FIFTH INTERNATIONAL CONGRESS ON SOUND AND VIBRATION
DECEMBER 15-18, 1997
ADELAIDE, SOUTH AUSTRALIA

ESTIMATION OF SURFACE MOBILITY OF AN INFINITE PLATE FOR A SQUARE CONTACT AREA WITH UNIFORM FORCE EXCITATION BY THE FINITE ELEMENT METHOD

Y.J. Li*, H.M. Williamson*, J. Dai* and C.J. Norwood**

* Acoustics and Vibration Unit, University College, The University of New South Wales, Australian Defence Force Academy, Canberra, ACT 2600, Australia.
** Ship Structures and Materials Division, AMRL, Defence Science and Technology Organisation, PO Box 4331, Melbourne, Victoria 3001, Australia

ABSTRACT

Finite element techniques were investigated for determination of surface mobility of an infinite plate excited over a square contact area by a uniform force distribution using the effective point mobility concept. Ordinary point and transfer mobilities were obtained using harmonic response analysis in FEM. Then surface mobilities were calculated for different sizes of square contact area, based upon these ordinary mobilities. The comparison between the numerical FEM results and theoretical prediction shows that the surface mobility obtained by both methods generally decreases with increasing area of the square.

1. INTRODUCTION

Classical vibration isolation models are based upon the point-like connection assumption and corresponding point mobility concept [1]. In practice, the contact area dimension can become comparable to the governing wavelength and accordingly excitations may have complicated spatial distribution at higher frequency. Thus, a multi-point connection model may be employed to determine effective point mobility and corresponding surface mobility [2, 3]. A theoretical model for the surface mobility of an infinite plate over a circular contact area was developed by Zhao et al [3, 4]. Similar models were extended for the surface mobility of an infinite plate excited over a rectangular contact area by Dai and Williamson [5]. This paper presents an investigation of the prediction of surface mobility of an infinite plate excited over a square contact area using the finite element method (FEM). This is quite an important step in this research because the theoretical models can only be used for very simple structures. Once confirmation of the method is achieved by comparison with analytical models, a FEM can be applied to more complicated practical structures in order to obtain better understanding of actual interface conditions between isolators and support structures.

2. SURFACE MOBILITY UNDER UNIFORM FORCE EXCITATION

For a multi-point coupled vibration isolation system the effective point mobility concept may be used to determine its surface mobility[2, 3]. The effective mobility at one point includes the contributions of all the other contact points. The point contact assumption requires that the connection point has dimensions which are only a fraction of the governing wavelength. Therefore, the effective point mobility M_i^e at point i can be defined as [2]

$$M_i^e = \frac{V_i^e}{F_i} = \frac{\sum_{j=1}^{N} M_{ij} \cdot F_j}{F_i} \tag{1}$$

where V_i^e is the velocity at connection point i, taking into account the effects of forces at all the other contact points. M_{ij} is ordinary point or transfer mobility between points i and j. F_i and F_j (i = j =1 to N) are forces at each connection point. This effective mobility concept together with the concept of complex power can be used to model interface between an isolator and supporting structure[3,4,5]. The net complex power injected into supporting structure through surface contact can be determined by

$$Q^s = \frac{1}{2}\sum_{i=1}^{N} F_i^* \cdot V_i^e = \frac{1}{2}\sum_{i=1}^{N} M_i^e \cdot |F_i|^2 = \frac{1}{2} M^s \cdot |F|^2 \tag{2}$$

where F is the total force acting on the contact area and $F_i = F/N$ for uniform force excitation. Hence, the surface mobility can be found as

$$M^s = \frac{\sum_{i=1}^{N} M_i^e \cdot |F_i|^2}{|F|^2} = \frac{1}{N^2}\sum_{i=1}^{N}\sum_{j=1}^{N} M_{ij} \tag{3}$$

For N point connections, N×N mobility measurements may be needed to calculate the surface mobility. Fortunately, the number of mobility measurements required for determining a surface mobility can generally be reduced. Firstly, since the system under consideration is passive and linear, the reciprocity principle is valid and accordingly the mobility matrix will be symmetric, ie. $M_{ij}=M_{ji}$. Secondly, for an uniform, conphase force distribution on the contact area over an infinite plate, the excitation force F_i and the point mobility are the same anywhere on the plate and the transfer mobility between two points is only dependent on the distance between the excitation and response points. These assumption reduces mobility measurements considerably.

3. PREDICTION OF ORDINARY MOBILITY BY FEM

The prediction of ordinary point mobility and transfer mobility using FEM is a vital step to determine the surface mobility and to study the characteristics of area contact of an isolator. Commercial FE packages normally does not contain an explicit mobility calculation. However, they can be used under some conditions to calculate point mobility and transfer mobility. FE harmonic response analysis is an analysis type in which the applied loads vary sinusoidally with a known amplitude and at a known frequency. The solution obtained is the steady-state response of the structure to these loads as a function of frequency. The resulting response spectrum is not a frequency response function. However, the harmonic response analysis can be used to determine the frequency response function of a structure to a particular harmonic forcing function, when it is made equivalent to the swept sine measurements under the condition in which the amplitude of excitation force is set to 1 Newton. Therefore, the frequency response function can be obtained directly from harmonic velocity response based upon the mobility definition.

Harmonic analysis solves the time-dependent equations of motion for linear structures undergoing steady-state vibration. Generally, two methods, the direct and modal superposition methods, can be used to do a harmonic analysis. In the direct approach all equations are solved simultaneously, ie., all equations are coupled. It is the easiest, but more expensive in terms of computing time and storage requirements, especially for a large structure. The modal superposition method uses the natural frequencies and mode shapes of the structure to compute the response to a sinusoidally varying forcing function. The solution to the ith modal equation can be obtained using applicable direct method and the complete solution in geometric coordinates can be obtained by summing modal response in each modes.

4. MODELLING OF AN INFINITE PLATE

The two types of FE models were employed to investigate the effect of mesh size on the harmonic response and surface mobility of a plate with physical dimension 1.2m by 2.4m. In the first model, the plate was modelled using shell elements with uniform meshes (0.05m×0.05m) as shown in Figure 1. In the second model as shown in Figure 2, a fine mesh (0.01m×0.01m) was used in the middle region of the plate where the responses are of most interest. A coarse mesh was used in the outer area in order to keep an appropriate aspect ratio. In both models, the infinite plate boundary was simulated by adding spring-damper elements around the boundary of the plate. It is noted that the number of elements per wavelength is supposed to be greater than 6 in order to characterise the dynamic response of a structure. For uniform mesh model, the maximum wavelength is 0.3 m, which corresponds to upper frequency limit of 112Hz. Likewise, the maximum wavelength and upper frequency limit for non-uniform model are 0.06m and 2670Hz respectively based on inner meshes. The model medium is assumed to be elastic and homogeneous, with following properties for aluminium: Young' s modulus, $E=70\times10^9$ N/m^2, Poisson' s ratio, $\mu=0.3$ and density, $\rho=2700$ Kg/m^3. The thickness of the plate, h=0.001m. The sinusoidal excitation force was applied at the centre of the plate with the amplitude of 1 Newton. The frequency range for this excitation was 0 to 2048Hz.

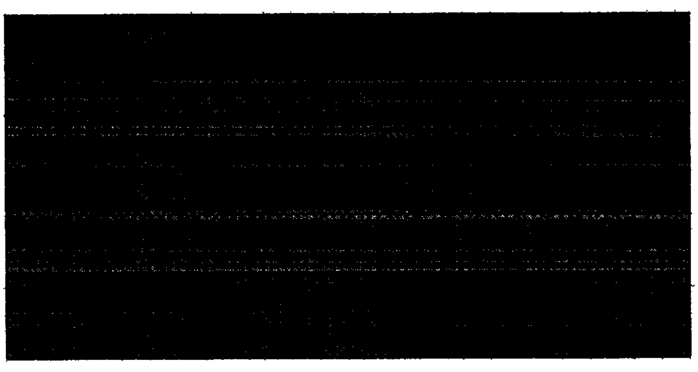

Figure 1 FE model of an infinite plate with uniform meshes

Figure 2 FE model of an infinite plate with non-uniform meshes

The surface mobilities were calculated over square contact area with 9 sub-regions (0.15m×0.15m), 16 sub-regions(0.20m×0.20m) and 25 sub-regions(0.25m×0.25m) as shown in Figure 3, 4 and 5.

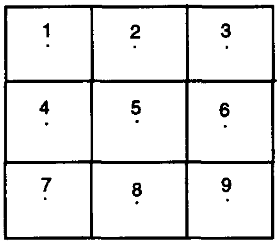

Figure 3 Square contact area (0.15m×0.15m) with 9 sub-regions

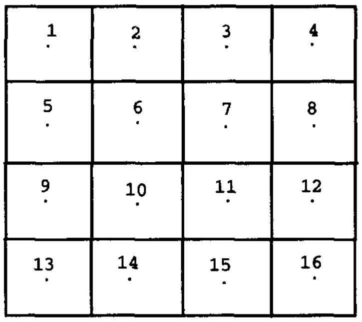

Figure 4 Square contact area (0.20m×0.20m) with 16 sub-regions

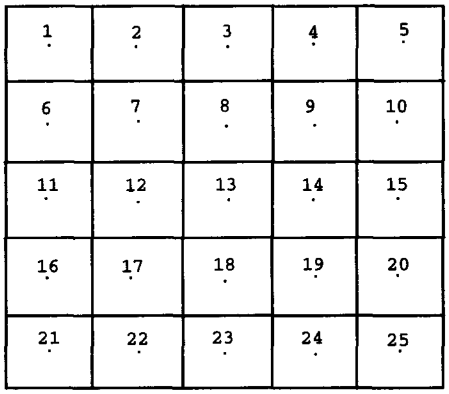

Figure 5 Square contact area (0.25m×0.25m) with 25 sub-regions

5. NUMERICAL RESULTS

5.1 The FE Model with Uniform Meshes

Original frequency response functions obtained are receptance. These responses were then converted to mobility. The calculated point mobilities are shown in Figure 6(a) and calculated transfer mobilities are shown in Figure 6(b), (c), (d), (e), and (f) in the ascending order of distances between excitation and response (D1=50mm, D2=70.7mm, D3=100mm, D4=111.8mm, D5= 141.4mm). It can be seen that the amplitude of frequency response decreased as the distance between the excitation and response point increases. The oscillation of mobilities at low frequency is probably due to the approximattion of an infinite plate. Note that calculated point mobility is not constant at all frequencies, compared with theoretical one (0.031m/sN).

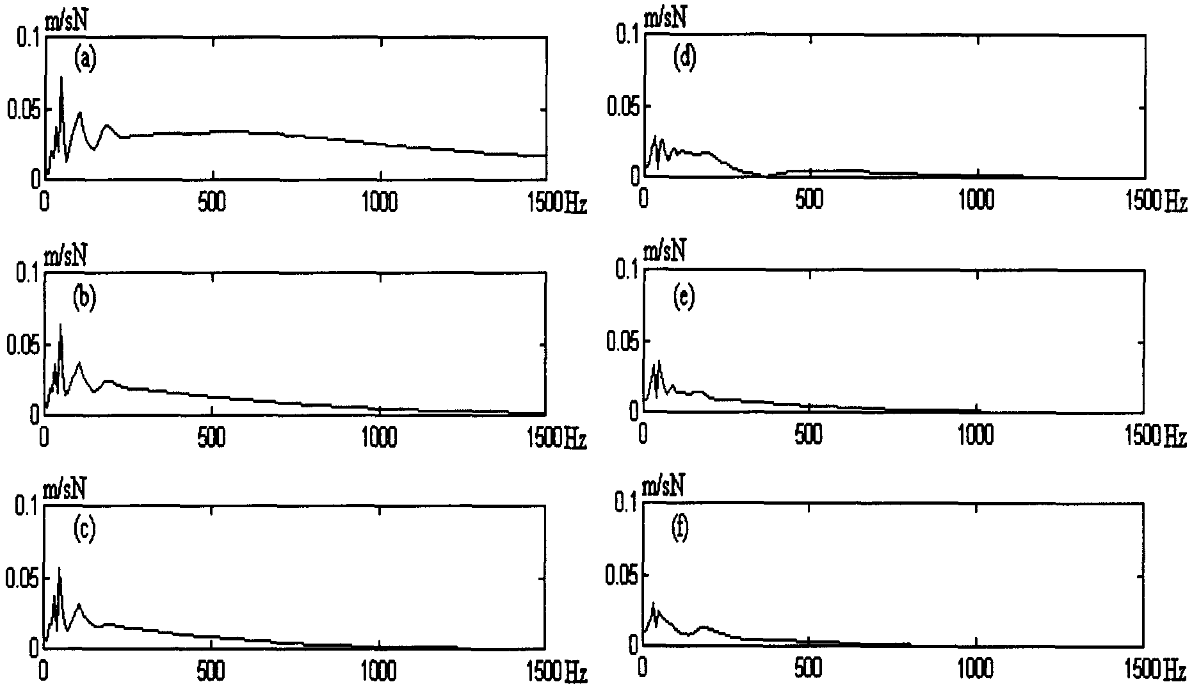

Figure 6 Calculated point and transfer mobility (uniform mesh model)

Then surface mobility was calculated for the contact areas, based upon the effective mobility concept as outlined in section 2. Figure 7(a), (b) and (c) shows calculated surface mobility over square contact areas with 9 sub-regions, 16 sub-regions and 25 sub-regions. A non-dimensional product kw/2, called Width-Based Helmholtz Number[5], is employed as the horizontal axis, where k is wavenumber and w/2 is the half-width of the square. These results can be compared to the analytical prediction of surface mobility for an infinite plate[5], as shown in Figure 7(d). It can be seen that calculated surface mobility over square contact area with 9 sub-regions and 16 sub-regions have only one dip, while the analytical prediction of surface mobility for an "infinite plate" has consecutive dips with the interval of about π. However, the calculated surface mobility over square contact area with 25 sub-regions have similar dips to analytical solution. The surface mobilities obtained from uniform mesh model generally decrease with increases of the Helmholtz number and have dips at intervals which are approximate multiples of π on the Helmholtz number axis. Moreover, the sidelobes of the surface mobility fall off at rate of 30dB per Helmholtz number of π.

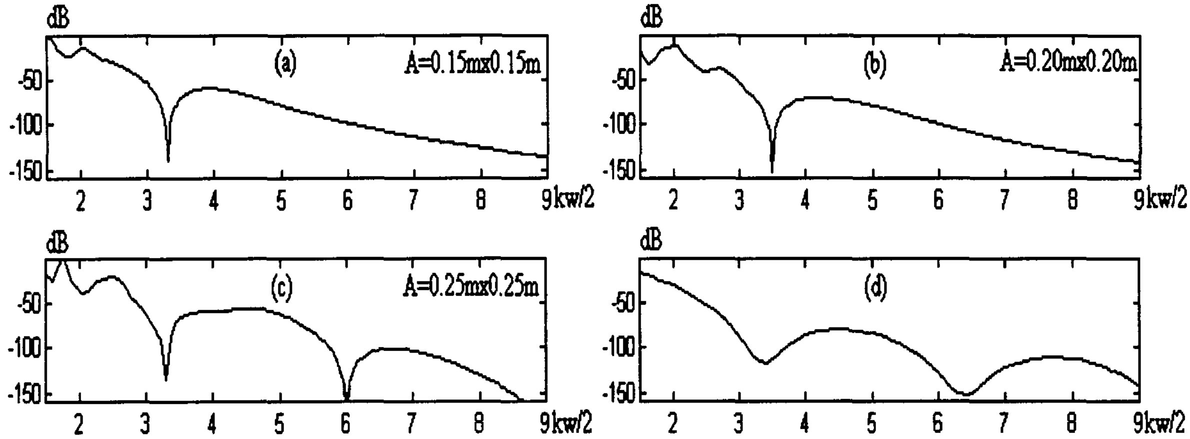

Figure 7 Predicted surface mobility of uniform mesh model with
(a) 9 sub-regions (FEM) (b) 16 sub-regions (FEM)
(c) 25 sub-regions (FEM) (d) Analytical prediction [5]

5.2 The FE Model with Non-Uniform Mesh

The similar analysis has been done on the FE model with non-uniform mesh. The calculated point mobilities are shown in Figure 8(a) and calculated transfer mobility are shown in Figure 8(b), (c), (d), (e), and (f) in the ascending order of distances between excitation and response point. These mobilities are similar to those from FE model with uniform mesh. Note that this model gives a better approximation for constant point mobility of an infinite plate.

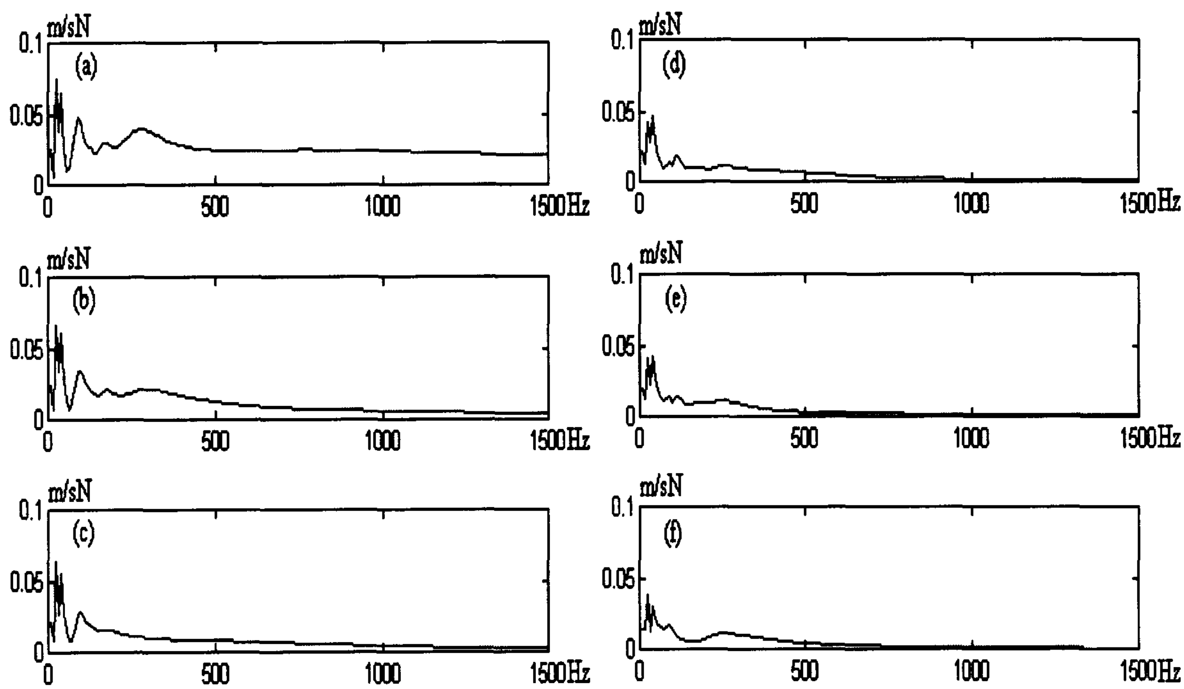

Figure 8 Calculated point and transfer mobility (non-uniform mesh model)

The calculated surface mobility for three contact area are shown in Figure 9(a), (b) and (c) which can be compared to the analytical prediction of surface mobility for an infinite plate[5], as shown in Figure 9(d). It can be seen that calculated surface mobility over square contact area with 9 sub-regions, 16 sub-regions and 25 sub-regions all have two dips, which is consistent with the analytical prediction of surface mobility for an infinite plate. However, the second dip in the calculated surface mobility over square contact area with 25 sub-regions has similar position as analytical one.

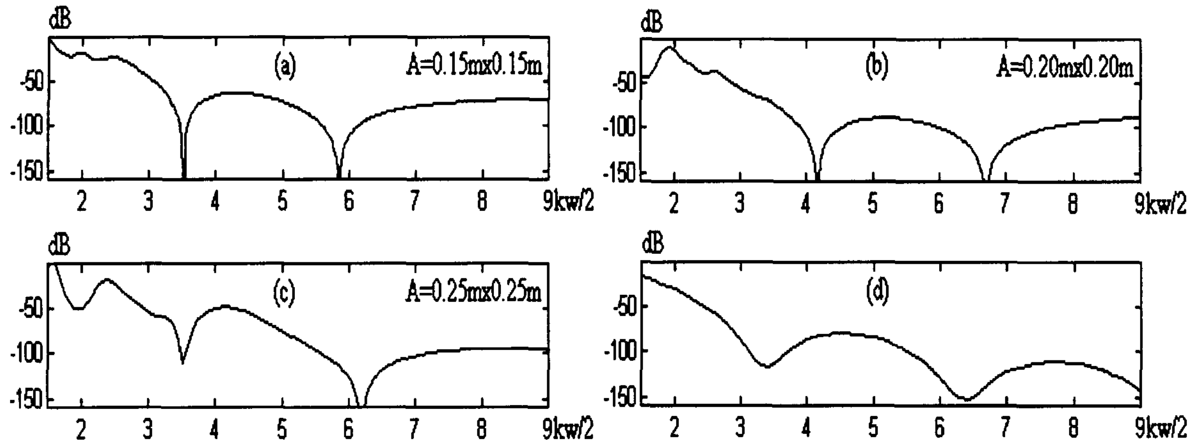

Figure 9 Predicted Surface mobility of non-uniform mesh model with
(a) 9 sub-regions (FEM) (b) 16 sub-regions (FEM)
(c) 25 sub-regions (FEM) (d) Analytical prediction [5]

6. CONCLUSIONS

It can be seen that the predicted surface mobility from non-uniform mesh model with 9 and 16 sub-regions are much better than that from uniform mesh model, compared with the analytical prediction. However, the position of two dips in the surface mobilities for both models with 25 sub-regions are close to each other and close to the analytical prediction. This means that the accuracy of numerical results increases as the number of sub-region in the contact area increase or mesh size in the FE model decreases. Furthermore, the second sidelobe of predicted surface mobility for non-uniform mesh model is higher than those from the uniform mesh model and analytical prediction. Generally speaking, the surface mobility obtained by two FE models decreases with increasing frequency and both results have dips at intervals which are approximate multiples of π on the horizontal axis. Moreover, the sidelobes of the surface mobility fall off at rate of 30dB per Helmholtz Number of π. This is in agreement with theoretical prediction and shows that the vibration transmission through the isolator with larger contact area would be greatly decreased at high frequency.

7. ACKNOWLEDGMENTS

The authors would like to express their gratitude to the Aeronautical and Maritime Research Laboratory (AMRL) of the Defence Science and Technology Organisation (DSTO) for their support of this research. This project has also been supported by an Australian Research Council grant.

8. REFERENCES

[1] E.E. Ungar and C.W. Dietrich, "High Frequency Vibration Isolation", J. Sound and Vibration, 1965, pp224-241.

[2] B. Petersson and J. Plunt, "On the Effective Mobilities in the Prediction of Structure-borne Sound Transmission Between a Source and A Receiving Structure, Part 1: Theoretical Background and Basic Experimental Studies," J. Sound and Vibration, 82, 517-529, 1982.

[3] J. Y. Zhao, H.M. Williamson and J.P.Baird, "Surface Excitation: Surface Mobility," Proc. 1993 Annual Conference, Aust. Acoustical Soc., p126-133, 1993.

[4] C. Norwood, H. Williamson and J. Zhao, "Surface Mobility of a Circular Contact Area of an Infinite Plate", Journal of Sound and Vibration, 202(1), 95-108, 1997.

[5] J. Dai and H.M. Williamson, "Surface Mobility for a Uniform Force over a Rectangular Excitation Area of an Infinite Thin Plate," AVC Report 9603, Acoustics and Vibration Centre, School of Aerospace and Mechanical Engineering, Australian Defence Force Academy, January 1996.