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BIAS ERRORS IN MEASUREMENT OF VIBRATORY POWER AND IMPLICATION FOR ACTIVE CONTROL OF STRUCTURAL VIBRATION

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ABSTRACT

Uncertainties in power measurements performed with piezoelectric accelerometers and force transducers are investigated. It is shown that the inherent structural damping of the transducers is responsible for a bias phase error, which typically is in the order of one degree. Fortunately, such bias errors can be largely compensated for by an absolute calibration of the transducers and inverse filtering that results in very small residual errors. Experimental results of this study indicate that these uncertainties will be in the order of one percent with respect to amplitude and two tenth of a degree for the phase. This implies that input power at a single point can be measured to within one dB in practical structures which possesses some damping. The uncertainty is increased, however, when sums of measured power contributions from more sources are to be minimised, as is the case in active control of vibratory power transmission into structures. This is demonstrated by computer simulations using a theoretical model of a beam structure which is driven by one primary source and two control sources. These simulations reveal the influence of residual errors on power measurements, and the limitations imposed in active control of structural vibration based upon a strategy of power minimisation.

1. INTRODUCTION

Measurement of structural power and power related quantities such as input mobilities and impedances are required in many areas of structural acoustics. These include the measurements of vibratory source power of machinery [1,2], structural intensity in components [3], structural damping by power injection methods [4], and most recently, the minimisation of structureborne sound power by adaptive active control [5-9]. Measurements are mostly performed with piezoelectric transducers, because they have very fine amplitude and phase responses which only deviate slightly from the true values. Such small deviations are of little importance in most vibration measurements, but they cannot be ignored in measurements of vibratory power input where the instrumentation requirements are very strict, especially with respect to the phase accuracy of the transducers. Fortunately, such bias errors can be largely compensated for by broadband calibration of the transducers and inverse filtering; this will improve the measurement accuracy of the mentioned quantities. Based upon simple models this paper gives a brief

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account of the probable causes to the bias errors and uncertainties of measurements performed with piezoelectric accelerometers and force transducers. The influence of preamplifiers and filters are not specifically considered herein, but this can be found elsewhere, eg in [10].

2. MEASUREMENT PRINCIPLES

The power fed into a structure by a point force $F(t)$ that generates a co-linear velocity $v(t)$ at the same point is given by their time averaged product. When the corresponding acceleration $a(t)$ is measured the power input can be determined from [11]

$$P = \overline{F(t)\int_{-\infty}^{\tau} a(\tau)\,\mathrm{d}\tau} = \frac{-1}{2\pi}\int_{f_1}^{f_2} \mathrm{Im}\{G_{Fa}(f)/f\}\mathrm{d}f \,, \tag{1}$$

where the overbar indicates time average of band-filtered signals; in the second, alternative expression $G_{Fa}(f)$ is the one-sided cross-spectral density of the force and acceleration signals as determined by a dual channel FFT analyser, and $\mathrm{Im}\{\cdot\}$ means the imaginary part.

Fig. 1a shows an arrangement suitable for 'power calibration' of a transducer set using a known mass, and Fig. 1b shows a typical situation for measuring input power to a structure.

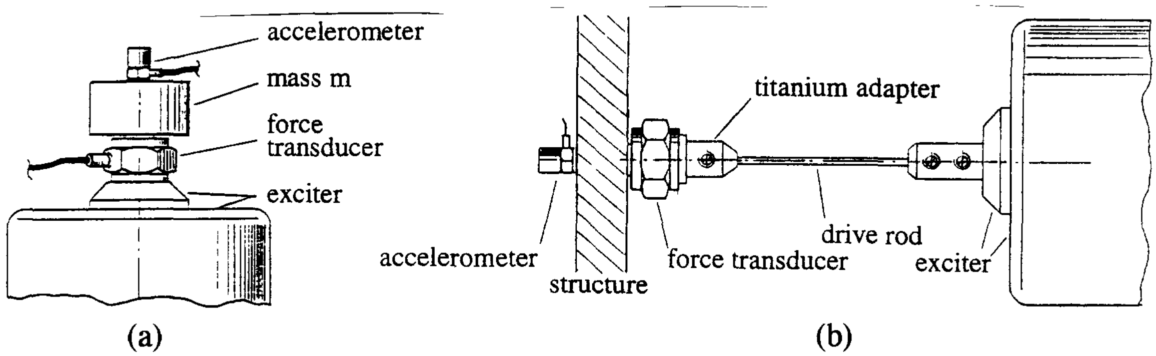

Figure 1. (a) Arrangement for calibration of an accelerometer-force transducer set, (b) measurement of input power to a structure.

Before addressing the accuracy in measurements of power we shall determine the bias errors in measurements of acceleration and force which utilise the piezoelectric transducer principle.

3. ACCELEROMETERS

An accelerometer is characterised by its blocked natural frequency $f_0 = (1/2\pi)(s/m)^{1/2}$, where m is the internal seismic mass and s is the stiffness of the piezoelectric transducer element and associated spring arrangements. The electric charge output Q_a from an accelerometer is governed by the deformation q_a of the piezoelectric element, as $Q_a = -K_a q_a$, where K_a is the transducer constant which depends upon the material properties of the piezoelectric element and the design and size of the accelerometer [10,12]. Moreover, this electric charge signal Q_a is proportional to the acceleration a at the mounting base of the accelerometer, that is, the quantity which is to be measured. Assuming harmonic motion at frequency f, the complex amplitude ratio $H_a(f) = Q_a/a$ thus defines the usable frequency range of the accelerometer and the expected deviations from the exact values in terms of amplitude and phase ϕ_a. This yields

$$H_a(f) = \frac{Q_a}{a} = \frac{-K_a}{(2\pi f_0)^2\left[\left(1-(f/f_0)^2\right)^2 + \eta_a^2\right]^{1/2}}\exp(\mathrm{i}\,\phi_a)\,, \quad \text{where} \quad \phi_a = \arctan\frac{-\eta_a}{1-(f/f_0)^2}\,, \tag{2}$$

where the damping in elastic elements of the accelerometer and friction at internal interfaces is accounted for by assuming a frequency-independent complex stiffness $s = s(1+\mathrm{i}\eta_a)$, in which η_a is the damping loss factor. At frequencies well below the blocked natural frequency f_0 of the accelerometer, that is, for $f < 0.1 f_0$, equations (2) reduce to

$$H_a(f) = \frac{Q_a}{a} \approx \frac{-K_a}{(2\pi f_0)^2}\exp(\mathrm{i}\,\phi_a)\,, \quad \text{where} \quad \phi_a \approx \arctan(-\eta_a) \approx -\eta_a\,, \tag{3}$$

which show that the amplitude is independent of the frequency when K_a is constant, and that there is a small constant phase lag caused by the damping. This may be deduced from the calibration chart curve by the amplification at resonance. For accelerometers (B&K types 4344 and 4375) with typical damping values of η_a=0.02, this corresponds to an absolute phase error of -1.15 deg. This model has been verified by broadband, back-to-back calibration using a quartz type reference accelerometer, type 8305, as shown in Fig. 2a. Examples of such measurements are given in Fig. 3 which shows relative phase errors of typically -0.7 deg. Since the absolute phase error of the reference accelerometer later on is shown to be -0.5 deg. this yields an absolute error of -1.2 deg. for the tested accelerometers. The excessive phase error occuring below 50 Hz is caused by the necessary clamping of signal leads which otherwise corrupts the sensitive measurements.

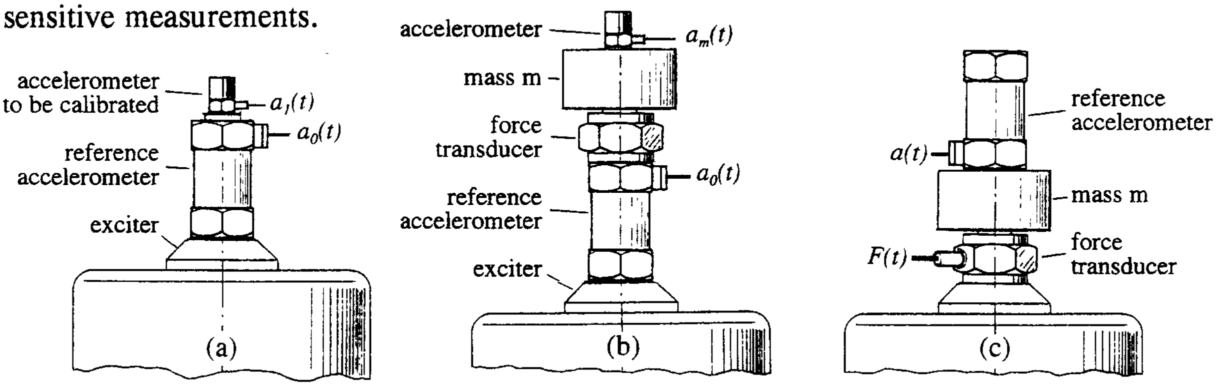

Figure 2. (a) Back-to-back calibration with a reference accelerometer, B&K type 8305; (b) testing of force transducer damping; (c) comparison test of reference accelerometer.

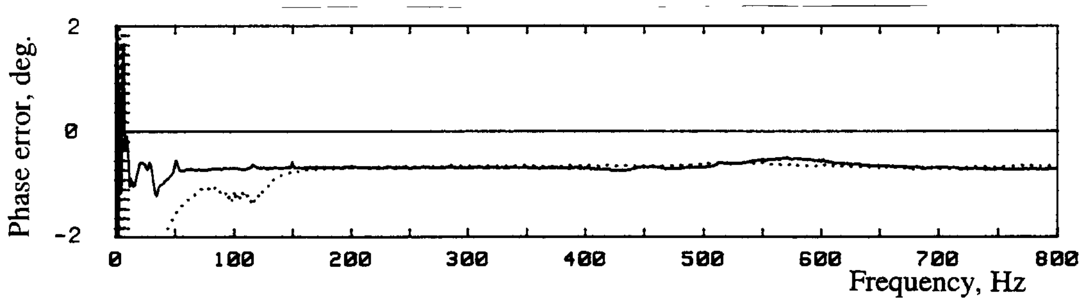

Figure 3. Phase spectra $\phi_{01}(f)$ of small accelerometers relative to a reference accelerometer, B&K type 8305, for :——— type 4375; ········ type 4344 .

The damping of a specific type of accelerometer may vary from one sensor to another due to production tolerances etc. Such variations and associated phase deviations are normally not available to the user, but a variation in the damping of about ±10% is very likely. This may explain the small, frequency-independent phase differences, which were found from tests and comparison of phase spectra of a number of nominally identical 3 grams accelerometers (B&K type 4375). These were mounted side-by-side in pairs on a vibration exciter (B&K type 4810), with one accelerometer of each pair being used as reference. The results of such measurements for three pairs of accelerometers showed phase differences with mean values of approximately 0.25 deg. [13]. These findings are in agreement with recent information provided by Brüel & Kjær [14], albeit for the more damped type of DeltaTron-accelerometers. At 160 Hz, the results from small samples of three different sizes of accelerometers showed phase deviations of typically -1.2±0.1 deg. relative to that of a reference accelerometer, B&K type 8305.

4. FORCE TRANSDUCERS

Similarly to the accelerometer the electrical charge output Q_F from a piezoelectric force transducer is governed by the deformation q_F of the transducer element as $Q_F = -K_F\, q_F$, where K_F is the transducer constant of the force transducer. Furthermore, since the force F over the piezoelectric element is proportional to the deformation and stiffness s_F of the element, the complex amplitude ratio $H_F(f) = Q_F/F$ thus becomes

$$H_F(f) = \frac{Q_F}{F} = \frac{-K_F}{s_F(1+\eta_F^2)^{1/2}} \exp(\mathrm{i}\,\phi_F)\,, \quad \text{where} \quad \phi_F = \arctan(-\eta_F) \approx -\eta_F\,, \tag{4}$$

where the characteristic of the piezoelectric force element is also represented by a complex stiffness $s_F = s_F(1+\mathrm{i}\eta_F)$, with η_F being the damping loss factor. This can be measured with the arrangement shown in Fig. 2b, since it is easily shown that the resonant amplification of the acceleration ratio $|a_m/a_0|$ across the crystal and mass is equal to $(1+\eta_F/2)/\eta_F \simeq 1/\eta_F$ for such a base excited 'oscillator'. With a quartz-type force transducer (B&K type 8200) a resonant amplification of 40.8 dB at 8429 Hz was measured with a total mass loading of 0.126 kg. This yields $s_F = 3.52\cdot10^8$ N/m and $\eta_F = 0.0091$, which corresponds to a phase deviation from the true value of -0.52 deg. This is in close agreement with the typical amplification of 40 dB (-0.57 deg.) measured by the manufacturer [14].

In eq.(4) it is assumed that the contact mass (ie, top mass) of the force transducer has a negligible influence, because it is usually chosen to be small in comparison with the local mass of the structural drive point in question. This may be compensated for, though, either by an electric circuit of mass cancellation [15] or by post-processing of the data.

5. DUAL SENSOR MEASUREMENTS

Dual channel measurements that employ both types of transducers are used for measuring mobilities and power input to structures. The bias errors of amplitude and phase inherent to such measurements can be quantified simply by combining the above relations for the individual transducers. In the case of measurements of mobility - or preferably the accelerance $A(f) = a/F$ ($\equiv G_{Fa}/G_{FF}$ for broad-band excitation) - this simplifies to $H_A(f) = Q_A/A$:

$$H_A(f) = \frac{Q_A}{A} \approx \frac{K_a\, s_F}{K_F\,(2\pi f_0)^2} \exp(\mathrm{i}\,\phi_A)\,, \quad \text{where} \quad \phi_A \approx \arctan(\eta_F - \eta_a) \approx \eta_F - \eta_a\,, \tag{5}$$

which apply for frequencies $f < 0.1 f_0$, and where $Q_A = Q_a/Q_F$ is the equivalent charge output corresponding to measured accelerance. This reveals that the amplitude deviation is negligible and that there is a small phase error corresponding to the difference between loss factors of the force transducer and accelerometer.

Having established above the damping and phase of the force transducer, eq. (5) may then be used for determining the unknown phase of the employed reference accelerometer, B&K 8305, by using the arrangement in Fig. 2c, again with charge preamplifiers (B&K 2635). The results in Fig. 4 show that the phase difference between the two transducers is very close to zero, with a value of 0.03 deg. in the first experiment, and 0.07 deg. with preamplifiers interchanged. Thus, the mean value determines the damping of the reference accelerometer to $\eta_{a,ref} = 0.0086$, and its absolute phase deviation to -0.49 deg. (The observed increase in phase at low frequencies is again caused by the clamping of signal leads, and the peak at 230 Hz is caused by residual angular motion due to imperfection in the exciter and test arrangement.)

The phase error ϕ_A with standard accelerometers is expected to be negative, because the loss factor of a quartz-type force transducer typically is smaller than that of a standard accelerometer, which uses a piezoelectric ceramic like lead zirconate titanate. This is also what has been found from absolute calibration measurements of Q_A/A using a known mass in the arran-

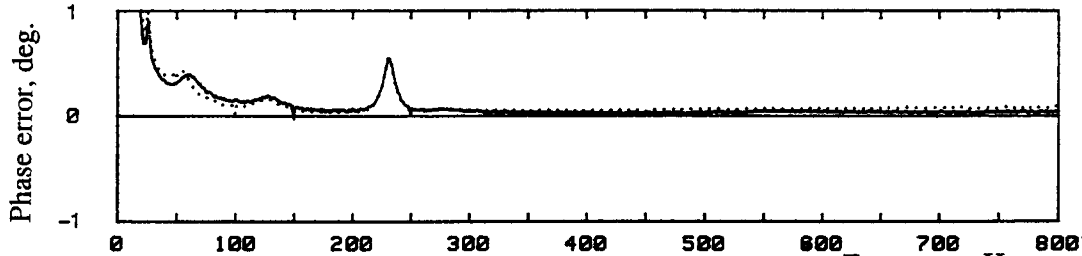

Figure 4. Phase spectra $\phi_A(f)$ of a reference accelerometer and a force transducer: ——,first measurement; ·········, preamplifiers interchanged.

gement shown in Fig. 1a. A typical result of the phase spectra obtained from these calibrations is given in Fig. 5 for a transducer set comprising a compression type accelerometer (B&K 4344) and a force transducer (B&K 8200), both using B&K 2635 preamplifiers. Phase errors found in this way are virtually independent of the frequency with values of typically -0.6 deg. This is also expected from eq. (4) using the predicted, absolute phase deviation of the accelerometer (-1.15 deg.) and the measured absolute phase of the force transducer (-0.52 deg.). Measurements of three transducer sets actually showed asymptotic values of: -0.5, -0.6 and -0.7 deg.

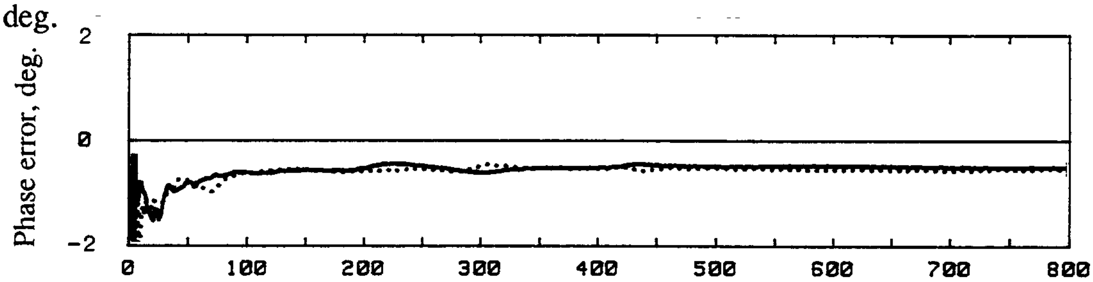

Figure 5. Phase spectra $\phi_A(f)$ of an accelerometer-force transducer set for power measurements.

5.1 Residual errors

The spectrum obtained by an absolute calibration which employ an ideal mass, as in Fig. 5, may readily be used as an inverse compensation-filter in order to improve the accuracy of dual channel measurements. When such a correction is made only small residual errors will remain, which reflect the uncertainty of amplitude and phase with respect to the calibration itself and the differences in mounting condition of transducers when these are attached to a structure. Let these errors be written explicitly for the force and acceleration measurements as relative amplitude errors δ_F and δ_a , respectively, and let the actual phase between the acceleration and force be designated θ with a residual phase error of Δ . By using these quantities one can determine the expected error on measurements of power input at a single point. The true power input from a harmonic point force is given in its time-average form by $P = (-1/4\pi f)\cdot \mathrm{Im}\{Fa^*\} = (1/4\pi f)\ |F|\,|a| \sin\theta$. This means that the measured power estimate P' becomes

$$P' = (1/4\pi f)\ |F|\,|a|(1+\delta_F)(1+\delta_a)\sin(\theta+\Delta), \tag{6}$$

and this yields $P'/P = (1+\delta_F)(1+\delta_a)(\cos\Delta + \cot\theta\ \sin\Delta)$. By neglecting second order terms the normalised error ϵ then becomes

$$\epsilon = P'/P - 1 = \delta_F + \delta_a + \Delta\cot\theta\ . \tag{7}$$

It is evident that this error becomes excessively large at frequencies where the phase θ between acceleration and force approaches a value of either zero or 180 deg., provided that $\Delta \neq 0$. This occurs when the structure is driven off-resonance at frequencies where it is predominantly either spring-like or mass-like. In that case the error becomes $\epsilon \approx \Delta \cot\theta = -\Delta\, Q/P$, where Q is the reactive power, which is governed by the real part of the accelerance. On the other hand, at resonance where $\theta = 90$ deg. the estimation error is only influenced by the sum of amplitude errors, that is, by $\epsilon = \delta_F + \delta_a$.

Eq. (7) can be used for quantifying the limitations on the actual phase angle θ for a specified maximum error ϵ. Let it be assumed that residual amplitude errors δ_F, δ_a are $\pm 1\%$, see [16], the residual phase error Δ is ± 0.2 deg. and that the uncertainty on power measurement is required to be less than ± 1 dB, ie $-0.20 < \epsilon < 0.25$. Then this implies that the conditions $1 < \theta < 179$ deg. have to be satisfied for measurements at a single point.

5.2 Multiple inputs

Further limitations are imposed upon the phase angle θ when power inputs from more forces have to be added and minimised, as in the active control experiments in ref. [6]. For three input forces and associated power contributions the 99% confidence limits of the phase angels thus become approximately $3 < \theta < 177$ deg. when the three sets of residual errors are assumed to have magnitudes as above. Whether this can be satisfied or not depends upon the modal properties and damping of the structure. In the simulations that follow and in the experiments reported in [6,7] these conditions are certainly violated in several frequency bands. This results in uncertainties on the power estimate that exceed ± 1 dB and which also becomes negative occasionally, because the observed phases take values that are either less than zero or larger than 180 deg. due to contributions from the phase errors Δ_i.

6. COMPUTER SIMULATIONS OF MEASUREMENT ERRORS IN ACTIVE CONTROL

Although the residual errors are hard to quantify for a specific set of transducers, it is possible to calculate the measurement accuracy required in relation to active power control if a theoretical model is available. This is the case for the beam experiments reported in [6] for which a fairly accurate analytical model has been established [5,7]. (Alternative control strategies have also been examined by Gardonio and Elliott [8,9].) The system considered is a lightly damped, finite beam structure which is driven at an interior point by a time-harmonic disturbance force and by two control forces at neighbouring positions. Maximum reduction of the total input power is accomplished by determining optimum values of the control forces with respect the disturbance force, ie by optimum feed-forward control. These values are governed by matrix expressions of the corresponding point and transfer mobilities (or accelerances) [5].

An extended version of this computer model has been employed in investigating the influence of measurement errors on the performance of the power control system. It should be noted that in the presence of such errors, it is the contaminated quantities that guide the adaptive power minimising algorithm, and not the true value of total power supplied to the structure by the three forces in this example.

Typical results from these control simulations are presented in Fig. 6, which shows the true power determined both before and after control. The controlled residual power for the ideal situation of no errors is represented by the lower dashed-line curve. Also shown is the degrading effect of increasing magnitudes of combined phase and amplitude errors. It is found that phase errors of 0.2 deg. in addition to amplitude errors of two percent are acceptable, if it is required that the true total power input must not increase under control at any of the applied excitation frequencies between 50 and 500 Hz. However, from the level difference between the full-line curve and the 'peaky' dashed-line curves it is seen that the control system generally is performing quite well, despite of the small errors imposed. Apart from the low frequencies it is seen that power reductions of about 15 dB are achieved at most excitation frequencies. Thus, the requirements to phase accuracy is most critical at the low frequencies, and it is actually the resonant condition at 55 Hz that determines this magnitude of phase

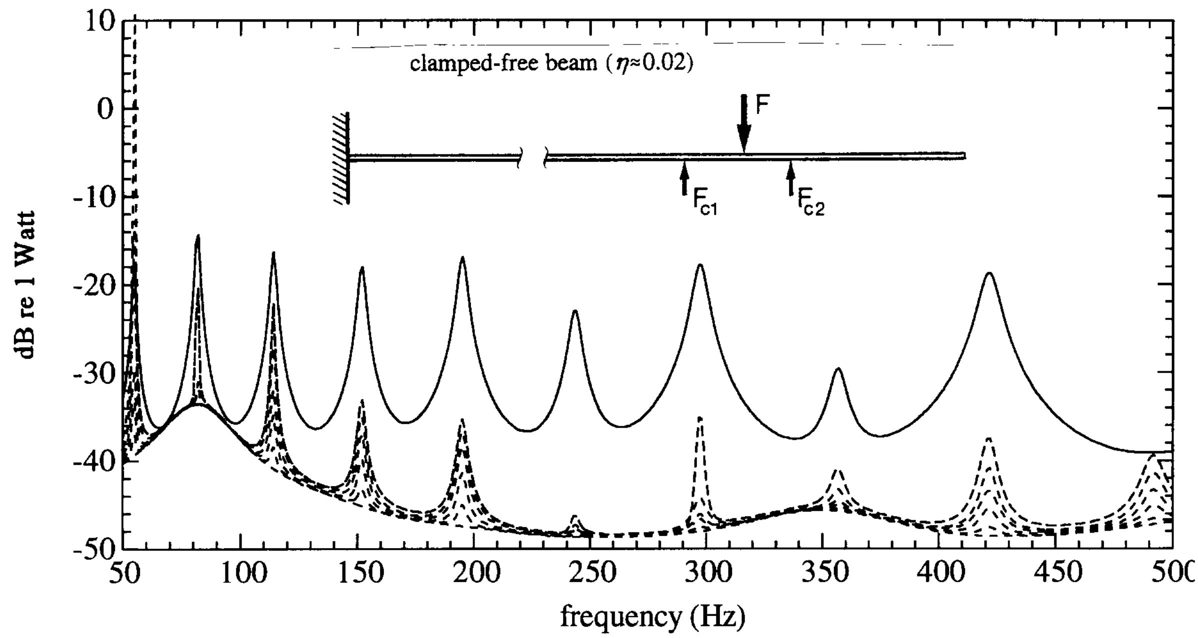

Figure 6. Calculations of total power input to a clamped-free beam: ——, without control; ------; with control, assuming combined phase errors and relative amplitude errors of magnitudes (0;0), (0.05;0.5), (0.1;1.0), (0.15;1.5), (0.2;2.0) and (0.25;2.5) in degrees and percent, respectively.

errors; at most other frequencies it was found that phase errors of up to 0.5 deg. could be tolerated in order to achieve 10 dB of power reduction.

Finally it is seen from Fig.6 that there is hardly any power reduction at 70 and 90 Hz. This is because these frequencies correspond to the quarter-wave-length node points of either of the two control forces, which are applied at positions almost symmetrically arranged with respect to that of the disturbance force.

7. CONCLUSION

A small bias error associated with the piezoelectric principle used in accelerometers and force transducers has been identified and quantified. By the use of simple models it is shown that the structural damping of these transducers is responsible for the phase error, which by far is the most serious one in measurements of vibratory power input and related quantities. From results of the limited number of accelerometers and force transducers tested herein these absolute phase errors are found to be independent of frequency and in the order of one degree and half a degree, respectively.

In dual channel measurements, however, it is not necessary to know the individual errors, since it is only the relative error between the measured acceleration (or velocity) and force that is important. For the examined sets of sensors and matched measurement chains the phase error has been found to be -0.6 ± 0.1 deg. Moreover, it is shown that this bias error can be compensated for both in amplitude and phase, at least to a first order, by performing a broadband calibration using an ideal mass and by inverse filtering. What then remain are minute residual errors, which reflect the uncertainty of the calibration itself and differences in transducer mounting when attached to a structure. Experimental results of this study indicates that these uncertainties will be in the order of ±0.2 deg. for the phase error and $\pm1\%$ for the amplitude error. If this accuracy is accomplished it is possible to measure input power to within ±1 dB in practical structures which possesses some damping, provided that transducers are well aligned and inertia effects are kept negligible by proper choice of transducer sizes and drive arrangement. The damping requirement implies that the true phase between the acceleration and drive force must be greater than one degree and less than 179 degrees.

Further limitations are imposed upon this phase if power contributions from more forces have to be added and minimised, like in the active control of structural vibration based on a power minimisation strategy. For a practical application, this means that use of passive damping treatment to the structure may be necessary when active power control is attempted. With a theoretical model of an active controlled beam structure, the calculations of control system performance and its degradation due to measurement errors also reveal that power control is very sensitive, especially to phase errors. From these simulations and the six-channel experiments reported in ref. [6,7] it is found that the power minimisation method cannot accept errors larger than 0.25 degree in phases and 2.5 percent in amplitudes. At low frequencies this is required solely to avoid that the total power input is <u>increased</u> by the control system. However, in the rest of the considered frequency range the control system generally works well with power reductions of about 15 dB.

Thus, the instrumentation requirements are very strict for active control based on a minimisation of measured power inputs. It seems that such requirements can be relaxed by using an alternative strategy that is based on a scalar minimisation of weighed sums of squared input forces and associated squared accelerations (or velocities), as suggested in ref. [8,9].

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