A DYNAMIC ESTIMATION METHOD OF LOW-FREQUENCY OBJECTIVE SOUND IN THE OUTDOOR MEASUREMENT CONTAMINATED BY WIND NOISE

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ABSTRACT

This paper describes a new method estimating an objective low-frequency acoustic sound contaminated with wind noise in the outdoor measurement. It is a dynamic estimation technique employing a reference band pressure level (BPL) of a particular frequency range of the observation as the basis of the wind noise estimation. First, a conditional probability distribution function (c.d.f) of the wind noise on knowing the reference BPL is shown. Next, a dynamic state estimation technique based on Bayes' filter is proposed by using the consecutively renewed parameters of c.d.f. of the wind noise as a trial. This method is fairly different from previously proposed state estimation method based on the use of the information on the wind speed observed in the vicinity of the microphone. Because the proposed method becomes comparatively complicated for the practical use, more simplified stochastic estimation methods of the objective sound are also proposed. Finally, the effectiveness of the proposed method is experimentally discussed by employing these simplified methods in this study, especially from the practical viewpoint.

INTRODUCTION

In the outdoor measurement of low-frequency acoustic sound under windy conditions, wind-induced noise (wind noise) appears as an inevitable disturbance. In general, it is difficult to remove the low-frequency components of the wind noise by using only usual
In such cases, some different types of countermeasures against the wind noise are proposed to undertake more accurate measurement: the method by putting a microphone near the ground surface,\(^5\) the synchronized integration method,\(^6\) the method using a coherent detector,\(^7,8\) the methods based on correlation techniques by using multiple microphones,\(^3,9,10\) and the dynamic estimation methods based on the Bayesian type digital filter\(^8,11,12,\) etc. We have also proposed two simple estimation methods of static and dynamic types in the time domain and another simple estimation method in the frequency domain from the different viewpoint of signal processing technique by using the wind speed observed in the vicinity of a microphone.\(^13-16\) Further, we have proposed a simple estimation method focusing the power spectral characteristics of the wind noise by using a particular frequency band pressure level (BPL) of the observed sound.\(^17\)

In this study, in order to estimate the objective sound successively with time, we discuss a new dynamic estimation method. First, we introduce the approximation of the fluctuation of the objective sound and the wind noise measured in the mean square sound pressure by Gamma distribution. The conditional probability distribution function (c.d.f.) of the wind noise is expressed as a function of the reference BPL of the particular frequency range. Next, the dynamic estimation method is discussed. The method is based on a recursive parameter estimation dynamically renewed on knowing the reference BPL. Though we intend to simplify the estimation algorithm form the beginning of this study, the derived equations are comparatively complicated for the practical use. Accordingly more simplified successive estimation method of the objective sound are also shown. Finally, the estimation results of the objective sound are shown by giving priority to the simplified methods in this study from the view point of the practical use.

**CONDITIONAL DISTRIBUTION OF WIND NOISE**

The SPL measurement with the averaging time \(T\) corresponding to the fast or slow response of sound level meters and the BPL measurement of which can be expressed as:

\[
\text{SPL} = 10 \log \frac{\overline{p^2}}{p_0^2} = 10 \log \frac{\frac{1}{T} \int_0^T p^2 dt}{p_0^2}, \quad \text{BPL} = 10 \log \frac{\overline{p_B^2}}{p_0^2} = 10 \log \frac{\frac{1}{T} \int_0^T p_B^2 dt}{p_0^2},
\]

where \(\overline{p^2}\) is the mean-square sound pressure, \(\overline{p_B^2}\) is the component of \(\overline{p^2}\) in a frequency band \(B\) and \(p_0 = 20(\mu\text{Pa})\) is the reference pressure.

Now, we consider the wind noise measured with a microphone in SPL or BPL. Generally, the wind noise consists of large level of low-frequency component\(^1,5,18\) and we verified some strong correlation between the BPL of a low-frequency range (for example, the range of 4-6 Hz) and the SPL of the wind noise.\(^17\) In this study, we intend to express the mean-square sound pressure of the wind noise by using the particular low-frequency band component of the wind noise itself. Let us call the particular BPL the reference BPL and let express the wind noise component as \(L_{r,B}\).
Let the mean square sound pressure of the wind noise be \( v \triangleq \frac{\Delta}{p^2} \) and let the average and the variance of the wind noise \( v_k \) at a time \( k \) be \( \overline{v_k} \) and \( V_k \) respectively:

\[
\overline{v_k} \triangleq <v_k>, \quad V_k \triangleq <(v_k - \overline{v_k})^2>.
\]

Now, we approximately express the c.d.f. of the wind noise \( v \) fluctuating only in the non-negative region by the following Gamma distribution on knowing \( L_{r BV,k} \):

\[
P(v_k | L_{r BV,k}) = \frac{1}{\Gamma(m_{v,k})} \frac{m_{v,k}^{m_{v,k}-1} e^{-\frac{v_k}{m_{v,k}}}}{S_{v,k} m_{v,k}} \]

where two characteristic parameters \( m_{v,k} \) and \( S_{v,k} \) can be estimated by the reference \( L_{r BV,k} \) observed at the time \( k \).

**A DYNAMIC ESTIMATION METHOD**

Let the mean square sound pressure of an objective sound (signal to be estimated) at \( k \) be \( x \triangleq \frac{\Delta}{p^2} \). In this study, we consider only a simplified case when estimating a constant level of the objective sound. Therefore, this objective sound is expressed by following system equation when the time proceeds to \( k + 1 \):

\[
x_{k+1} = x_k.
\]

Then, the observation \( y_k \) of the objective sound under the wind noise \( v_k \) is expressed by reasonably supposing the independent relationship between the sound and the noise as:

\[
y_k = x_k + v_k.
\]

Here, let a set of the observation be \( \mathcal{Y}_k \triangleq \{y_1, y_2, \ldots, y_k\} \) until the time \( k \), and we notice the well-known Bayes’ estimation principle:

\[
P(x_k | \mathcal{Y}_{k-1}) = \frac{P(x_k, y_k | \mathcal{Y}_{k-1})}{P(y_k | \mathcal{Y}_{k-1})} = \frac{P(x_k | y_k, \mathcal{Y}_{k-1})}{P(y_k | \mathcal{Y}_{k-1})} P(x_k | \mathcal{Y}_{k-1}).
\]

Now, we introduce two kinds of orthonormal functions \( \phi_m^{(1)}(x_k) \) and \( \phi_n^{(2)}(y_k) \) with two kinds of weighting functions of \( \phi_m^{(1)}(x_k) \) and \( \phi_n^{(2)}(y_k) \) respectively:

\[
\int \phi_m^{(1)}(x_k) \phi_m^{(1)}(x_k) P(x_k | \mathcal{Y}_{k-1}) dx_k = \delta_{mm'},
\]

\[
\int \phi_n^{(2)}(y_k) \phi_n^{(2)}(y_k) P(y_k | \mathcal{Y}_{k-1}) dy_k = \delta_{nn'}
\]

where \( \phi_0^{(1)}(x_k) = 1 \) and \( \phi_0^{(2)}(y_k) = 1 \). Then the joint probability distribution \( P(x_k, y_k | \mathcal{Y}_{k-1}) \) can be expressed in the following series expansion form:

\[
P(x_k, y_k | \mathcal{Y}_{k-1}) = P(x_k | \mathcal{Y}_{k-1}) P(y_k | \mathcal{Y}_{k-1}) \sum_{m=0}^{\infty} \sum_{n=0}^{\infty} A_{mn} \phi_m^{(1)}(x_k) \phi_n^{(2)}(y_k).
\]

Upon using the equations (7) and (5), the expansion coefficients can be evaluated as:
where \( A_{m0} = A_{0n} = 0 \) \((m, n \neq 0)\) \( A_{00} = 1\) and \( A_{11}\) corresponds to the linear correlation between \( x_k \) and \( y_k \). Thus, the Bayes’ estimation principle can be expressed in the expansion form:

\[
P(x_k | Y_k) = P(x_k | Y_{k-1}) \{1 + \sum_{(m,n)\neq(0,0)} A_{mn} \phi_m^{(1)}(x_k) \phi_n^{(2)}(y_k)\} \quad (9)
\]

This equation shows how to improve the estimate \( P(x_k | Y_k) \) from the prediction \( P(x_k | Y_{k-1}) \) by obtaining the newly observed data \( y_k \). Further, the estimation accuracy can be increased when the statistical parameters of the wind noise are successively estimated and renewed with the use of the relation in eq. (3) by obtaining the reference \( L_{y,rB} \simeq L_{v,rB} \) at the same time.

Finally, following to the well-known Weierstrass’ polynomial approximation theorem, let us introduce a polynomial function \( f_N(x_k) \) of \( N \)-th order to express the arbitrary function of \( x_k \) as:

\[
f_N(x_k) = \sum_{i=0}^{N} C_{Ni} \phi_i^{(1)}(x_k) \quad (10)
\]

With help of two equations (10) and (11), this arbitrary function \( \hat{f}_N(x_k) \) can be successively estimated in the expansion form with the observation \( y_k \):

\[
\hat{f}_N(x_k) \triangleq \langle f_N(x_k) | Y_k \rangle = \sum_{n=0}^{\infty} \sum_{i=0}^{N} C_{Ni} A_{in} \phi_n^{(2)}(y_k).
\]

**Estimation equations for Gamma distribution**

Now, similar to the approximation in eq. (3), let us express the c.d.f. of the objective sound \( x_k \) and the observation \( y_k \) as the Gamma distribution:

\[
P(x_k | Y_{k-1}) = P_T(x_k; S^*_{x,k}, m^*_{x,k}) , \quad m^*_{x,k} = (x_k^*)^2 / X_k , \quad S^*_{x,k} = x_k^* / m^*_{x,k} ,
\]

\[
x_k^* \triangleq \langle x_k | Y_{k-1} \rangle , \quad X_k = <(x_k - x_k^*)^2 | Y_{k-1}>\quad (13a)
\]

\[
P(y_k | Y_{k-1}) = P_T(y_k; S^*_{y,k}, m^*_{y,k}) , \quad m^*_{y,k} = (y_k^*)^2 / Y_k , \quad S^*_{y,k} = y_k^* / m^*_{y,k} ,
\]

\[
y_k^* \triangleq \langle y_k | Y_{k-1} \rangle , \quad Y_k = <(y_k - y_k^*)^2 | Y_{k-1}> \quad (13b)
\]

Then, two kinds of orthonormal functions in eq. (7) required in eq. (12) are expressed by using well-known Laguerre polynomial. The expansion coefficients \( C_{Ni} \) in eq. (12) can be derived by obtaining the statistical Laguerre expansion expression of \( x_k \) after by employing the polynomial expressions into eq. (12). Further, the linear and non-linear correlation quantity \( A_{in} \) in eq. (12) can also be derived by using the Laguerre polynomial expression of \( y_k \) with eq. (7). Here, let us show only the derived equations necessary for the estimation of \( x_k \):

\[
\hat{x}_k = C_{10} + C_{11}\{A_{11}\sqrt{\frac{\Gamma(m_{y,k})}{\Gamma(m_{y,k} + 1)}} L_1^{(m_{y,k}-1)}(\frac{y_k}{S^*_{y,k}}) + A_{12}\sqrt{\frac{2\Gamma(m_{y,k})}{\Gamma(m_{y,k} + 2)}} L_2^{(m_{y,k}-1)}(\frac{y_k}{S^*_{y,k}})\}.
\]
\[ C_{10} = m_{x,k}^* s_{x,k}^* , \quad C_{11} = -\sqrt{m_{x,k}^* s_{x,k}^*} , \quad A_{11} = \sqrt{\frac{m_{x,k}^*}{m_{y,k}^*} \left( \frac{s_{x,k}^*}{s_{y,k}^*} \right)} , \]
\[ A_{12} = \sqrt{\frac{2m_{x,k}^*}{m_{y,k}^*} \left( \frac{s_{x,k}^*}{s_{y,k}^*} \right)} (m_{y,k} + 1) \left\{ m_{y,k}^* (m_{y,k} + 1) - m_{y,k}^* \left( \frac{s_{y,k}^*}{s_{y,k}^*} \right) - (m_{x,k}^* + 1) \left( \frac{s_{x,k}^*}{s_{y,k}^*} \right) \right\} . \] (14)

Though the distributions are approximated to the Gamma distribution, the derived estimation equations becomes rather complicated for the practical use.

**Simplified estimation methods**

In order to estimate the objective sound successively with more practical methods, let us consider following two simplified methods. Let the average level of \( L_{rBv,k} \) for adequate duration be \( a_{v} L_{rBv} \), the average of estimated wind noise conditioned by \( a_{v} L_{rBv} \) be \( a_{v} \hat{v} \) and the average of observation \( y_k \) for the same duration be \( a_{v} y \):

\[ a_{v} L_{rBv} = 10 \log \frac{1}{n p_0^2} \sum_{j=1}^{n} p_{rBv,j} , \quad a_{v} \hat{v} = m_{v} S_{vl} |_{avL_{rBv}} , \quad a_{v} y = \frac{1}{n} \sum_{j=1}^{n} p_{v,j} , \] (15)

were \( m_{v} S_{vl} |_{avL_{rBv}} \) is the parameters of \( P_{r}(v_k; S_{v,k}, m_{v,k}) \). Then, the estimate \( a_{v} \hat{x} \) of the average objective sound for the same duration can be calculated as following simple equation:

\[ a_{v} \hat{x} = a_{v} y - a_{v} \hat{v} = a_{v} y - m_{v} S_{vl} |_{avL_{rBv}} . \] (16)

Further, let the estimated average power spectrum of objective sound, that of wind noise and that of observation for the adequate time be \( a_{v} \hat{S}_x(f) \), \( a_{v} \hat{S}_v(f) |_{avL_{rBv}} \) and \( a_{v} S_y(f) \) respectively. Here, we use the conditional average\(^{17} \) of the power spectrum of the wind noise on knowing \( L_{rBv} \). Then, as in the same way in the reference,\(^{17} \) the average power spectrum of objective sound can be estimated successively as:

\[ a_{v} \hat{S}_x(f) = a_{v} S_y(f) - a_{v} \hat{S}_v(f) |_{avL_{rBv}} . \] (17)

**EXPERIMENTS AND CONCLUSIONS**

An experimental result of the relative frequency distribution of the wind noise conditioned by \( L_{rBv} \) is shown in Fig. 1. The calculated parameters by approximating the frequency distribution in Fig. 1 to Gamma distribution with their least-squares fit of parabolas are shown in Fig. 2. Now, we can approximately express the c.d.f. of the wind noise as shown in Fig. 3 by estimating these parameters on knowing \( L_{rBv} \) in Fig. 2,

Only the estimated results of the objective sound under the wind noise by using two kinds of simplified estimation methods are shown here in Figs. 4 and 5. The estimated average levels of the objective sound for 50 min. in these two methods are not so good by comparing with the long time averages estimated in the previous two static methods.\(^{16,17} \)
Fig. 1
Relative frequency distribution of the wind noise in the mean square sound pressure conditioned by the reference BPL. The wind speed varied between about 0.8 to 10.6 m/s around its average of about 3.4 m/s.

Fig. 2
Parameters of Gamma distribution by calculating the frequency distribution in Fig. 1 with their least-squares fit of parabolas as a function of the reference BPL.

Fig. 3
Approximate expression of the c.d.f. of wind noise by using Gamma distribution as a function of the reference BPL.
The result suggests the difficulty of the successive estimation of the objective sound in the variable of the mean square sound pressure which distribute in non-negative area.

The issues to derive more simplified estimation equations on the dynamic estimation method and to find out more accurate simplified methods are rest to study.

REFERENCES


