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AN ESTIMATION METHOD OF ORIGINAL STOCHASTIC INFORMATION IN NON-GAUSSIAN RANDOM SIGNALS WITH AMPLITUDE LIMITATIONS

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ABSTRACT

In this paper, an estimation method of the original stochastic information in random noise signals of which amplitude fluctuation is limited by upper and/or lower levels is proposed for the generalized case of non-Gaussian random signals. First, an explicit expression of the probability density function for the random signal with amplitude limitations is introduced by using a combination of the well known statistical Hermite orthonormal expansion form and the Dirac's delta function. The original stochastic information is closely related to all of expansion coefficients (i.e., distribution parameters) in the probability density expression, and accordingly can be evaluated through these coefficients. That is, by use of this expression, the first, second, ..., nth order moments can be expressed by using the above expansion coefficients, the definite integration values of the standard Gaussian distribution and the levels of amplitude limitations. Thus, the expansion coefficients directly connected with the original case with no amplitude limitations can be evaluated by comparing experimental values of the statistical moments based on the limited observation data with the theoretically calculated moments. The original stochastic information for the case with no effect of the amplitude limitation can be estimated through these expansion coefficients.

1. INTRODUCTION

As is well-known, in the usual case of evaluating random noise signals, it is very important to study some stochastic information, such as, the mean value, the standard deviation, the evaluation quantity for the sound environment, L_x, and the correlation function connected with the amplitude probability density function. In such cases, the standard methods directly connected with the original definition of these statistical quantities are usually employed. In real measurement conditions, however, the random noise signals are sometimes recorded with amplitude limitations at upper and/or lower levels, caused, for example, by an unsuitable selection of the amplitude gain or some maladjustment of the bias voltage of the data recorder. Accordingly, it is clear that the true values of the stochastic information for original signals with no amplitude limitation cannot be obtained only based on these standard methods, without finding any further signal processing method of such amplitude limited signals. From the above points of view, some signal processing methods for estimating the original statistical quantities from such distorted observation signals have already been proposed for a Gaussian and a non-Gaussian random processes[1,2]. In this paper, some synthetic type signal processing methods are proposed in addition to some new results.

First, an explicit expression on the probability density function for random signals with amplitude limitation is introduced by using a combination of the well-known statistical Hermite orthonormal expansion form, taking the standard Gaussian distribution as the first expansion term. Next, by using this expression, the expansion coefficients for objective random signals can be evaluated, through the moments of the probability density function calculated by using the limited actual observation data. As a result, one can see that the above mentioned stochastic information in the case with no effect of the amplitude limitations can be concretely estimated through the above expansion coefficients.

Finally, the effectiveness of the proposed method is confirmed experimentally by applying it to the actual noise signals.

2. ESTIMATION OF EXPANSION COEFFICIENTS THROUGH AN OBSERVED SIGNAL WITH AMPLITUDE LIMITATIONS

2.1 Estimation of the Mean Value and the Standard Deviation in case of a Gaussian Random Process

Let $x(t)$ be a random signal having a mean value μ and a standard deviation σ, in case of a Gaussian process. When the random signal is observed with an amplitude limitation at a lower level α_1 and an upper level α_2, then the mean value and the standard deviation of the original signal can be determined by the following procedures.

First, the probability density function of the actually observed random signal with an amplitude limitation can be expressed by using the Gaussian distribution $N(x)$ and Dirac's delta function $\delta(x)$ as follows[1]:

$$p_t(x) = N(x)u(x) + \beta_1\delta(x-\alpha_1) + \beta_2\delta(x-\alpha_2) \tag{1}$$

with

$$u(x) \equiv \begin{cases} 1 & \alpha_1 < x < \alpha_2 \\ 0 & x \le \alpha_1, \quad x \ge \alpha_2 \end{cases}, \quad N(x) \equiv \frac{1}{\sqrt{2\pi}\sigma}\exp\Big\{-\frac{(x-\mu)^2}{2\sigma^2}\Big\},$$

$$\beta_1 \equiv \int_{-\infty}^{\alpha_1} N(x)dx \text{ and } \beta_2 \equiv \int_{\alpha_2}^{\infty} N(x)dx. \tag{2}$$

From equation (1), the first order moment m_1 can be expressed by its definition as follows:

$$m_1 = \int_{-\infty}^{\infty} xp_t(x)dx = \sigma^2\{N(\alpha_1) - N(\alpha_2)\} + \mu(1-\beta_1-\beta_2) + \beta_1\alpha_1 + \beta_2\alpha_2. \tag{3}$$

In the same way, the second order moment m_2 can be calculated as follows:

$$m_2 = \int_{-\infty}^{\infty} x^2 p_t(x)dx$$

$$= \sigma^2 N(\alpha_1)(\alpha_1+\mu) - \sigma^2 N(\alpha_2)(\alpha_2+\mu) + (\sigma^2+\mu^2)(1-\beta_1-\beta_2) + \beta_1\alpha_1^2 + \beta_2\alpha_2^2. \tag{4}$$

In equations (3) and (4), α_1 and α_2, are known in advance, and m_1, m_2, β_1 and β_2 are directly calculated by use of the observed data. Accordingly, one can solve the non-linear simultaneous equation (5) with respect to μ and σ, e.g., by introducing some improved Newton-Raphson method, and the mean value and the standard deviation, without influence of an amplitude limitation, can be obtained.

$$\left.\begin{aligned} f(\mu,\sigma) &= \sigma^2\{N(\alpha_1) - N(\alpha_2)\} + \mu(1-\beta_1-\beta_2) \\ &\qquad +\beta_1\alpha_1 + \beta_2\alpha_2 - m_1 = 0 \\ g(\mu,\sigma) &= \sigma^2 N(\alpha_1)(\alpha_1+\mu) - \sigma^2 N(\alpha_2)(\alpha_2+\mu) \\ &\quad +(\sigma^2+\mu^2)(1-\beta_1-\beta_2) + \beta_1\alpha_1^2 + \beta_2\alpha_2^2 - m_2 = 0. \end{aligned}\right\} \tag{5}$$

2.2 Estimation of Hermite Expansion Coefficients for an Observed Random Signal with Amplitude Limitations

When $x(t)$ shows an arbitrary non-Gaussian distribution, then the probability density function of the random signals can be expressed in the general form of a statistical Hermite polynomials, taking the standard Gaussian distribution as the first expansion term as shown in equation (6),

$$p(x) = N_0(x)\left\{1 + \sum_{n=1}^{\infty} A_n H_n(x)\right\} \tag{6}$$

with

$$N_0(x) \equiv \frac{1}{\sqrt{2\pi}}\exp\left(-\frac{x^2}{2}\right), \quad A_n \equiv \frac{1}{n!}\Big\langle H_n(x)\Big\rangle.$$

At that time, the differences between the true values and the assumed values ($\mu = 0$, $\sigma = 1$) for the mean value and the standard deviation, in the ideal case of assuming that the random signal has the standard Gaussian distribution, are reflected in the expansion coefficients of the probability density expression.

Here, when more than the $(N + 1)$th order expansion coefficients can be neglected, this probability density function can be approximately expressed by using a finite Hermite polynomials up to the Nth order in the expansion form. In the same manner as Gaussian random process, the probability density function of the actually observed random signal with amplitude limitations can be expressed by using equation (6) and the Dirac's delta function as follows[2]:

$$p_t(x) = p(x)u(x) + \beta_1\delta(x - \alpha_1) + \beta_2\delta(x - \alpha_2), \tag{7}$$

with

$$\beta_1 \equiv \int_{-\infty}^{\alpha_1} p(x)dx \text{ and } \beta_2 \equiv \int_{\alpha_2}^{\infty} p(x)dx. \tag{8}$$

From this equation, the first, second, $\cdots$, Nth order moments m_1, m_2, $\cdots$, and m_N are calculated respectively as follows:

$$\left.\begin{array}{l} m_1 = I_1 + A_1 I_2 + A_2(I_3 - I_1) + \cdots + \beta_1\alpha_1 + \beta_2\alpha_2, \\ m_2 = I_2 + A_1 I_3 + A_2(I_4 - I_2) + \cdots + \beta_1\alpha_1^2 + \beta_2\alpha_2^2, \\ \quad \vdots \qquad\qquad \vdots \qquad\qquad \vdots \\ m_N = I_N + A_1 I_{N+1} + A_2(I_{N+2} - I_N) + \cdots + \beta_1\alpha_1^N + \beta_2\alpha_2^N \end{array}\right\} \tag{9}$$

with

$$I_n = \int_{\alpha_1}^{\alpha_2} x^n N_0(x)dx.$$

In these equations, α_1 and α_2 can be known in advance, and m_1, m_2, $\cdots$, m_N, β_1 and β_2 can be directly calculated too from these original definition by use of the observed data. Moreover, the nth integral, I_n, of the standard Gaussian distribution can be calculated in advance by using the recursive relations as

$$I_0 = \frac{1}{\sqrt{2\pi}}\int_{\alpha_1}^{\alpha_2} \exp\left(-\frac{x^2}{2}\right)dx, \;\; I_1 = N_0(\alpha_1) - N_0(\alpha_2)$$

and

$$I_n = \alpha_1^{n-1}N_0(\alpha_1) - \alpha_2^{n-1}N_0(\alpha_2) + (n-1)I_{n-2}; \quad (n = 2, 3, \cdots, N).$$

Accordingly, the expansion coefficients A_1, A_2, $\cdots$, and A_N without influence of the amplitude limitations can be obtained as the solution of the following Nth simultaneous equations:

$$\left.\begin{array}{l} K_{11}A_1 + K_{12}A_2 + K_{13}A_3 + \cdots + K_{1N}A_N = B_1, \\ K_{21}A_1 + K_{22}A_2 + K_{23}A_3 + \cdots + K_{2N}A_N = B_2, \\ \quad \vdots \qquad\quad \vdots \qquad\quad \vdots \qquad\quad \vdots \qquad\quad \vdots \\ K_{N1}A_1 + K_{N2}A_2 + K_{N3}A_3 + \cdots + K_{NN}A_N = B_N, \end{array}\right\} \tag{10}$$

where

$$K_{i1} \equiv I_{i+1}, \quad K_{i2} \equiv I_{i+2} - I_i, \;\; K_{i3} \equiv I_{i+3} - 3I_{i+1},$$

$$K_{iN} \equiv I_{i+N} - \frac{N(N-1)}{2} I_{i+N-2} + \frac{N(N-1)(N-2)(N-3)}{2 \cdot 4} I_{i+N-4} - \cdots,$$

and

$$B_i \equiv m_i - I_i - \beta_1 \alpha_1^i - \beta_2 \alpha_2^i \quad (i = 1, 2, \cdots, N).$$

2.3 Estimation of the Expansion Coefficients based on Re-evaluating the Values of β_1 and β_2

In the above procedures, β_1 and β_2 are estimated through a ratio of the number of sampled data affected by upper and lower levels to the number of whole sampled data. But, finite number of sampled data can be practically used in this estimation. So, the estimated values of β_1 and β_2 include some errors, and naturally are distributed around the true values. Accordingly, one can find an improved estimation procedure based on the re-evaluation of β_1 and β_2 as follows:

step 1.

Estimate the values of β_1 and β_2 through the observed data.

step 2.

Repeat the following procedures 1 and 2:

1. Solve equation (5) or equation (10) and determine the mean value and the standard deviation, or the expansion coefficients $A_1, A_2, \cdots$, and A_N.

2. Substitute the above values (the mean value and the standard deviation, or the expansion coefficients) into the right side of equation (2) or (8), and re-evaluate the values of β_1 and β_2.

3. EXPRESSION OF STOCHASTIC INFORMATION BY USING THE HERMITE TYPE EXPANSION COEFFICIENTS

After determining expansion coefficients based on the above mentioned procedure, some stochastic information connected with the original random noise can be evaluated through the expansion coefficients. First, the noise evaluation quantity, L_x, for the sound environment can be easily calculated by a cumulative probability distribution $Q(L_x)$. Namely, L_x can be determined by solving the following equation:

$$Q(L_x) = \int_{-\infty}^{L_x} p(\xi) d\xi$$

$$= \int_{-\infty}^{L_x} N_0(\xi) \left\{ 1 + \sum_{n=1}^{N} A_n H_n(\xi) \right\} d\xi = (100 - x)/100. \tag{11}$$

Next, the correlation information can be expressed based on the conditional average without using the total data of the instantaneous values as follows[3]:

$$\rho(\tau) = \int_{-\infty}^{\infty} k(\xi) \frac{m(\xi) - \mu}{\xi - \mu} d\xi \tag{12}$$

with

$$k(\xi) = \frac{p(\xi)}{\sigma^2}(\xi - \mu)^2.$$

Furthermore, when the random signal is approximated by a typical Gaussian distribution, one can use the first expansion term in equation (12) as the linear correlation function in a simplified well-known form:

$$\rho(\tau) = \frac{m(\xi) - \mu}{\xi - \mu}. \tag{13}$$

4. EXPERIMENTAL CONSIDERATIONS

In this section, some test results are shown, as obtained by applying the proposed method to the actual noise data measured at a hydroelectric plant as a Gaussian random process. The data have been sampled at 1 millisecond intervals and the number of the sampled data is 10000. The sampled data have been normalized by the mean value and the standard deviation, and artificially limited in the computer. The mean value of the data is estimated by using the proposed procedure in sections 2.1 and 2.3. Figure 1 shows the estimation errors versus iteration number. In Figure 1, the estimated mean value by using the re-evaluation surely approaches to the true value as the iteration number increase.

Next, the proposed procedure is applied to actual road traffic noise observed at a suburban district in Hiroshima City. The data have been sampled at 1/3 second intervals and the number of the sampled data is 2000. The sampled data have been artificially limited in the computer at the lower and the upper levels or only at the upper level. Result are shown in Figures 2, 3 and Table 1.

Figure 2 shows the estimation errors of the mean value and the standard deviation for the non-Gaussian random signal, when only the upper limitation level α_2 changes from 71 dBA to 59 dBA. As is evident from the figure, the errors calculated by the proposed procedures expressed in section 2.2 do not increase further even if the above mentioned ratio increases, differing from

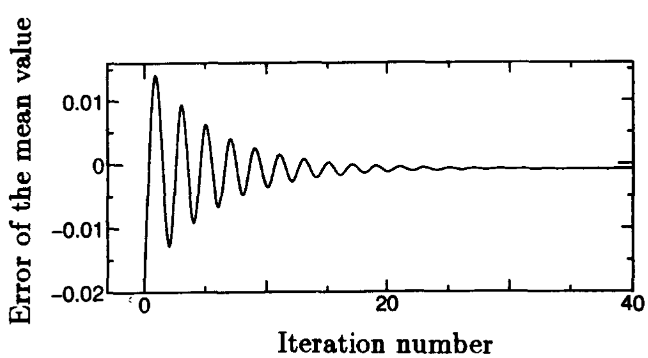

Figure 1 Estimation errors of the mean value versus iteration number in the case of a Gaussian process.

the errors calculated without any countermeasure (direct method).

The mean value and the standard deviation and some noise evalution quantities for the sound environment, such as, L_5, L_{10}, L_{25}, L_{50}, L_{90} and L_{95} are estimated by the direct method without employing any countermeasure and by the proposed method based on equation (11)(α_1 = 66 dBA, α_2 = 80 dBA, β_1=0.153, β_2=0.151). The results are shown in Table 1. In Table 1, the experimental values obtained through the original sampled data and the theoretically estimated values(fourth approx.) based on the proposed method are close enough to each other within an error of 1.0 dBA allowed in a usual environmental standard.

Figure 3 shows the correlation functions. In this figure, the dash-dotted line and dotted line show the correlation functions with the amplitude limitations calculated by the direct and the proposed methods respectively (α_2 = 65 dBA, β_2=0.54). The correlation function of the original data without amplitude limitations is shown in the figure as true values by the solid line.

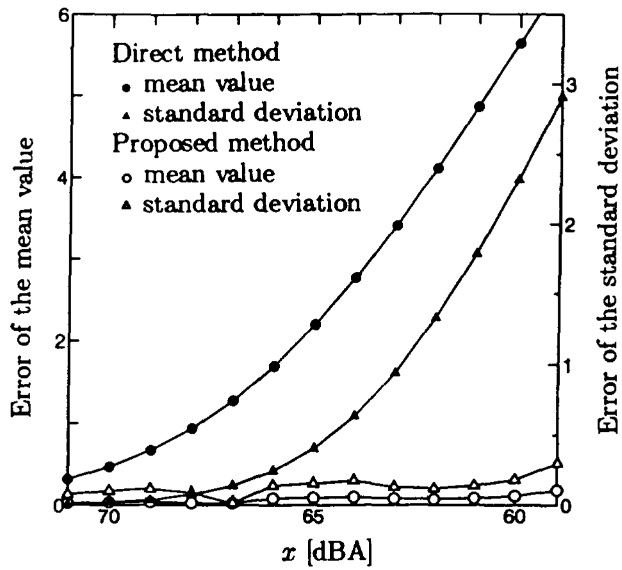

Figure 2 Estimation errors of the mean value and the standard deviation in the case of a non-Gaussian process.

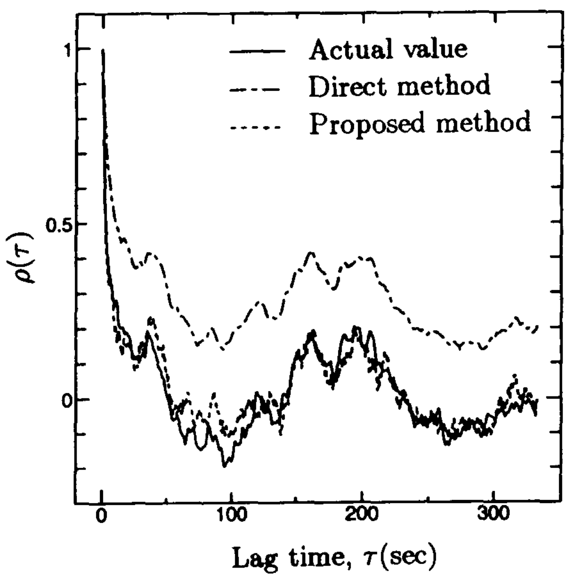

Figure 3 A comparison of autocorrelation function for the actual road traffic noise.

It is evident that the values estimated by use of the proposed method agree successfully with the actual results in comparison with the direct method.

Table 1 A comparison of the mean value, the standard deviation and the evaluation quantities for the sound environment, L_x, between the experimental values, the direct method and the proposed method (fourth approximation).
$\alpha_1 = 66$dBA, $\alpha_2 = 80$dBA, $\beta_1 = 0.153$, $\beta_2 = 0.151$
(dBA)

	μ	σ	L_5	L_{10}	L_{50}	L_{90}	L_{95}
Experimental values	73.5	6.31	82.7	81.0	74.3	63.1	60.9
Direct method	73.7	4.84	*	*	74.3	*	*
Proposed method	73.8	6.12	83.5	81.5	74.6	63.7	61.1

* Uncertain value

5. CONCLUSIONS

In this paper, some signal processing methods for estimating the stochastic information of random signals of which amplitude fluctuation is limited by the upper and/or lower levels have been proposed for both the idealized case of a Gaussian random process and the generalized case of a non-Gaussian random process. The proposed methods can determine the original stochastic information, such as the mean value, the standard deviation, evaluation quantity L_x and the correlation function based on the expansion coefficients without use of a whole information about the distorted observation signal data. It must be pointed out that the instantaneous values of random signal can not be originally estimated but the statistical distribution given as the expansion coefficients can be determined as some stable information supported by many sampled data. In the experiment, the effectiveness of the proposed methods has been confirmed by applying it to actual noise under an artificial setting of the amplitude limitation. And, one can conclude that the proposed method is effective in estimating the stochastic information despite requiring only a fairly small number of data uneffected by amplitude limitations.

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