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A PRECISE ESTIMATION METHOD MATCHED TO DIGITAL LEVEL MEASUREMENT FOR THE PROBABILITY DISTRIBUTION OF SOUND, LIGHT AND ELECTROMAGNETIC WAVE FLUCTUATIONS LEAKED FROM VDT

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ABSTRACT

In an actual measurement of random fluctuations, e.g., sound, light and electromagnetic (abbr., EM) fluctuation waves, the measured data are very often processed in a level-quantized form at discrete time intervals. In this paper, a unified explicit expression for the probability function with level-quantized random variables for an arbitrary random fluctuation wave is theoretically derived in a general form of statistical orthogonal expansion series. Next, based on this general theory, a new procedure for estimating the precise level distribution on the basis of information on the moment statistics calculated especially from roughly quantized level data is proposed. The effectiveness of the proposed theoretical method is experimentally confirmed by applying it to the actual sound, light and EM fluctuation waves measured in front of VDT.

1. INTRODUCTION

When the statistical evaluation and/or prediction problems in the sound, light and EM environments are discussed, especially from the methodological viewpoint, the following points are essentially important :

(i) In the above statistical problems, since there still remain many present-day problems not only from the viewpoint of the effect on instruments but also from that of vital effect, it is necessary to find the accurate measurement method [1-4].

(ii) These fluctuations which we encounter in our daily life exhibit various types of probability distribution forms, apart from a standard Gaussian distribution, due to the diversified natural, social and/or human causes of fluctuation.

(iii) In the actual observation, the measured data have become to be very often processed in a quantized form at discrete time intervals. This is because various statistical evaluations and extraction of the lower and/or higher order statistical informations (e.g., mean, variance, median, higher order moments, 90% range, etc., sometimes a quasi-peak value) become easier by use of a digital computer.

(iv) In this case, it increases in importance, especially from a methodological viewpoint not only in a usual frequency domain but also especially in a time domain, to establish a unified statistical treatment for evaluating the probability distribution function of level-quantized state variables, and to find some simplified signal processing method for the reduction of computational complexity.

Based on these practical points of view, in this paper, a unified explicit expression for the probability function with level-quantized random variables for an arbitrary random fluctuation is theoretically derived in the general form of a statistical orthogonal expansion series and of unifying analogue level data processing. Here, a probability function which can be arbitrarily and artificially chosen in advance based on the operational viewpoint is taken into the first expansion term, and the effect of the statistical property on the whole fluctuation is reflected in the second and higher order terms in the expansion expression. Next, based on this general theory, a new procedure for estimating the precise level distribution on the basis of information on the moment statistics calculated from the roughly quantized level data is proposed. The effectiveness of the proposed theoretical method is experimentally confirmed by applying it to the actual sound, light and EM fluctuation waves measured in front of VDT.

2. THEORETICAL CONSIDERATION

Let $z(t)$ be an instantaneous value at an arbitrary sampling time t for the sound, light or EM fluctuation waves with an arbitrary probability function form. Also, let z_i be the ith quantized level value of $z(t)$ at a time t fluctuating over the amplitude range $\left[z_{M_0}, z_{N_0}\right]$ with a quantized level difference interval h.

The probability function $P(z)$ for a level-quantized random variable z can be generally expressed in the statistical orthogonal expansion form as follows :

$$P(z) = P_0(z)\sum_{n=0}^{\infty} A_n \phi_n(z), \quad \left[A_0 = 1,\ \phi_0(z) = 1\right], \tag{1}$$

where $P_0(z)$ is a probability function for the quantized random variable with a quantized level difference interval h, which can be artificially or arbitrarily established in advance from an operational viewpoint [Accordingly, the well-known standard type probability distributions like the binomial or Poisson types [5] can be very often employed as $P_0(z)$]. Here, $\{\phi_n(z)\}$ is a set of orthonormal polynomial functions satisfying the relationship :

$$\sum_{i=M_0}^{N_0} P_0(z_i)\phi_n(z_i)\phi_m(z_i) = \delta_{mn} \tag{2}$$

with

$$z_{N_0} = N_0 h, \quad z_{M_0} = M_0 h. \tag{3}$$

At this time, the expansion coefficient A_n $(n = 0,1,2,\cdots)$ can be given as :

$$A_n = \langle \phi_n(z) \rangle , \tag{4}$$

where $\langle * \rangle$ denotes an averaging operation with respect to the random variable *. The orthonormal polynomial of the nth order $\phi_n(z)$ can be determined by use of the pre-established probability function $P_0(z)$ as follows :

$$\phi_n(z) = [1/P_0(z)]\nabla^n \{P_0(z)V_n(z)\} . \tag{5}$$

Hereupon, $V_n(z)$ is determined so that $\phi_n(z)$ satisfies an orthonormal condition [Eq.(2)], once after the specific form of the probability function $P_0(z)$ is selected [6]. And, ∇ denotes the following backward difference operator [7] :

$$\nabla f(z) \equiv (1/d)\{f(z) - f(z-h)\} \tag{6}$$

with two arbitrary constants, d and h. In Eq.(6), the constant d is introduced for the purpose of unifying both the discrete level type expression (d=1) and the continuous level type one ($d=h \to 0$). After substituting Eq.(5) into Eq.(2) and using the well-known formula (see Appendix) :

$$f(z)\nabla g(z) = \nabla\{f(z)g(z)\} - \nabla f(z) \cdot g(z-h), \tag{7}$$

we easily obtain :

$$\sum_{i=M_0}^{N_0} \nabla^n [P_0(z_i)V_n(z_i)]\phi_m(z_i)$$
$$= \frac{1}{d}\left\{\nabla^{n-1}[P_0(z_i)V_n(z_i)]\phi_m(z_i)\right\}\Big|_{i=M_0-1}^{N_0} - \sum_{i=M_0}^{N_0} \nabla^{n-1}[P_0(z_i-h)V_n(z_i-h)\nabla\phi_m(z_i)]. \tag{8}$$

By applying Eq.(7) repeatedly (n-1) times to the second term of the above expression, we can find the following relationship :

$$\sum_{i=M_0}^{N_0} \nabla^n [P_0(z_i)V_n(z_i)]\phi_m(z_i) = \sum_{j=0}^{n}(-1)^j \frac{1}{d}\left\{\nabla^{n-1-j}[P_0(z_i - jh)V_n(z_i - jh)]\nabla^j \phi_m(z_i)\right\}\Big|_{i=M_0-1}^{N_0} = 0 \quad (\text{for } m < n) \tag{9}$$

and

$$(-1)^n \sum_{i=M_0}^{N_0} P_0(z_i - nh)V_n(z_i - nh)\, \nabla^n \phi_m(z_i) = 1 \quad (\text{for } m = n). \tag{10}$$

From Eq.(9), we find that $V_n(z)$ must satisfy the following relationship :

$$P_0(z_{M_0} - jh)V_n(z_{M_0} - jh) = 0 \quad (j = 1, 2, \cdots, n) \tag{11}$$

and

$$P_0(z_{N_0} - jh)V_n(z_{N_0} - jh) = 0 \quad (j = 0, 1, 2, \cdots, n-1). \tag{12}$$

On the other hand, from the definition of differences and polynomial, the following difference equation must be satisfied :

$$\nabla^{n+1}\phi_n(z) = 0, \tag{13}$$

since $\phi_n(z)$ is the nth order polynomial with respect to z. Here, Eq.(10) is the normalization condition for $\phi_n(z)$ and $V_n(z)$ satisfying Eqs.(11) and (12), and can be used as the boundary condition for Eq.(13).

As a specific example of $P(z)$ in Eq.(1), the generalized binomial distribution especially with a quantized level difference interval h can be chosen as a basic probability function $P_0(z)$ as follows :

$$P_0(z) = B(z; N, p, h) \equiv \binom{N/h}{z/h} p^{z/h}(1-p)^{(N-z)/h} \tag{14}$$

with

$$p = \langle z \rangle / h, \quad z_{M_0} = 0 \quad \text{and} \quad N = z_{N_0} = N_0 h. \tag{15}$$

The functional form of $V_n(z)$ in Eq.(5) can be determined by use of the boundary conditions shown in Eqs.(11) and (12), as follows :

$$V_n(z) = C_n(-1)^n (N-z)^{(n)}, \tag{16}$$

where $(N-z)^{(n)}$ denotes the nth order factorial function defined as :

$$x^{(n)} \equiv x(x-h)\cdots[x-(n-1)h] \quad (n \geq 1) \tag{17}$$

and

$$x^{(0)} \equiv 1. \tag{18}$$

Also, the above coefficient C_n is determined in order to satisfy the normalized condition shown in Eq.(10), as follows :

$$C_n = \frac{d^n}{h^n} \frac{1}{\left(N_0^{(n)} n!\right)^{1/2}} \left(\frac{p}{1-p}\right)^{n/2}. \tag{19}$$

Thus, substituting Eqs.(14), (16) and (19) into Eq.(5), we can derive the following expression of $\phi_n(z)$ as :

$$\phi_n(z)=\frac{1}{h^n\left(N_0^{(n)}n!\right)^{1/2}}\left(\frac{1-p}{p}\right)^{n/2}\sum_{i=0}^{n}\binom{n}{i}(-1)^{n-i}\left(\frac{p}{1-p}\right)^{n-i}(N-z)^{(n-i)}z^{(i)}. \tag{20}$$

Upon averaging $\phi_n(z)$ over the whole range of z, each expansion coefficient A_n can be accurately calculated according to Eq.(4). Thus, the probability function $P(z)$ can be explicitly expressed in the form of Eq.(1) with use of Eq.(20).

We are aware of the fact that the usual moment statistics are fairly stable quantities supported by the averaging operation over a large number of data even for the roughly observed level data. The small value of quantized level difference interval h in the resultant expression of the probability function $P(z)$ can be changed and be appropriately determined, according to the research objective of our estimation problem on a precise measurement. The main purpose of this report is to estimate a precise level distribution form with this small value of level difference interval h, especially based on the roughly quantized instantaneous values with large values of the quantized level difference interval h' $(h' > h)$.

3. EXPERIMENTAL CONSIDERATION

To confirm the effectiveness of the proposed theory, we have applied it to the sound level data, the illumination data and the EM data measured in front of VDT. Figure 1 shows the block diagram of experimental arrangement. The sound level fluctuation has been measured by using a precision sound level meter. The electric field has been measured by using Holaday's VLF radiation survey meter with true rms. detector. The illuminance has been measured by using a illumination meter. The total number of the measured data is respectively 500. The quantized level difference interval of the sound level is 1 [dB]. The roughly quantized level data has been purposely obtained in advance by setting the quantized level difference interval to 3 [dB]. On the other hand, the quantized level difference interval of illumination data is 0.5 [lux]. The roughly quantized level data has been obtained in advance by setting the quantized level difference interval to 1 [lux]. Moreover, the quantized level difference interval of the electric field data is 0.1 $[V/m]^2$. The roughly quantized level data has been obtained in advance by setting the quantized level difference interval to 0.5 $[V/m]^2$.

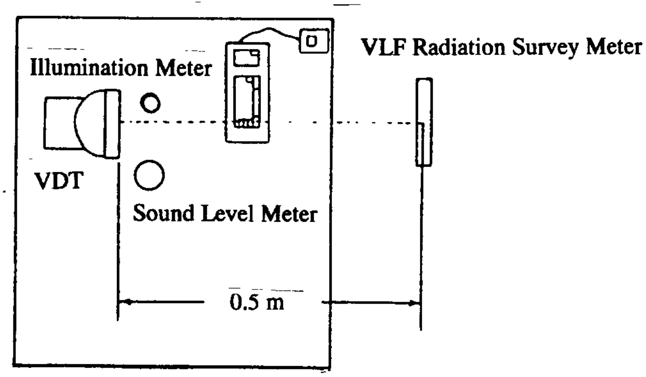

Fig. 1 Block diagram of experimental arrangement.

Figure 2 shows a comparison between the theoretical values by using the proposed unified probabilistic expression and the experimental curve of the cumulative probability distribution for the roughly observed sound level fluctuation data. From this figure, it is obvious that the successive addition of higher order expansion terms moves the theoretical values closer to the experimental curve. In addition, by using the proposed precise estimation method based on the derived probabilistic expression of the unified type, the estimated results have been obtained. Figure 3 shows the estimated results for this case. From this figure, the theoretical values are in satisfactory agreement with the experimental curve.

Figure 4 shows a comparison between the theoretical values by using the unified probabilistic expression and the experimental curve of the cumulative probability distribution for the roughly observed EM fluctuation data. From this figure, it is obvious that the successive addition of higher order expansion terms moves the theoretical values closer to the experimental curve. Figure 5 shows the estimated results by using the proposed precise estimation method. That is, from these figures, it is obvious that the successive addition of higher order expansion terms moves the theoretical values closer to the experimental curve. So, the effectiveness of the proposed precise estimation method has been experimentally confirmed.

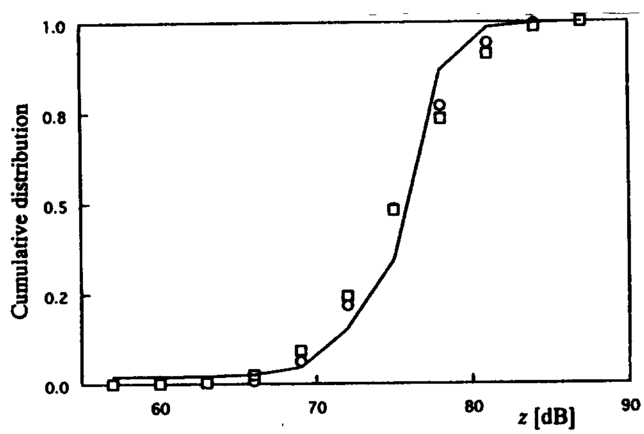

Fig.2 A comparison between the theoretical values by use of the unified probability expression and the experimental curve of cumulative distribution for the roughly observed sound level data. ——— : experimental curve. Theoretical values : □ first term ; ○ third approximation.

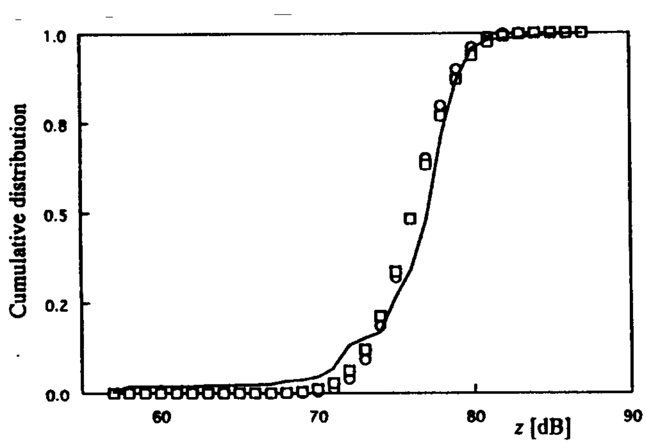

Fig.3 A comparison between the theoretical values by use of the proposed precise estimation method and the experimental curve of cumulative distribution for the sound level data. ——— : experimental curve. Theoretical values : □ first term ; ○ third approximation.

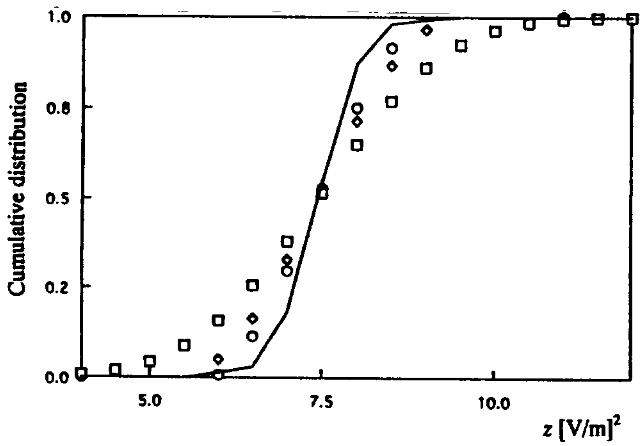

Fig.4 A comparison between the theoretical values by use of the unified probability expression and the experimental curve of cumulative distribution for the roughly observed EM data. ——— : experimental curve. Theoretical values : □ first term ; ◇ first approximation ; ○ second approximation.

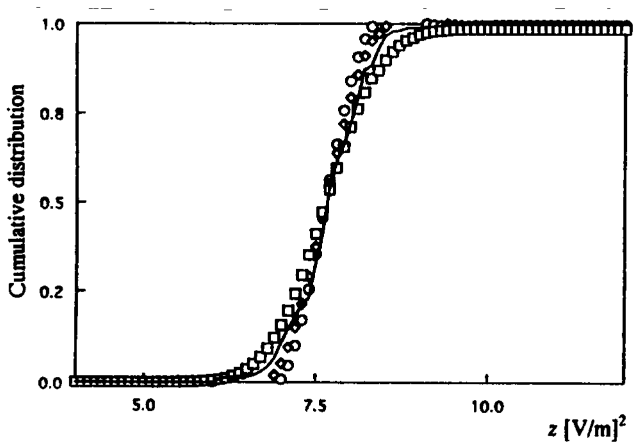

Fig.5 A comparison between the theoretical curves by use of the proposed precise estimation method and the experimental curve of cumulative distribution for the EM data. ——— : experimental curve. Theoretical values : □ first term ; ◇ first approximation ; ○ second approximation.

4. CONCLUSION

The main point of this study is to establish a new precise estimation method of the probability function form for the sound, light and EM fluctuation waves, especially based on roughly quantized level data from a methodological viewpoint.

First, a unified expression of the probability distribution function with a quantized level difference interval has been derived in the form of a statistical orthonormal series expansion, especially in a style of unifying the analogue level data. Next, a new estimation method of the precise distribution based on the roughly observed data with a large value of the quantized level difference interval has been established by using the above general theory.

Finally, the effectiveness of the proposed method has been experimentally confirmed by applying it to the actual sound, light and EM fluctuation waves measured in front of VDT.

The theoretically estimated results have been in good agreement with the actually observed data.

Since the proposed method is at an early stage of study, there still remain many future researches. For example, in order to confirm further practical effectiveness, the proposed method must be applied to the other wave motion environment. Moreover, it is also necessary to find the reasonable method for selecting an optimum order of theoretical expansion expression.

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APPENDIX : PROOF OF EQ.(7)

By using Eq.(6), we can easily obtain :

$$\begin{aligned}\nabla\{f(z)g(z)\} &= (1/d)\{f(z)g(z) - f(z-h)g(z-h)\} \\ &= (1/d)\left[\{f(z)g(z) - f(z)g(z-h)\} + \{f(z)g(z-h) - f(z-h)g(z-h)\}\right] \\ &= f(z)(1/d)\{g(z) - g(z-h)\} + (1/d)\{f(z) - f(z-h)\}g(z-h) \\ &= f(z)\nabla g(z) + \nabla f(z)\cdot g(z-h). \qquad \text{(A1)}\end{aligned}$$

Thus, Eq.(7) can be directly derived from the above equation.