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Characteristic boundary conditions and its application to resonance tube

Seung-Chul Kwak and Ho-Seon Rew
Fluid Machinery Team
Living System Research Laboratory , LG Electronics Inc.
327-23, Kasan-dong, Keumchun-gu , Seoul 152-020 , Korea

Duck-Joo Lee
Department of Aerospace Engineering
Korea Advanced Institute of Science and Technology
371-1,Kusong-dong,Yusong-gu , Taejon 305-701 , Korea

Abstract - To investigate the pressure pulsation in a resonance tube, a computer program based on the unsteady compressible Euler equations has been developed. The convective terms are constructed using a Roe's approximate Riemann solver and MUSCL for high accuracy is employed. To limit oscillations near shocks, Koren's differentiable limiter is adopted and an explicit two-step Runge-Kutta method is used for time integration. Several characteristic boundary conditions suggested by Thompson, Watson-Myers and Hwang were tested and discussed to understand their physical meanings. Through the above tests, appropriate boundary conditions are selected and applied to simulate the pressure pulsation in the resonance tube. The calculation results are compared with the experiment by Merkli and Thomann. The computed amplitude and the period of pressure pulsation showed good agreement with experiment except peaks. It is believed that the practical engineering problems concering acoustics can be directly simulated using Euler equations.

1. Introduction

The rapid developments of computational fluid dynamics (CFD) enables us to directly compute the small acoustic waves without any assumptions and have brought out a new research area so called Computational Aero-Acoustics (CAA). Recently CAA has been intensively studied for the practical applications because of limitations of simple linear wave equation. Although there are many difficult problems concerning with numerical accuracy, computation time, and boundary conditions, we believe that CAA can provide us valuable information. This justifies continuous CAA studies in spite of many drawbacks.

As mentioned before, one of important issues in CAA is a boundary condition treatment. The characteristic boundary condition assumes locally 1-Dimensional and accounts for the direction of wave propagation by eigen values and is treated using calculation variables under proper physical conditions, i.e. nonreflecting, sound source and so on. There are several boundary conitions; Thompson[1]'s nonreflecting boundary condition(NRBC), full reflective boundary conditions(FRBC), Hwang[2]'s transparent acoustic source condition(TAS), Watson-Myers[3]'s acoustic source condition excited by pressure(WMP), Watson-Myers's acoustic source condition excited by velocity(WMU). Several boundary conditions based on the theory of characteristics will be investigated and their physical meanings will be discussed in this study. Using the two-dimensional compressible Euler equation, direct numerical simulation for acoustic problems will be carried out. After basic study, numerical simulation of a resonance tube was performed using the characteristic boundary conditions. To verify the present computations, the calculation result was compared with the those of Merkli and Thomann[4].

2. Governing Equations

The 2D axisymmetric compressible Euler equations are written as follows [5];

$$\frac{\partial Q}{\partial t}+\frac{\partial E}{\partial x}+\frac{\partial F}{\partial y}+\alpha H=0 \qquad (1)$$

where $Q=\left[\rho,\rho u,\rho v,e\right]^T$, $E=\left[\rho u,\rho u^2+p,\rho uv,u(e+p)\right]^T$, $F=\left[\rho v,\rho uv,\rho v^2+p,v(e+p)\right]^T$

$H=\frac{1}{y}\left[\rho v,\rho uv,\rho v^2,v(e+p)\right]^T$, $p=(\gamma-1)\left[e-\rho\left(u^2+v^2\right)/2\right]$

if a=0 : 2D planar, a=1 : axisymmetric

The governing equation can be transformed from physical space to computational domain using chain rule and the following relations:

$$\tau=t,\ \xi=\xi(t,x,y),\ \eta=\eta(t,x,y) \qquad (2)$$

Thus the governing equations in the computational space are as following:

$$\frac{\partial \hat{Q}}{\partial \tau}+\frac{\partial \hat{E}}{\partial \xi}+\frac{\partial \hat{F}}{\partial \eta}+\alpha\hat{H}=0 \qquad (3)$$

where $\hat{Q}=Q/J$, $\hat{E}=\left[\xi_t Q+\xi_x E+\xi_y F\right]/J$

$\hat{F}=\left[\eta_t Q+\eta_x E+\eta_y F\right]/J$, $\hat{H}=H/J$

3. Numerical Methods

Time integration of Eq.(3) is performed by an explicit manner using 2 step Runge-Kutta method.

$$Q^{n+1/2}=Q^n-\frac{\Delta t}{2}\cdot RHS^n \qquad (4)$$

$$Q^{n+1}=Q^n-\Delta t\cdot RHS^{n+1/2}$$

The above time integration method is 2nd order accurate. Because the convective terms in Eq.(3) significantly affect the accuracy and the stability of numerical calculation, special attention must be paid. In this study, the numerical flux are constructed using the method known as Roe's approximate Riemann solver[6].

$$\hat{E}_{i+1/2}=\frac{1}{2}\left[\hat{E}(Q_L)+\hat{E}(Q_R)+\hat{R}_{i+1/2}\hat{\Phi}_{i+1/2}/J_{i+1/2}\right] \qquad (5)$$

It should be noted that since Eq. (5) is just first order accurate, excessive dissipation errors may smear all the small acoustic waves. Thus some kind of accuracy enhancing methods must be introduced. In this study, MUSCL(Monotone Upwind Schemes for Conservation Laws) introduced by Vanleer[7] is employed and to prevent oscillations near discontinuities

such as a shock, Koren's differentiable limiter[8] is introduced to insure TVD property. The formulas used in this study are following:
MUSCL :

$$Q^L_{i+1/2} = \left\{1 + \frac{\psi_i}{4}\left[(1-\kappa)\nabla + (1+\kappa)\Delta\right]\right\}Q_i \ , \ Q^R_{i+1/2} = \left\{1 - \frac{\psi_{i+1}}{4}\left[(1+\kappa)\nabla + (1-\kappa)\Delta\right]\right\}Q_{i+1} \quad (6)$$

Limiter :

$$\psi_i = \frac{3\nabla Q_i \Delta Q_i + \varepsilon}{2(\Delta Q_i - \nabla Q_i)^2 + 3\nabla Q_i \Delta Q_i + \varepsilon} \quad (7)$$

The numerical flux in Eq. (5) is 2nd order accurate if k=-1 and 3rd order accurate for k=1/3 in Eq. (6). In this study, k was 1/3. The constant ε in Eq. (7) is a very small number to keep from dividing by zero and 10^{-6} used.

4. Boundary conditions

Before the presentation of the calculation results, it is important to understanding the physical meaning of various characteristic boundary conditions; nonreflecting boundary condition, full reflective boundary conditions, transparent acoustic source condition, acoustic source condition excited by pressure, acoustic source condition excited by velocity. In this section, formula of these five boundary conditions will be described and discussed.
Assume that flow at the boundary is one-dimensional then 1D characteristic equation is derived from Eq.(3) as following:

$$\frac{\partial \hat{W}}{\partial \tau} + \hat{\Lambda}\frac{\partial \hat{W}}{\partial \xi} = 0 \quad (8)$$

where $\delta Q = P\delta W$, Q : conservative variable, Q : Characteristic variable. Variations of characteristic variables are defined as following:

$$\delta\hat{W} = \begin{bmatrix} \delta\rho - \delta p/c^2 \\ \delta q_t \\ \delta p + \rho c \delta q_n \\ \delta p - \rho c \delta q_n \end{bmatrix} \quad (9)$$

The eigenvalues of Eq. (8) are $\hat{\Lambda} = Diag(U, U, U + c\nabla\xi, U - c\nabla\xi)$ (10)

where U is contravariant velocity.

4-1. Nonreflecting boundary condition

This boundary condition condition was suggested to realize that the outgoing waves are reflected back into domain without phase and amplitude changes. The nonreflective condition by Thompson[1]can be expressed by dividing the characteristic equation as in Eq. (10) then setting the eigenvalues corresponding to incoming waves to be zero (Sommerfeld radiation condition). The mathematical form is

$$\begin{aligned} &\frac{\partial \hat{W}}{\partial \tau} + \hat{\Lambda}_{in}\frac{\partial \hat{W}}{\partial \xi} = 0 \\ &\frac{\partial \hat{W}}{\partial \tau} + \hat{\Lambda}_{out}\frac{\partial \hat{W}}{\partial \xi} = 0 \\ &\hat{\Lambda}_{in} = 0 \end{aligned} \quad (11)$$

4-2. Full reflective boundary conditions

In contrary to NRBC, this condition was suggested to reflect all the incoming waves at an infinite impedance solid wall. Therefore, the outgoing waves are reflected back into domain without phase and amplitude changes. With no normal velocity, isentropic and zero vorticity conditions, FRBC is derived as following:

$$\frac{\partial \rho}{\partial \tau} = \frac{1}{c^2}\frac{\partial p}{\partial \tau},\ \frac{\partial q_t}{\partial \tau} = 0,\ \frac{\partial p}{\partial \tau} = -\lambda_4\left(\frac{\partial p}{\partial \xi} - \rho c\frac{\partial q_n}{\partial \xi}\right),\ \frac{\partial q_n}{\partial \tau} = 0 \tag{12}$$

4-3. Transparent acoustic source condition

This is an sound source condition regarding the nonreflective characteristic of the outgoing wave. Hwang[2] derived this condition and effectively applied to a single expansion muffler. To estimate the transmission loss of the muffler in his study, TAS was applied at exciting end and the other end was modelled by NRBC. Using these two conditions, anechoic terminating condition was satisfied. TAS condition is also derived from the characteristic equation using isentropic and plane wave relations as following:.

$$\begin{gathered}\frac{\partial p}{\partial \tau} = \varepsilon\cos(\omega\tau),\ \frac{\partial \rho}{\partial \tau} = \frac{1}{c^2}\frac{\partial p}{\partial \tau} \\ \frac{\partial q_n}{\partial \tau} = \pm\frac{1}{\rho c}\frac{\partial_\tau p}{\partial \tau}(+:wrigt,\ -:left),\ \frac{\partial q_t}{\partial \tau} = 0\end{gathered} \tag{13}$$

4-4. Acoustic source condition excited by pressure

This boundary condition is for an acoustic source condition excited by pressure and was suggested by Watson and Meyer[3] using the characteristic relation. This boundary condition can describe the strength of acoustic wave by pressure. With isentropic and zero vorticity assumptions for inflow boundary, WMP having sinusoidal pressure variation at the boundary can be derived as following:

$$\frac{\partial p}{\partial \tau} = \varepsilon\omega\cos(\omega\tau),\ \frac{\partial \rho}{\partial \tau} = \frac{1}{c^2}\frac{\partial p}{\partial \tau},\ \frac{\partial q_n}{\partial \tau} = \left[\frac{\partial_\tau p}{\partial \tau} + \lambda_4\left(\frac{\partial p}{\partial \xi} - \rho c\frac{\partial q_n}{\partial \xi}\right)\right]\Big/\rho c,\ \frac{\partial q_t}{\partial \tau} = 0 \tag{14}$$

4-5. Acoustic source condition excited by velocity

In the case of the resonance tube having piston movement, the excitation force described by velocity may be helpful. Therefore a normal velocity component is chosen as an excitation source in this boundary condition. This condition is very similar to WMP but major difference between WMP and WMU is an exciting source pattern. With isentropic and zero vorticity assumptions for inflow boundary, WMP having sinusoidal velocity variation at the boundary can be derived as following:

$$\begin{gathered}u' = \varepsilon\sin(\omega\tau),\ \frac{\partial p}{\partial \tau} = \rho c u' - \lambda_4\left(\frac{\partial p}{\partial \xi} - \rho c\frac{\partial q_n}{\partial \xi}\right) \\ \frac{\partial \rho}{\partial \tau} = \frac{1}{c^2}\frac{\partial p}{\partial \tau}\frac{\partial q_n}{\partial \tau} = \left[\frac{\partial_\tau p}{\partial \tau} + \lambda_4\left(\frac{\partial p}{\partial \xi} - \rho c\frac{\partial q_n}{\partial \xi}\right)\right]\Big/\rho c = u',\ \frac{\partial q_t}{\partial \tau} = 0\end{gathered} \tag{15}$$

5. Results and discussion

Before the discussion on a practical application, wave propagation problems with/without acoustic sound source were intensively studied to understand the physical meaning of various boundary conditions. To investigate the characteristics of TAS-NRBC and TAS-FRBC, TAS was given at inlet while NRBC or FRBC was alternatively adopted at outlet. Figure 1 shows the schematic of tested problems and boundary conditions. The pressure variation at inlet can be expressed as following;

$$\partial_\tau p' = \begin{cases} \varepsilon\omega\cos(\omega\tau) & \text{if } 0 \le \omega\tau \le \pi \\ 0 & \text{if } \omega\tau \succ \pi \end{cases} \tag{16}$$

where nondimensionalized angular velocity(ω) and amplitude(ε) in Eq. (16) are 0.5 and 0.001, respectively. The converge histories of TAS-NRBC and TAS-FRBC without acoustic sound source were displayed in Fig. The residuals of TAS-NRBC and TAS-FRBC in Fig. 2 are remarkably decreased over 1200 and 2000 iteration, respectively. The remarkable decrease of residual in Fig. 2 is closely associated with the wave motion(see Figs. 3 and 4). The boundary condition which does not generate any reflective wave is of great importance. In the case of TAS-FRBC, the reflective characteristic of FRBC is distinctly observed as shown in Fig.4. Note that the double amplitude in Fig. 4 results from the duplication of the outgoing and reflective wave at the outlet boundary.

To study the effects of sound sources, a problem similar to Fig.1 was chosen. But the boundary conditions were changed. TAS was used at inlet while TAS, WMP and WMU conditions at outlet were alternatively selected.

The pressure variations at inlet and outlet can be described following:

At inlet : same with Eq.(16)

At outlet : $\partial_\tau p' = \varepsilon\omega\cos(\omega\tau)$ (17)

where non-dimensionalized angular velocity and amplitude in Eq. (17) are 0.5 and 0.001, respectively. Figures 5~7 display time histories of acoustic pressure at various locations. The calculation results at three different locations in Figs. 5~7 show nearly same behavior of the acoustic wave. However the remarkable differences can be found when the wave generated at inlet passes the observation point. It should be noticed that the locations of Figs. 5~7 corresponds to inlet, center and outlet, respectively. This means that TAS, WMU and WMP have different reflective properties. TAS has nonreflective characteristic while WMP and WMU do generate reflective wave. TAS can be thought as a nonreflective acoustic source condition[2] while WMP and WMU as a reflective one. The fundamental difference between WMP and WMU is the excitation method and they satisfy characteristic equation. Therefore it is desirable to apply an exact boundary condition to simulate acoustic problem correctly. For example, in the case of a speaker located far away from observation point, TAS leads to right calculation since the reflections are negligible. In the case of numerical simulation of a resonance tube having piston moving with small amplitude, WMU is proper boundary condition because there will be reflective waves and the known excitation source is not pressure but velocity.

5-4. Resonance tube

One of important practical applications of acoustics is the resonance tube. Figure 8 shows the schematic of Merkli and Thomann's experiment[4]. Because the length of the resonance tube is much larger than the amplitude of the piston, it can be assumed that the motion of piston is negligible and the effect of the piston is modeled by WMU condition as mentioned before. Therefore the deforming mesh was not used. Since the opposite tube end is solid wall, FRBC is used. Furthermore the acoustic wave motion is considered as axisymmetric. The grid number in this calculation is 51 x 9. Since the piston is oscillated at the resonance frequency of the tube, the 180 degree phase difference of two pressure signals at the piston head and the tube end is observed in Fig. 9. Also, we can find that gradual acoustic wave superposition with time(see 10) increases the pressure gradient thus leads to shock wave. We could find that the average pressure of the tube increases with time due to the wave reflection. Ridder and Beddini[9] shifted their results by the amount of mean pressure to compare their calculation result with the experimental result[4]. To verify the present calculation, we directly compared with the experiment without shift. The present calculation result shows good agreement except peaks as shown in Fig. 11. Especially, the phase and amplitude show remarkable good agreement with experiment. It should be noted that the approach method based on the classical linear theory cannot predict the abrupt pressure increase as shown in Fig. 11. Therefore we believe that fundamental and realistic approach using nonlinear governing equations such as Euler or Navier-Stokes equations is possible and helpful.

6. Conclusions

Using the two-dimensional compressible Euler equations, direct numerical simulation on acoustic problems was carried out. Several boundary conditions based on the theory of characteristics were investigated to fully understand their physical meanings by successful calculations and discussed. We found that TAS has nonreflective characteristic while WMP and WMU do generate reflective wave. The fundamental difference between WMP and WMU is the excitation method and they satisfy characteristic equation. The wave motion in a resonance tube is modeled and calculated using the characteristic boundary conditions. Also the results are compared with those of Merkli and Thomann's experimental result. The present calculation showed very good agreement with experiment except peaks thus we believe that this approach is valid.

7. References

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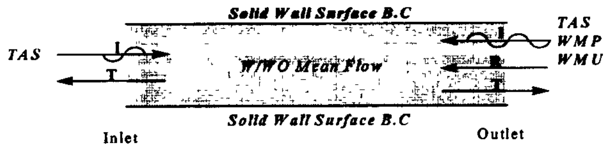

Fig.1 Schematic of test of acoustic sources

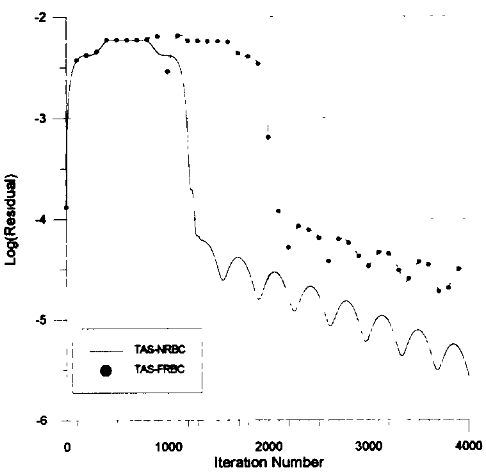

Fig. 2. Convergence histories for TAS-NRBC and TAS-FRBC

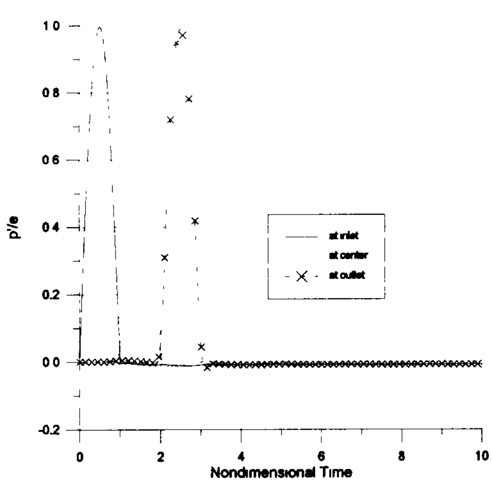

Fig. 3 Time histories of pressure for TAS-NRBC

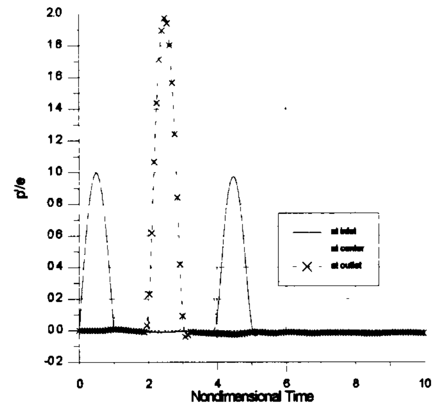

Fig. 4 Time History of Pressure for TAS-FRBC

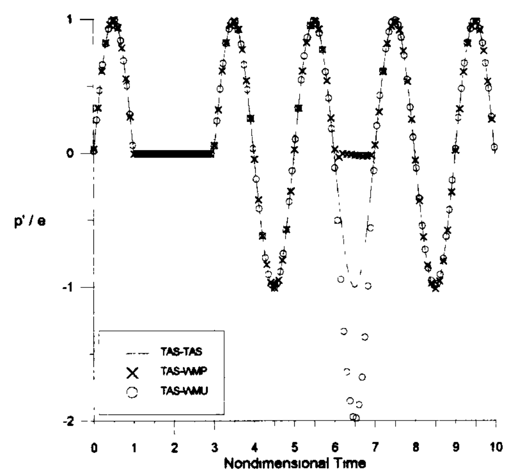

Fig.5 Time histories of acoustic pressure at inlet

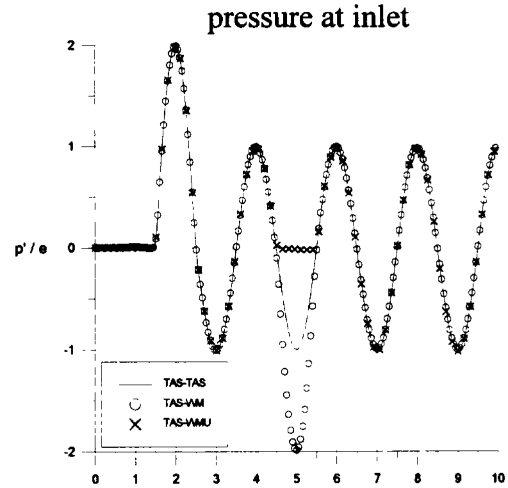

Fig.6 Time histories of acoustic pressure at center

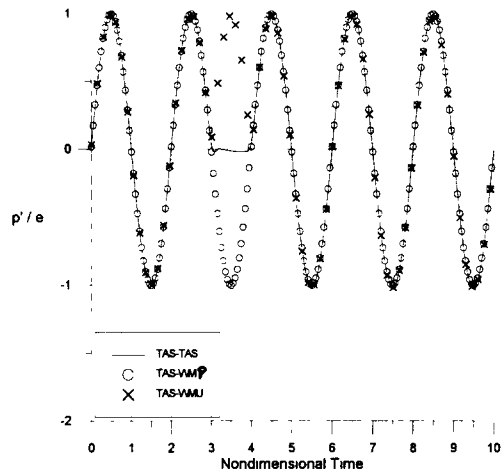

Fig.7 Time histories of acoustic pressure at outlet

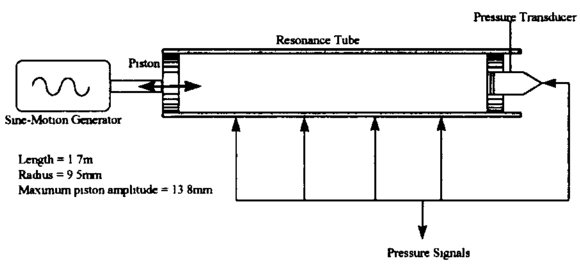

Fig.8 Schematic of Merkli andThomann's experiment

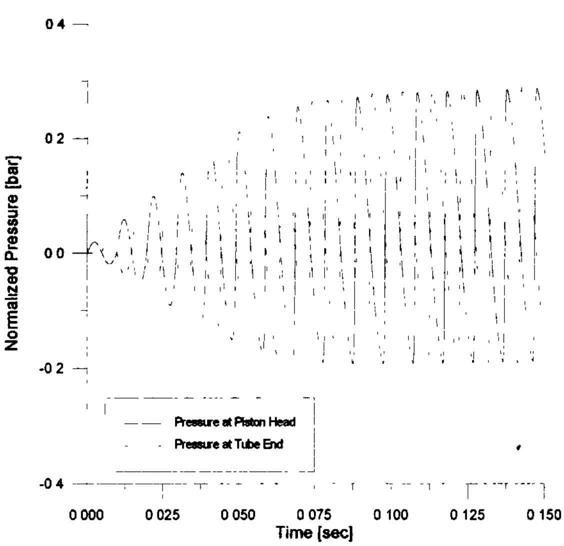

Fig.9 Pressure response at resonant frequency (102Hz); piston amplitude=13.8mm

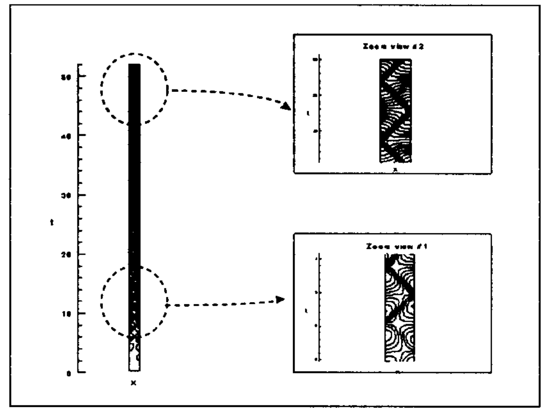

Fig.10 x-t Diagram of Resonance Tube at resonance frequency.

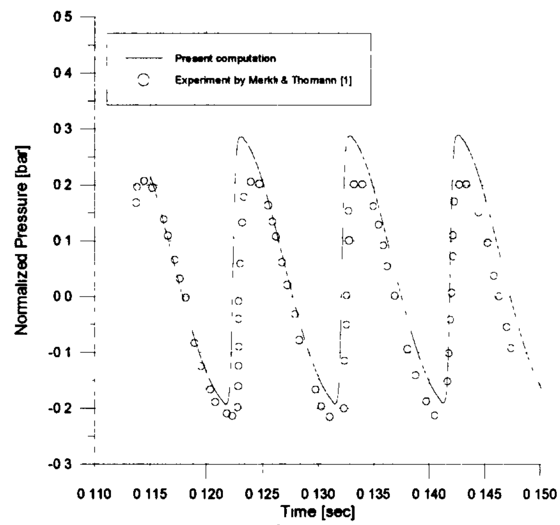

Fig.11 Comparison of computational and experimental results; f=102Hz; piston amplitude=13.8mm