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Vibratory conveyance of granular materials

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The flow of granular materials on vibratory conveyors was studied analytically. Vibratory conveyors have proved useful for widely differing duties from feeding small and delicate components on automatic assembly lines to the transportation of large quantities of raw materials. There is a considerable descriptive data of vibratory conveyors, but for simplicity of analysis much previous theoretical analysis only predicts the motion of a particle on a vibrating plate. For optimum operating conditions, the behavior of granular materials during vibration must be known. In the present work, the discrete element method proposed by Cundall was adapted for a dynamic analysis of vibratory conveyance of granular materials. Granular materials are first assumed as ellipse models by considering shape anisotropy. Then, the contact forces are modeled by mechanical units such as springs, dashpots and friction sliders. It is shown that the mean velocity of transport of granular materials depends on the frequency and amplitude of the vibratory input as well as various physical parameters. The flow patterns obtained by this method appear realistic.

1. Introduction

The vibratory conveyance of granular materials can change the flow pattern easily by the control of the input vibratory energy and has been widely used for many years. The behavior of granular materials subjected to vibrations has often been investigated [1, 2], theoretically and experimentally, however, it is not clearly understood. This fact is not surprising, since almost every previous work has been focused on a particle's motion on a vibrating plate.

Recently, the discrete element method has been used to investigate the gravity flow of particles, such as the discharge from a hopper and the rock avalanche [3, 4]. In this method, the behavior of the entire system is determined by the integration of ordinary differential equations of motion of the individual particles over time. In the present work, we studied the vibratory conveyance using an improved discrete element method, in which granular materials are treated as elliptical models by considering the shape anisotropy. It is shown that the mean velocity for transport of granular materials depends on the frequency and amplitude of the vibrating vessel as well as on other various physical parameters.

2. Vibratory conveyance

Figure 1 shows the model of a vibratory conveyor. The vibrating vessel undergoes translational harmonic vibrations at an angle β to the horizontal plane. The granular materials in the vessel are conveyed with repeated sliding and free flight. It is supposed that the motion of the vessel is independent of the flow pattern of the granular materials. Using the x-y coordinate system as shown in Fig. 1, the motion of the vibrating vessel at time t, is represented by

$$X = \mathrm{a} \cos\beta \sin\omega t, \quad Y = \mathrm{a} \sin\beta \sin\omega t \qquad \cdots \quad (1)$$

where X and Y are the displacements of the vessel and a and ω are the amplitude and the angular frequency of the vibrations, respectively. In this figure, x_i, y_i are the coordinates of the particle i.

3. Discrete element method using the elliptical model

In the present work, the two-dimensional motion of cylindrical bodies with

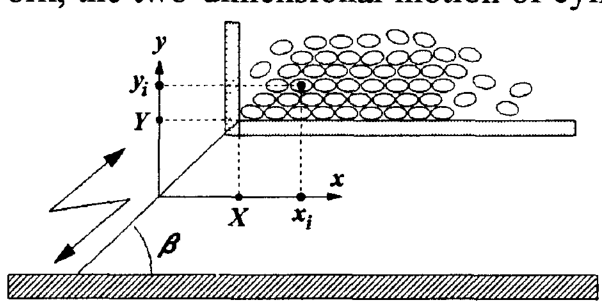

Fig. 1 Model of a vibratory conveyor

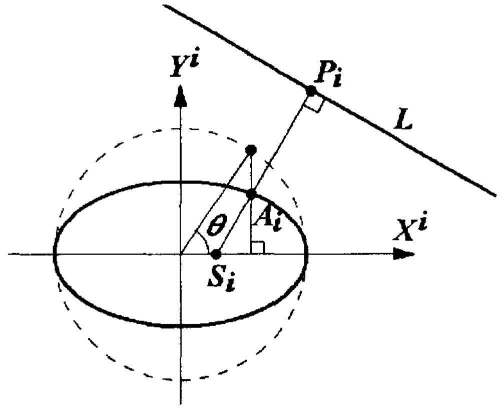

Fig. 2 Distance between an ellipse element and the vessel

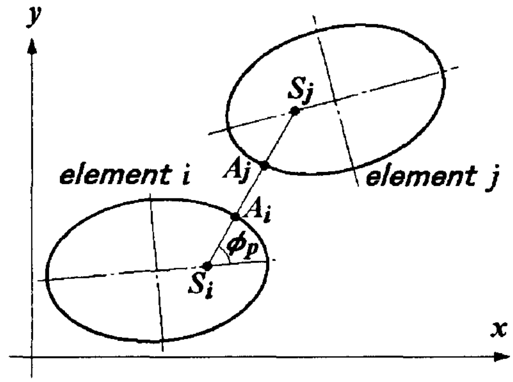

Fig. 3 Distance between two ellipse elements

elliptic cross sections are treated, taking into account the dependence of shape anisotropy on the flow of granular materials. The cylindrical body of an elliptic cross section is termed 'ellipse particle' in the following sections.

3.1 Contact judgement

(a) Contact between an ellipse particle and the vessel

Consider the ellipse particle i with the local coordinate X^i-Y^i as shown in Fig. 2. The major axis and the flat ratio (=minor axis / major axis) of the ellipse particle i are $2r$ and p, respectively. In this figure, the broken circle has a radius of r. An arbitrary point A_i is chosen on the boundary of the ellipse particle i. The line normal to the ellipse curve at point A_i crosses the X^i-axis at point S_i. Assume that the vibrating vessel is shown as a line L, then the realization of contact between the ellipse particle and the vibrating vessel is affirmed by the following procedure. Both points A_i and S_i are initially determined in such a way that line A_iS_i is perpendicular to line L at point P_i. Then, the realization of contact is confirmed by the condition that A_iS_i is longer than S_iP_i, i.e.

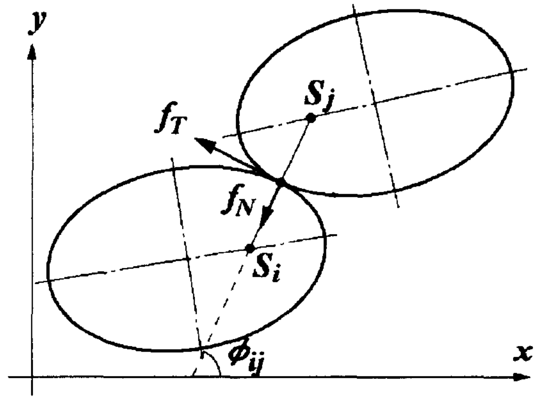

Fig. 4 Contact Force

$$|A_i S_i| \geq |S_i P_i| \qquad \cdots (2)$$

(b) Contact between two ellipse particles

Consider the contact problem between two ellipse particles i and j as shown in Fig. 3. An arbitrary point A_i is chosen on the boundary of the ellipse particle i. The line A_iS_i, perpendicular to the ellipse at this point crosses the major axis at point S_i. Points A_j and S_j of particle j are determined as before. Therefore, the two ellipse particles are taken to be in contact if line A_iS_i coincides with line A_jS_j and the following condition holds, i.e.

$$|S_i S_j| \leq |S_i A_i| + |S_j A_j| \qquad \cdots (3)$$

3.2 Contact force

Fig. 4 shows the contact between two particles i and j. The contact force acting on the particle i can be divided into two components, i.e. the normal force f_N and the tangential force f_T. In order to determine the contact force, the angle ϕ_{ij} between line the S_iS_j and the x-axis is considered.

$$\cos\phi_{ij} = (x_{sj} - x_{si})/L_{ij} \qquad \cdots (4)$$

$$L_{ij} = \sqrt{(x_{sj} - x_{si})^2 + (y_{sj} - y_{si})^2} \qquad \cdots (5)$$

The normal component of the contact force established using an elastic spring and a dashpot is given in Eq. (6). The tangential force is given in Eq. (7), considering the Coulomb-type friction law.

$$f_N = P_N + c_N \dot{\delta}_N = k_N \delta_N + c_N \dot{\delta}_N \qquad \cdots (6)$$

$$f_T = \mu f_N \dot{\delta}_T / |\dot{\delta}_T| \qquad \cdots (7)$$

$$\delta_N = \sqrt{(x_{ai} - x_{ai})^2 + (y_{aj} - y_{ai})^2} \qquad \cdots (8)$$

$$\dot{\delta}_N = (u_{xi} - u_{xj})\cos\phi_{ij} + (u_{yi} - u_{yj})\sin\phi_{ij} \qquad \cdots (9)$$

$$\dot{\delta}_T = -(u_{xi} - u_{xj})\sin\phi_{ij} + (u_{yi} - u_{yj})\sin\phi_{ij} \qquad \cdots (10)$$

where subscripts N and T refer to the normal and the tangential direction, respectively. u_{xi} and u_{xj} are the horizontal components of the velocities at the center of particles i and j, respectively. u_{yi} and u_{yj} are the vertical components of the velocities at the center of particles i and j, respectively. δ is the displacement, k_N is the spring constant, c_N is the damping coefficient, μ is the coefficient of friction and the dot denotes a time derivative. It is noted that the tension force is not considered.

It is supposed that the contact problem for cylindrical bodies with elliptic cross sections is the same as that for cylinders in the vicinity of the contact area, then the relation between the normal force F_N and displacement δ_N is defined according to the Hertzian contact theory.

To obtain the damping coefficient, the contact phenomena is modeled by the mass-spring-dashpot system with a single degree of freedom. Therefore, the damping coefficient is expressed by [5]

$$c_N = \frac{2\sqrt{mk_N}}{\sqrt{1+(-\pi/\ln e)^2}} \quad \cdots (11)$$

where e is the coefficient of restitution.

3.3 Equation of motion

When ellipse particle i is in contact with particle j, the total force acting on particle i is obtained by the summation of the contact forces, as

$$F_{xi} = \sum_j (-f_N \cos\phi_{ij} - f_T \sin\phi_{ij}),\quad F_{yi} = \sum_j (-f_N \sin\phi_{ij} + f_T \cos\phi_{ij}) \quad \cdots (12)$$

in which Σ represents the sum over all particles j in contact with particle i. The summation of the torque caused by the forces is given by

$$T_i = \sum_j \{X_A{}'(f_T \cos\phi_p - f_N \sin\phi_p) + Y_A{}'(f_N \cos\phi_p + f_T \sin\phi_p)\} \quad \cdots (13)$$

where (X_A^i, Y_A^i) is the coordinates of point A_i in the local coordinates X^i-Y^i. ϕ_p is the angle between line S_iS_j and the major axis of particle i (See Fig. 3). Therefore, the equations of translational and rotational motions for particle i are given by

Table 1

Number of particles	n = 105
Particle length	80mm
Spring constant	
Particle – particle	knp = 5.8×10^6N/m
Particle – vessel	knw = 11.5×10^6N/m
Coefficient of friction	
Particle – particle	μ p = 0.52
Particle – vessel	μ w = 0.25
Coefficient of restitution	
Particle – particle	e p = 0.62
Particle – vessel	e w = 0.76

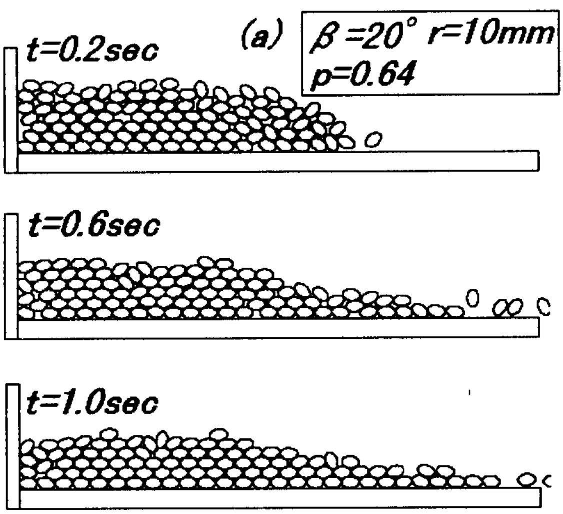

Fig. 5 Flow pattern (T=0.01sec, a=0.1mm)

$$m\ddot{x}_i = F_{xi}, \quad m\ddot{y}_i = F_{yi} - mg, \quad I\ddot{\theta}_i = T_i \qquad \cdots (14)$$

where m is the mass of a particle, g is the acceleration due to gravity and I is the moment of inertia of the particle.

4. Calculated results

The computational conditions and the physical properties of the ellipse particles for the simulation are shown in Table 1. The particles used in the calculation are uniform in size and density for simplification during calculations.

Fig.5 shows the effect of the angle β of the vibrating vessel on the flow pattern of the ellipse particles. Initially, the particles are piled up evenly on the left-hand side of the vibrating vessel. The flow occurs from left to right. The

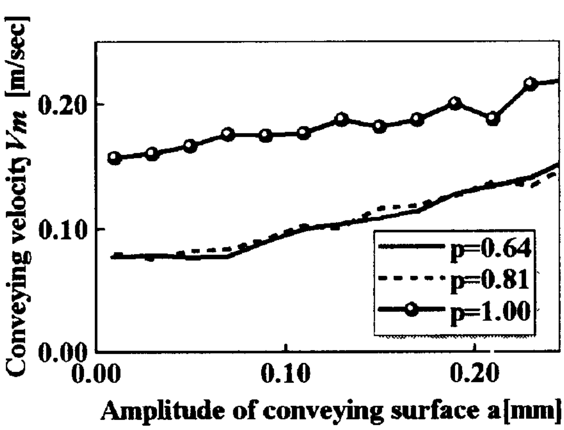

Fig. 6 Effect of flat ratio p (T=0.01sec, β=20°)

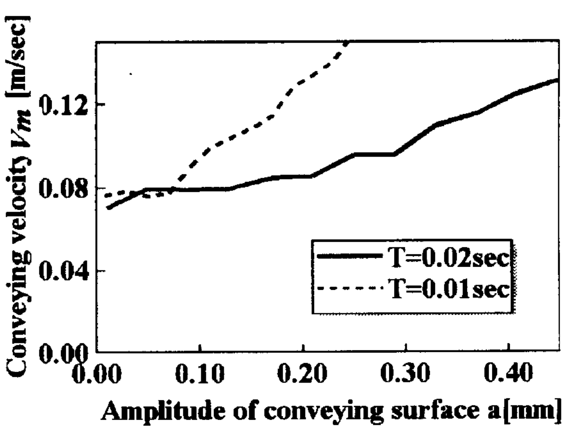

Fig. 7 Effect of period T (p=0.64, β=20°)

flow pattern is very realistic. From a comparison between Fig.5(a) and (b), the conveying displacements of the ellipse particles appear to increase with the decrease of the angle β of the vibrating vessel.

Figs. 6 and 7 show the relation between the conveying velocity *Vm* and the amplitude of the conveying vessel. In order to examine the effect of the shape anisotropy, three different types of particles have been considered in Fig. 6. We define the conveying velocity *Vm* to be

$$V_m = \sum_{i=1}^{n} \frac{\Delta x_i}{n} \quad \cdots (15)$$

where Δxi is the conveying displacement of the particle per second and n is the total number of particles. As shown in the figures, the conveying velocity increases with the increase of the amplitude of the vibrating vessel. In the case of flat particles (p=0.64, 0.81), the conveying velocity is smaller than that in the other case. The reason seems to be that the flat particles are difficult to rotate.

Fig. 7 also shows the effect of period T of the vibrating vessel on the conveying velocity. The increasing ratio of the conveying velocity increases as T decreases.

5. Conclusion

The vibratory conveyance of granular materials was investigated by means of an improved discrete element method, in which granular materials are treated as elliptical models by considering the shape anisotropy. In the case of flat particles, the transport of granular materials is more difficult to than in the case of disk shaped particles. In addition, it was shown that the mean velocity for transport of granular materials depends on the frequency and amplitude of the vibrating vessel.

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