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TUNING MARIMBAS USING NUMERICAL OPTIMISATION

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The transverse modes generated when a uniform beam is struck between its two supports are not harmonic. In order to tune the beam so that at least the lowest of these modes are harmonically related, parabolic arches are cut on the underside of the beams. The actual dimensions of the undercut are an empirical design determined by tradition, and relate the subtleties of the sound produced with those favoured by the human ear. The current paper takes a very general approach to the problem. Finite element analysis is used to determine the optimal undercut required to tune the beam. However, no assumption is made regarding the shape of the cut beyond ensuring that material is not added and that the structure remains a single beam. An optimisation approach is then used to calculate the profile of the beam to satisfy the prescribed frequencies. The calculated profile depends on the optimisation criterion specified. The paper presents a variety of profiles for which the optimization criteria vary from ease of modelling to ease of construction. The results often do not resemble the traditional cut.

INTRODUCTION

The optimisation of a musical instrument is generally the task of the maker. Historically, the task has been undertaken using not much more than a sense of touch and sound. Structures are carved and crafted so that they are playable and respond to excitation with sounds that please. In many cases this is synonymous with producing overtones of the fundamental that are harmonically related, although in the case of some instruments, there are intermediate structures, such as the back plate of a violin, that may require quite different tunings. Whatever the system, the craft is expressed in much the same way; the structure is carved roughly at first according to a set pattern and then more finely until the optimal tuning is obtained. The art of removing material, in the correct amounts and physical positions, to tune an instrument is often made easier by the existence of tuning curves that map the response of each of the modes to particular adjustments.

The results of this empirical art have been the subject of numerous studies in musical acoustics. However such investigations have generally been restricted to confirmation of existing designs [1]. The aim of our work is to design new profiles, that conform to the musical expectations using numerical techniques. Whilst the idea is general, in this paper we have restricted its application to the tuning of idiophones such as marimbas.

For a musically pleasing sound the lowest three transverse modes, generated by striking the marimba bars, are required to be close to harmonically related [2]. However, a rectangular wooden beam responds with nonharmonic overtones when struck. Accordingly, the marimba maker typically carves a parabolic arch on the underside of the beam. Application of one-dimensional vibrational analysis techniques is sufficient to show that this results in a significant reduction to the frequency of the fundamental mode with a lesser and variable effect on the higher modes [3]. However in this paper we don't wish to analyse what has already been produced, but rather to design suitable undercuts with no a-priori assumptions as to the shape of the undercut. This approach allows a wide range of geometries of the undercut to be explored. Hence, we shall examine a range of geometries for their suitability and ease of construction.

GENERAL APPROACH

We begin by adopting a suitable theory that can describe the motion with sufficient accuracy, and by describing the physical characteristics of the structure and the boundary conditions.

In the case of the marimba we have used the one-dimensional Timoshenko beam theory, which is the simplest extension of the classical theory that can account for shear deformation. This results in a fourth-order system of coupled differential equations, namely

$$kGA(w^{\prime\prime}-\theta^{\prime})=-\rho A\Omega^2 w$$

$$EI\theta^{\prime\prime}+kGA(w^{\prime}-\theta)=-\rho I\Omega^2\theta$$

where k is the shear correction factor, G is the shear modulus, A is the cross-sectional area, ρ is the density, Ω is the natural frequency, and w and θ are the displacement and rotation amplitudes of the beam. A prime denotes a differentiation with respect to x.

The beam is assumed to have a rectangular cross-section with width b and varying height $h(x)$, so that the sectional properties A and I are also functions of x, that is

$$A(x)=bh(x)$$

$$I(x)=\frac{bh(x)^3}{12}$$

The beam is supported by two spring supports positioned at L_1 and $L_1 + L_2$ as shown in Figure 1. Hence the length of the overhang is L_1 and the length of beam between the two supports is L_2.

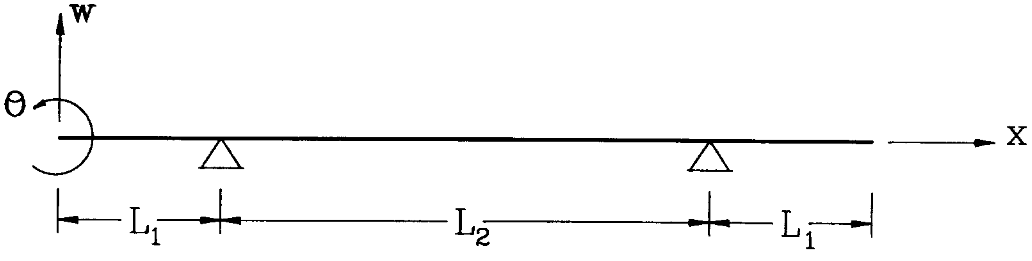

Figure 1. Geometry and sign convention

PROBLEM FORMULATION

The goal is to design a beam that responds with specified frequencies. We do this by defining the function *h(x)*, without reference to the traditional parabolic undercut. Hence, the geometry of the beam is described by a number of parameters that are the primary unknowns for the problem. An optimisation procedure is then used to determine the values of the parameters such that the beam has the desired frequency characteristics and other specified criteria are satisfied. Therefore, we introduce an optimisation function *f(h)*, which is a function of the primary unknowns, and optimise it subject to constraints *g(h)*, which in this case are the desired natural frequencies together with some manufacturing constraints. It is worth noting that the choice of optimising criteria can be fairly arbitrary and in the following examples we attempt only a local minimisation. Hence, any one solution may not represent a true global optimal solution. For simplicity, the beam is assumed to be symmetrical about its midlength point and its width is kept constant. Clearly a wide range of geometries could be adopted within this general framework.

We shall present four distinct geometries below and discuss their relative merits. In each case we have restricted the cut section to be the section of the beam between the two spring supports. As an example, for each shape we shall assume that we are tuning a marimba bar made from Australian hardwood such that the first three frequencies are 128 Hz, 512 Hz and 1280 Hz. This is a traditional tuning regime for a marimba. The properties of the timber were assumed to be $E = 21$ GPa, $\nu = 0.3$ and $\rho = 648$ kg.m^{-3}. The width of the bar was 64 mm and its uncut height was 18.9 mm. The lengths of the bar were taken as $L_1 =$ 120 mm and $L_2 = 300$ mm. The beam is supported on material with a spring stiffness of 2.88 x 10^4 N.m^{-1}.

PIECEWISE-CONSTANT HEIGHTS

The beam is discretised into N sections each of constant length, with height h_i, $i = 1....N$. The response of the beam is thus a function of h_i. The constraints are the first three natural frequencies together with restrictions on the value of the heights. For ease of construction the height variations are restricted so that no material is added and hence a maximum height h_{max} is stipulated. Furthermore, to ensure that the beam remains structurally sound, a minimum height h_{min} is also stipulated. Hence, the variables h_i are bound within the range

$$h_{min} \le h_i \le h_{max}, \quad i=1.....N$$

A number of optimisation criteria have been explored. As a first choice we required that the volume of wood removed from the bar be minimised, so that

$$f(h_i) = \sum_i^N (h_{max} - h_i)$$

since the width of the beam and the length of the sections remain constant.

Alternatively, the optimisation criterion can be adapted to generate smooth profiles. This was achieved by stipulating that the differences between two adjacent heights be minimised. Hence,

$$f(h_i) = \sum_{i=1}^{N-1} (h_{i+1} - h_i)^2 + \alpha \sum_{i=1}^{N-2} (h_{i+2} - h_i)^2$$

where α is a selectable weighting factor. For either optimising function, the possible solutions are numerous. Figures 2(a) and 2(b) show an example of each, where $N = 6$ and $\alpha = 0.5$.

Experimental investigations [4] indicated that smoother profiles showed closer agreement between theory and results. This was attributed to two-dimensional effects that cannot be accounted for by a one-dimensional beam theory. However, we can attempt to minimise these effects by choosing smoother variations for the beam profile. A number of such options are discussed below.

PIECEWISE-LINEAR HEIGHTS

The simplest extension of the previous geometry is to require that there are no jumps in the profile of the beam. Hence, the beam is again discretised into six sections each of constant length, however this time the disretisations have linearly varying heights. The optimisation criteria used was such that there was minimum difference in slope between adjacent sections. Figure 2(c) shows an example of this discretisation.

SINUSOIDAL FUNCTIONS

Further smoothness of the profile can be generated by introducing more complex geometries such as piecewise quadratic or cubic functions. However, we have chosen to use sinusoidal profiles to achieve the same aim. The beam is hence no longer discretised in appearance, although finite elements are still used to evaluate the response. To provide a general approximation to the height, we have assumed that the height is a combination of a linear function plus a truncated sine series. Hence, the height of the beam between the two supports is written as

$$h(x)=\frac{2a_0x}{L_2}+\sum_{n=1}^{N} a_n\sin\frac{n\pi x}{L_2}$$

where a_n are the amplitudes of the sinusoids.

The optimisation criterion used ensured that the curvature of the cut was minimised. Hence, f was taken as

$$f(a_n)=4\sum_{n=1}^{N} n^4a_n^2$$

where f represents the sum of the squared second derivatives. Figure 2(d) depicts an example of this approach.

EASE OF CONSTRUCTION

Clearly, there is a wide variety of geometries that can be used to describe the shape of the cut. The process is general and any of the shapes can be optimised to produce solutions. However, the original problem formulation made the assumption that we are solving a one-dimensional problem. Sudden jumps in the height of the cut introduce regions of stress concentration that cannot be fully accounted for by the one-dimensional theory. We have therefore two ways in which we can minimise these effects. The first is to optimise the cut so that there is minimal discontinuities and the second is to use a higher order theory. The latter option is currently under investigation while in this paper we have chosen to avoid increasing the order of the theory. Hence we have looked to smoother cuts, and so progress from the relatively easy to construct shape in Figure 2(a) to the smoother, but more difficult to construct shapes in Figures 2(c) and (d). Eventually we get to the point of asking "What should the shape be so that the cut is smooth and easy to construct?" Hence the final geometry.

In this case we construct the beam by drilling holes through the centre of the beam. The number of holes is specified and the size and distance between the holes are optimised to give the required tuning regime, with restrictions to ensure ease of construction (no overlapping holes, minimum and maximum size of hole). A solution is shown in Figure 2(e).

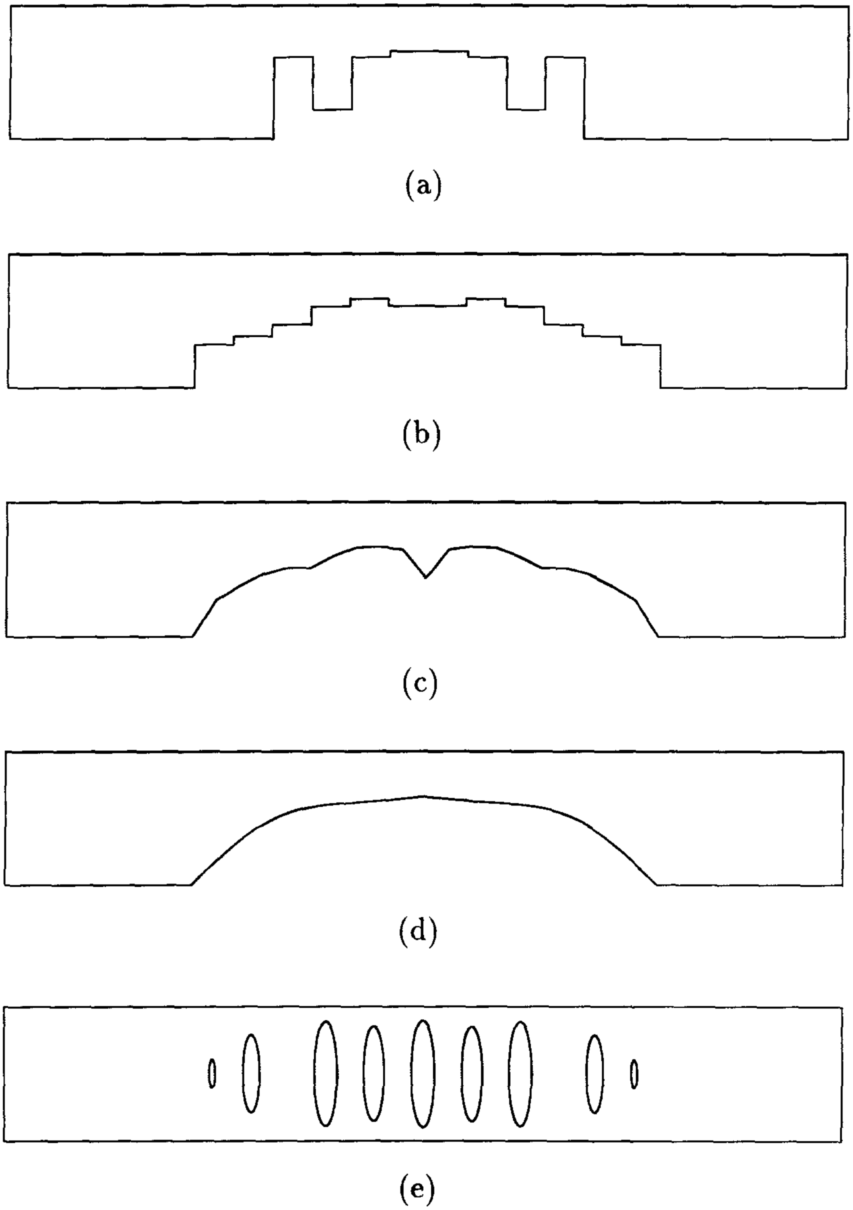

Figure 2. Predicted profiles for a marimba bar made from Australian hardwood. Note that in each case the scale for length and height differ by a factor of six. (a) Beam was discretised into six piecewise-constant heights, volume of wood removed was minimised (b) Beam was discretised into six piecewise-constant heights, height difference between two adjacent discretisations was minimised (c) Beam was discretised into six piecewise-linear heights, slope difference between adjacent heights was minimised (d) Sinusoidal cut with minimum curvature (e) Beam with circular holes.

CONCLUSION

This paper has discussed the general problem of designing marimba bars for a specified frequency response using numerical optimisation techniques. The results show that there are many possible profiles that the bar can be cut to, all of which satisfy the required tuning regime. Hence, it is not necessary to restrict the profiles to the traditional parabolic cuts. However, previous experimental work has shown that the one-dimensional theory is less accurate when the profile has sudden jumps. Hence, it is preferable to use smoother profiles. The general procedure is nevertheless applicable to all cuts and enables the instrument maker to choose a shape that is reasonably easy to construct.

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