the body turns from one side of the string to the other for its upward journey the yo-yo inertia keeps it moving downward thus stretching the string. The string elasticity then comes into play as there exists a spring mass system with the mass having conceptually an initial velocity. Measurements of string elasticity gave an effective stiffness of approximately 64 Nm^{-1} and calculating a natural frequency using a 54 gm mass and a stiffness of 64 Nm^{-1} gives a natural frequency of 5.47 Hz. A half period for this frequency is 91.4 ms. Note that this is a ball park figure when compared to 65 ms. As the stiffness can be greater than 64 Nm^{-1} due to the stretch created by the motion of the yo-yo as string elasticity is non linear with stretching of the string, thus both slip and elasticity of the string affect the peak force and force duration. It can be assumed that larger peak forces are associated with a classical yo-yo with a very stiff string and with a rotary inertia J as small as possible in comparison to mr^2.

REFERENCES

1. W. Bürger, The Yo-Yo : A Toy Flywheel, American Scientist, V72, March-April 1972, 137-142.

2. W. Bürger, Das Jojo-ein Physikalisches Spielzeug. Physikalische Blätter, 39, December 1983, 401-404.

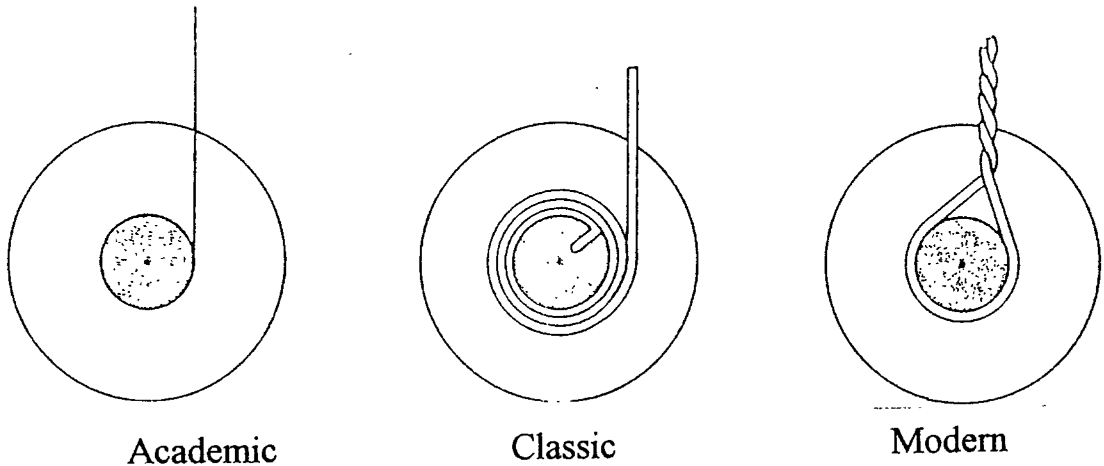
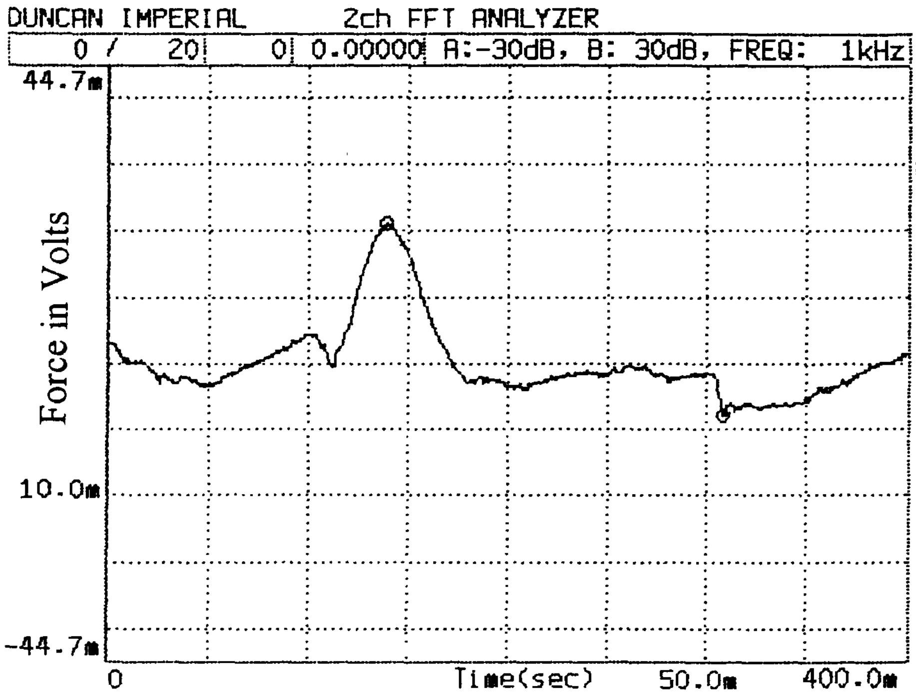


Figure 1 Classification of Yo-Yo types.



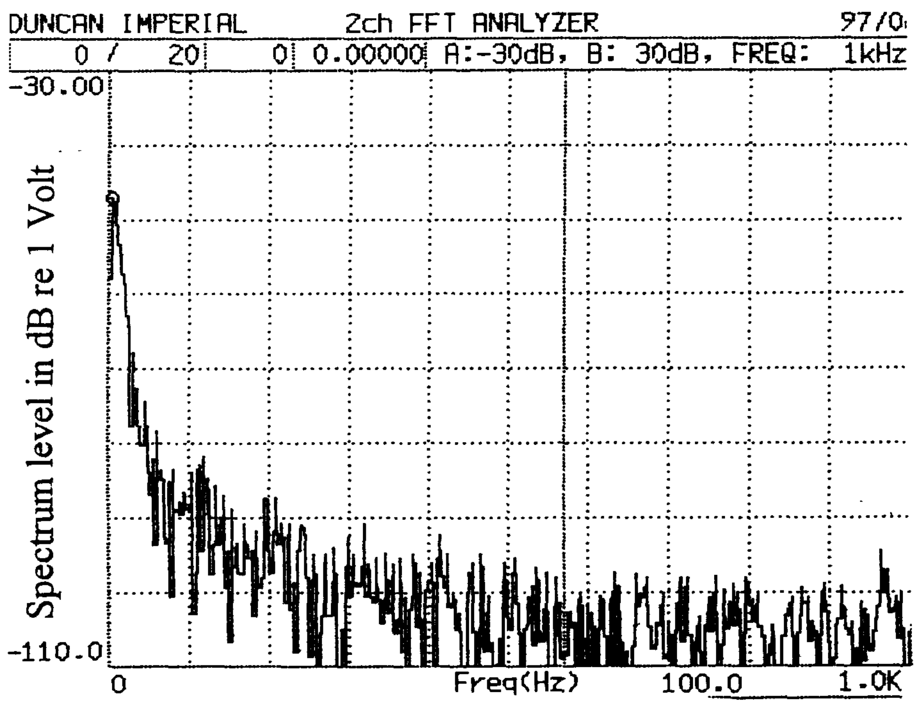
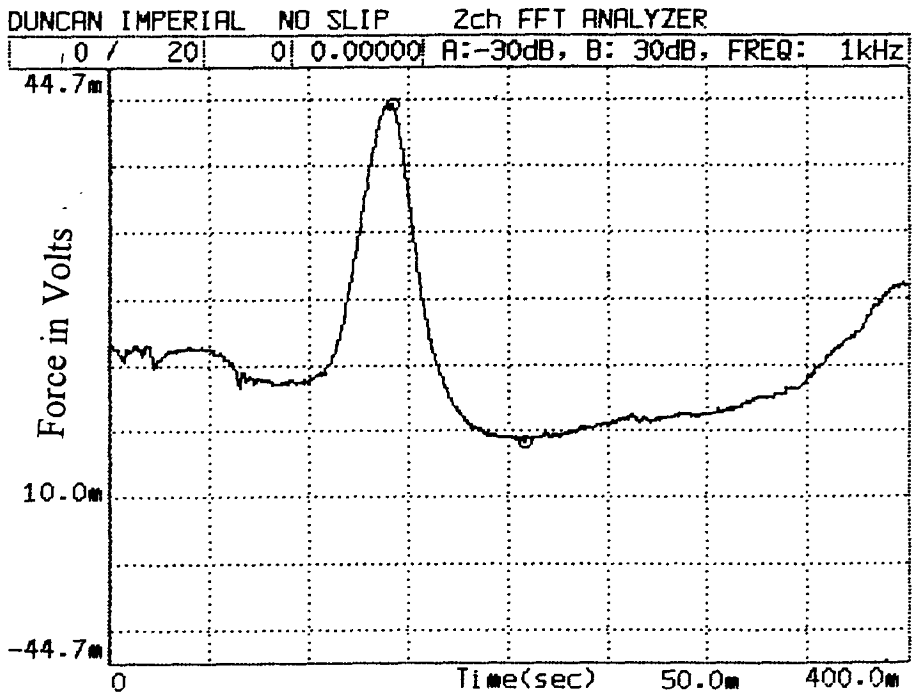


Figure 2 Force time history and spectrum of a Duncan Imperial (modern) Yo-Yo. Peak force is 0.21 N and with a duration of 65 ms.



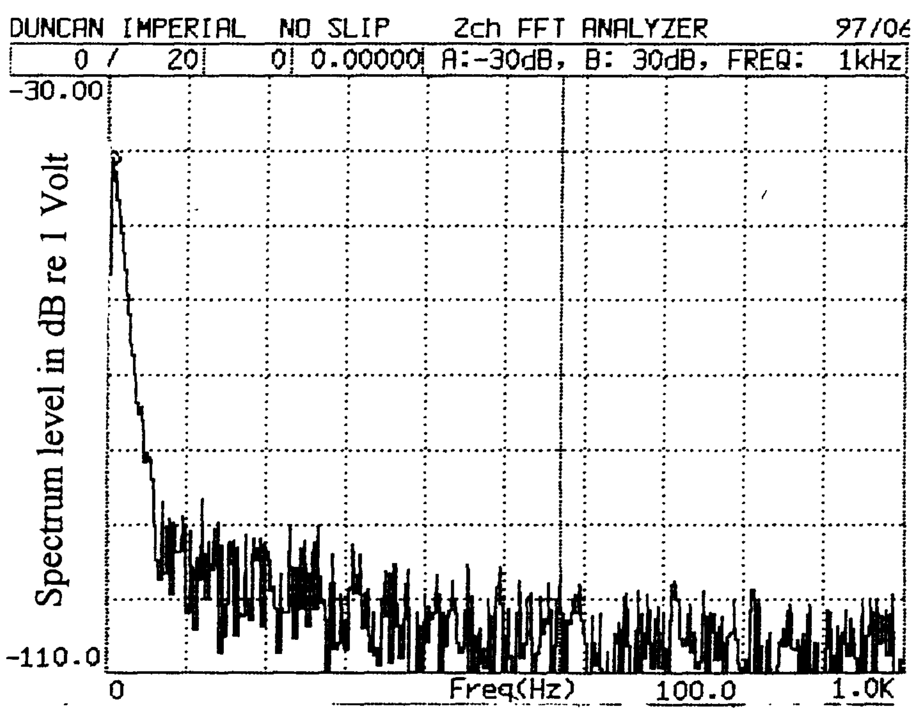
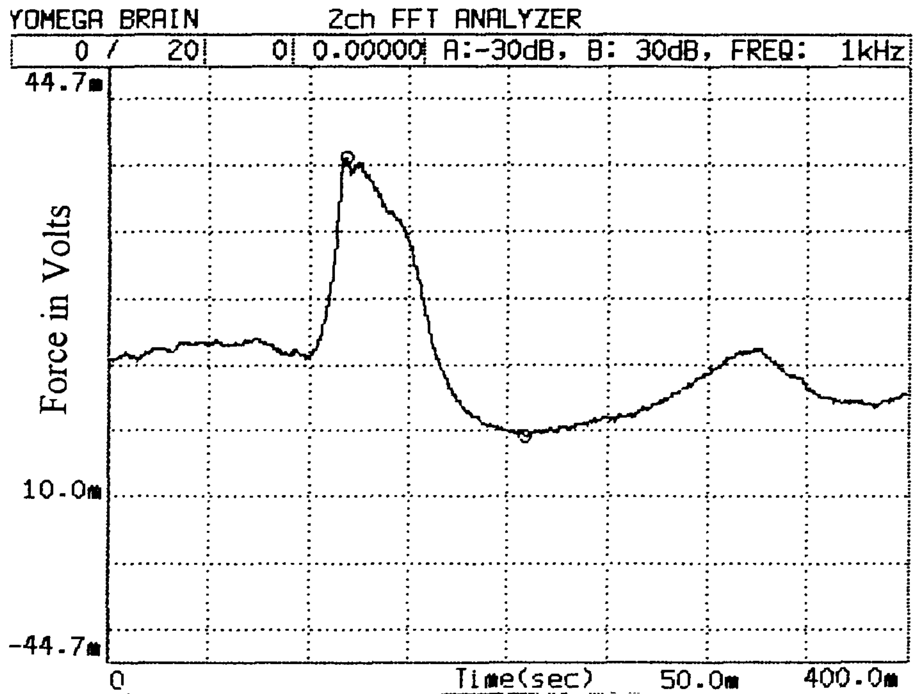


Figure 3 Force time history and spectrum of a Duncan Imperial (classical) Yo-Yo. Peak force is 0.39 N with a duration of 64 ms.



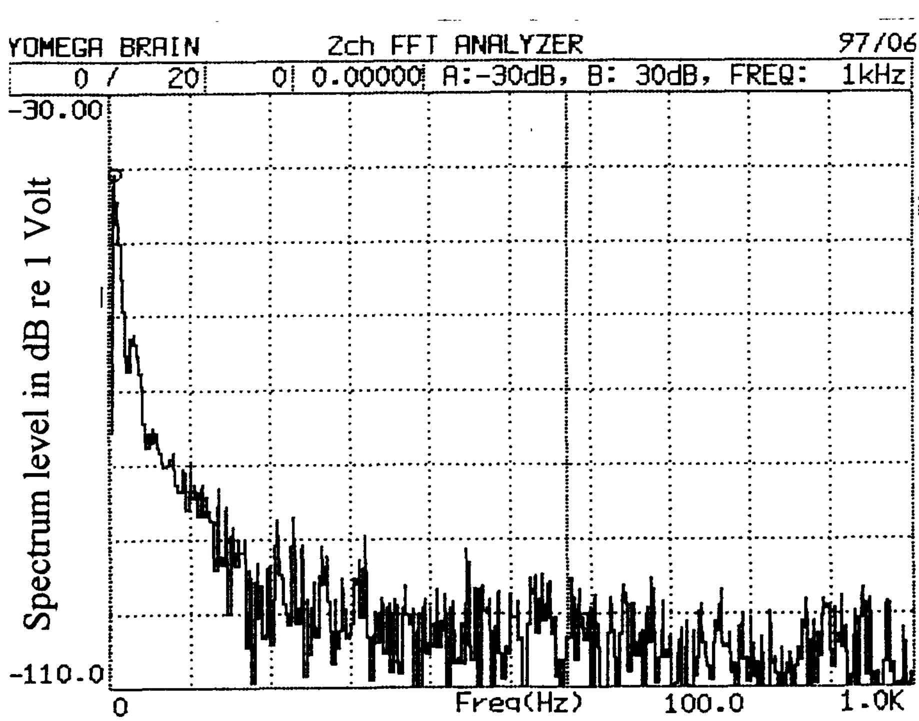
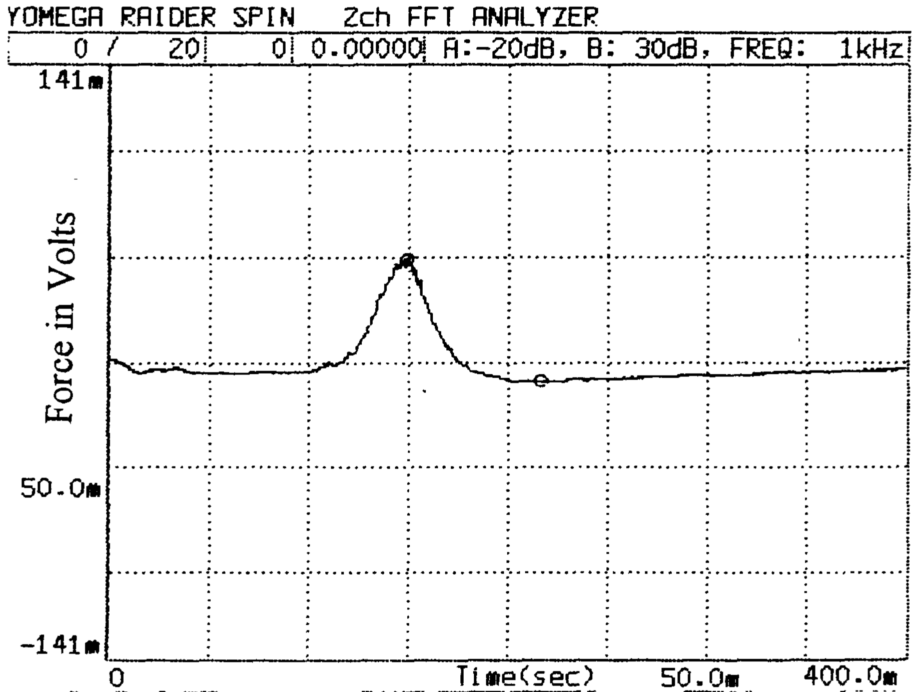


Figure 4 Force time history and spectrum of a Yomega Brain Yo-Yo. Peak force is 0.31 N with a duration of 65 ms.



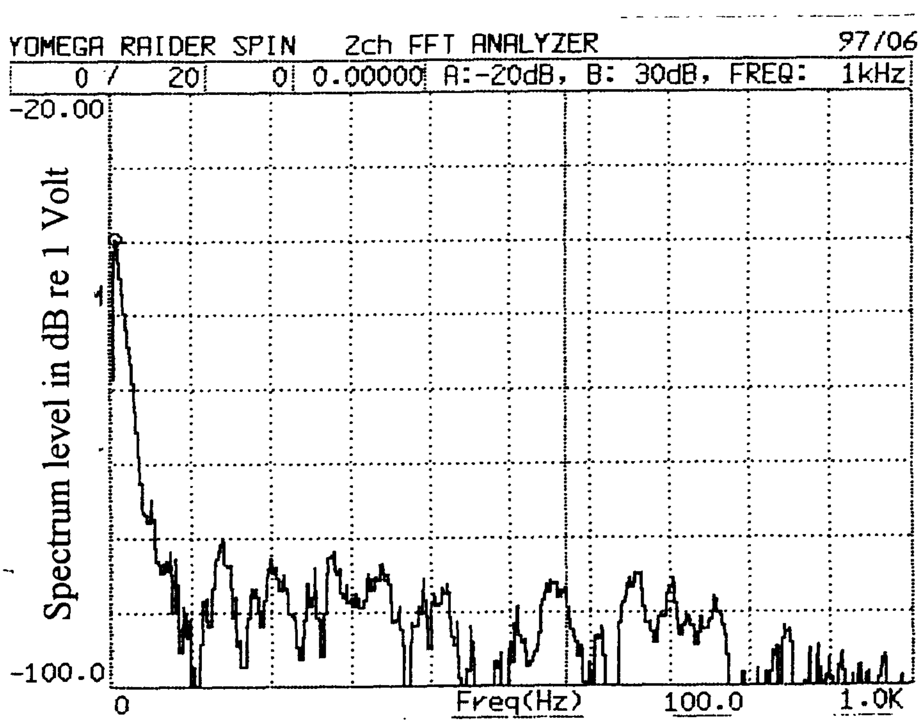


Figure 5 Force time history and spectrum of a Yomega Raider Yo-Yo. Peak force is 0.49 N with a duration of 70 ms.

TABLE 1 : DATA FOR CALCULATION OF AVERAGE FORCE FROM EQUATION (8)

A.

1.	Spool radius	-	3mm
2.	Maximum radius	-	13mm
3.	Length 1	-	1000mm
4.	Mass (m)	-	54 gm
5.	Rotary inertia (J)	-	2.22×10^{-5} kg m^2

B. Calculated items

1.	Final Velocity	-	0.58 ms^{-1}
2.	Time for turn (τ)	-	1.48×10^{-2}s
3.	mΔv	-	6.23×10^{-2} kgs
4.	Force	-	4.22 N.

TABLE 2 : MEASURED DATA FOR FORCE

YO-YO	PEAK FORCE N	FORCE PERIOD ms	MASS - gm
Duncan Imperial Modern	0.21	65	54.3
Duncan Imperial Classical	0.39	64	54.3
Yomega Brain (Clutch)	0.31	65	65.5
Yomega Raider (Ball Bearing)	0.49	70	58.4