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VIBRATION SUPPRESSION THROUGH SMART DAMPING

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ABSTRACT

This work examines the use of piezoceramic devices, as well as viscoelastic elements in various configurations as damping treatments to suppress unwanted vibrations in structural elements such as satellite components, bladed disk assemblies and circuit boards. The combining of the use of piezoelectric materials with viscoelastic damping treatments is a form of hybrid damping referred to here as smart damping. Here we examine various configurations for combining viscoelastic and piezoceramic damping treatments in view of modeling viscoelastic behavior. The various designs and configurations available are based on trying to take advantage of the best properties of passive damping treatments and of active damping provided by smart structures' actuation. These materials allow the construction of unobtrusive sensors and actuators fully integrated into a structural system along with any viscoelastic material. One advantage of using piezoceramic elements combined with viscoelastic elements is the possibility of using the active component to compensate for the temperature and frequency dependence of a viscoelastic element rendering a treatment that is insensitive to these effects. Previous work in the area is presented which is followed by a discussion of modeling issues.

INTRODUCTION

Here several methods of adding damping to structures using piezoceramics and viscoelastic materials are described. Several configurations are possible for vibration suppression. These include passive constrained layer damping treatments, piezoceramic shunts, active control, active constrained layer damping treatments and hybrid combinations of these. Each of these are introduced here. This is followed by a treatment on modeling the viscoelastic effects using an internal variable method and model reduction. Each method has its strengths and weaknesses. All are able to effectively damp resonance.

The paper starts by reviewing the modeling definitions for viscoelastic materials and ends with more effective method of modeling viscoelastic and hysteretic behavior over a large frequency range. In addition, the literature in active constrained layer damping is reviewed.

PASSIVE DAMPING

The use of passive damping treatments forms a mature technology which is a well established means of reducing unwanted vibrations in structures (Johnson, 1995). The majority of applications employ add on, or designed in viscoelastic materials (VEM), however other materials are currently under consideration. Viscoelastic damping treatments have been modeled using the concept of a complex modulus and also in terms of modal strain energy. These models, while primitive and limited to the steady state response calculations, are well developed and allow the design of highly damped systems in applications ranging from noise control to fatigue. Here we review these techniques and define a new modeling approach which takes advantage to the internal variable approach for modeling hysterisis common to rubber like materials.

Materials used in passive damping treatments that exhibit a viscoelastic behavior are polymers, rubber, pressure sensitive adhesives, urethanes, epoxies and enamels. Adding these materials to a structure or material system improves the vibration response by

- reducing the resonant peak response
- reducing the settling time of the response
- reducing noise transmission
- reducing the rattle space required for isolation

Each of these effects are importanat considerations in designing a system. The first two are measures of performance that are manifested in the transient response region while the models commonly used for VEM are not compatable with the transient response. Also, the first two contradict each other in the sense that decreasing the settling time often increases the overshoot in the transient response. This is used to motivate the use of active control.

Damping is often measured and discussed using a terminology derived from various disciplines. These various characterizations are often used interchangably which is incorrect as they are only related at a resonant frequency in the steady state. Structural damping is usually denoted by g, quality or magnification Factor is denoted by Q, the loss factor denoted by η, the viscous damping ratio denoted by ζ, the critical damping factor denoted by c_{cr} and the logarithmic decay denoted by δ. The critical damping ratio is of course defined based on a single degree of freedom spring-mass-damper system described by

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = f(t) \tag{1}$$

Dividing through by the mass leads to the definition that $c_{cr} = 2m\omega_n$ where ω_n is the usual natural frequency (k/m). Then if the forcing function $f(t)$ is harmonic at ω_n the following hold

$$\zeta = c/c_{cr}$$

$$\eta = g = 2\zeta$$

$$Q = 1/\eta$$

$$\delta = 2\pi\zeta = ln(x(t)/x(t+T)), \text{ where } T \text{ is a period of oscillation}$$

The loss factor is also defined from measurements in the steady state as (2π times) the ratio of the energy lost per cycle to the maximum stored energy. The damping ratio is measured from either looking at the free decay and computing δ from measurements of the response separated by a period, or by looking at the magnitude plot from a swept sine test and measuring the width of the peak around resonance (Q).

The loss factor is often associated with a complex modulus approach to modeling energy dissipation. This description results from assuming a solution to equation (1) of the form $X\exp(J\tilde{\omega}_n t)$ where j is the square root of negative one and X is the amplitude of the steady state response. The damping term in equation (1) then becomes $jc\omega_n X$ which is combined with the stiffness term (kX) to produce a displacement coefficient of $k(1 + j\eta)$ called the complex stiffness or in terms of the modulus, G, the complex modulus: $G* = G(1 + j\eta)$.

Typically, viscoelastic materials have low shear modulus, but high loss factors ($\eta = 1.8$) and hence dissipate a lot of energy (in the form of heat). The complex modulus description is frequency dependent because measurements of η at various frequencies yield different values. Likewise, the loss factor is also temperature dependent decreasing with increasing temperature. An example of these dependencies is given in figure 1.

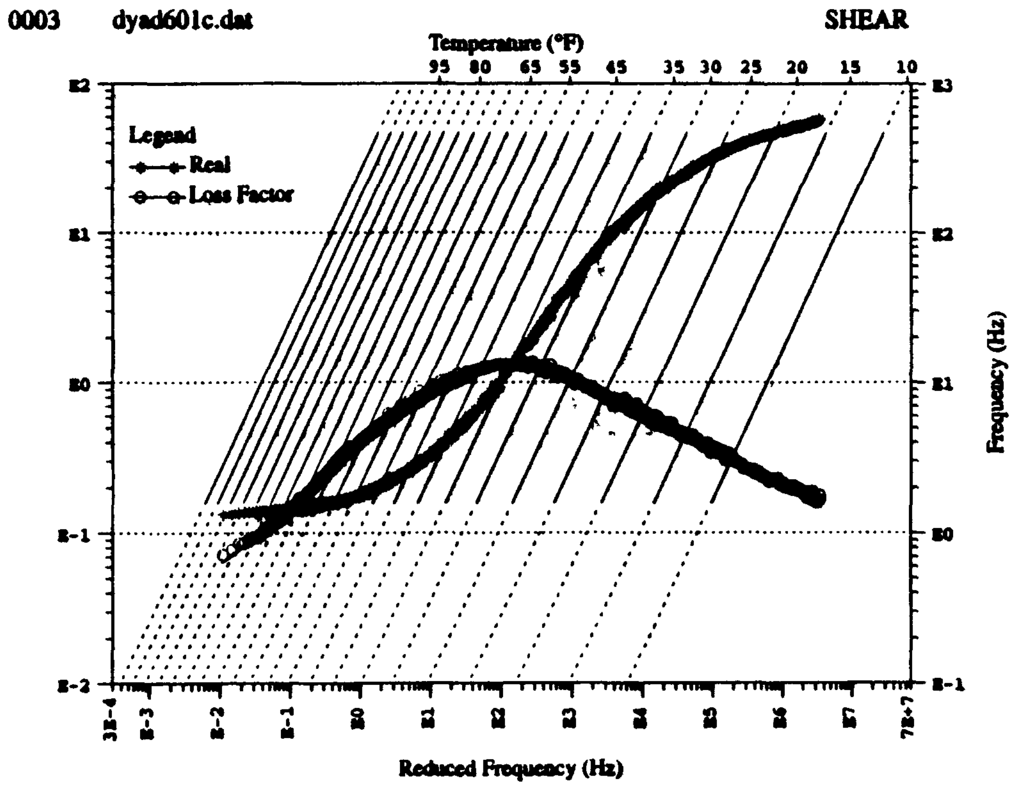

FIGURE 1 Loss factor as a function of frequency and temperature

While these effects are notable, the curves illustrate clearly that a single passive damping treatment is capable of providing significant damping over a broad range of frequencies. However the modeling is only valid in the steady state, making the prediction of time responses difficult. Here we offer alternative modeling techniques as well as illustrating various combinations of passive and active elements to improve the global damping properties.

Passive damping may also be obtained by using piezoelectric shunts. Rather then applying a viscoeleastic layer to a host structure a piezoelectric device (usually a piezoceramic) is layered into or on a host structure and shunted to a resister or resister and inductor. As the host vibrates, the piezoelectric effect changes the induced strain into a voltage which is then dissipated as heat through the shunt circuit. The result is a system that produces a loss factor versus frequency curve much like that of figure 1. Only loss factors of about 0.45 for longitudinal vibration and just 0.08 in the transverse direction are obtained when practical values of resistance and inductance are used. However, the peak value of the loss factor can be easily change from one value of frequency to another providing increased design flexibility. In addition, the shunted piezoceramic system is not as temperature dependent as the viscoelastic counter part, and is much stiffer then a VEM. Thus shunted piezoceramics do not directly compete with VEM for adding damping, but do offer more design flexibility and temperature stability.

ACTIVE DAMPING

Here we restrict our attention to active damping methods that consist of adding imbedded or surface mounted piezoceramic devices as actuators and an active control system to the host structure. Many researchers, including Inman (1995) have shown piezoceramic based, smart structural control systems to be effective means of adding damping to structures. In fact damping ratios as high as $2(\eta = 4)$ have been measured depending on the choice of control law. Thus, the use of embedded piezoceramic materials is an extremly competitive source of added damping. They also enjoy the same temperature stability of the shunted systems and can be easily tuned to troublesome modes. The difficulties are the added complexity of the closed loop system and the associated electronics.

An added level of complexity in the design of an active damping treatment is the choice of closed loop control law. Many different control laws can be used and new schemes are continually being developed. One important point to note however is that the most commonly used control law (PD: velocity and position feedback) is not the most effective. Basically in order to justify the use of active control it must provide something more than is obtained by passive treatments. Essentially velocity feedback suffers from the same restrictions that passive treatments do. The addition of damping can reduce settling times but cannot simultaneously reduce the overshoot. However the use of a feedback compensator can accomplish this by adding extra states (much like and absorber) to reduce both settling time and overshoot. Thus, the use of more exotic control laws can make a substantial difference as indicated in table 1 which shows the settling time for a cantilevered beam controlled by a self sensing piezoceramic system (Inman, 1995) for several different control laws.

Control Law	ζ
PD	0.92
LQR	0.82
PPF	2.0
Optimal PPF	2.13

Table 1 The effect of control law on damping ratio for a simple system

Much work exists in the literature on the use of piezoceramic based control systems for vibration suppression. The review article by Rao (1994) captures most of the literature. There are significant issues in modeling piezoceramic devices. Here we use a rather simple model because the hysterisis and temperature effects introduced by the piezoceramic are not as sever as those introduced by the VEM, we ignore them and use the model put forth by Crawley and de Lius (1987), and Hagood, et al (1990), often refered to as a pin-force model.

The pin-force model models the piezoceramic layer as providing a moment between the ends of the patch proportional to the control input voltage, the distance from the nuetral axis (moment arm) and the piezoceramic material properties (d_{31}) as detailed in Dosch et al. (1992). The sensor signal from the piezoceramic is proportional to the stain induced by the motion of the piezoceramic layer as computed by a bridge like circuit (Dosch, et. al., 1994). The actuation provide a moment to the beam

$$M(x,t) = k_a v_a(t)[h(x - x_1) - h(x - x_2)] \tag{2a}$$

where M is the moment produced, k_a is a constant determined by the beam thickness, the actuator thickness and width and physical parameter of the piezoceramic material, v_a is the control input voltage, h is the heaviside step function and the actuator lies between x_1 and x_2. The sensor provides a voltage

$$v(t) - k_s[y'(x_2,t) - y'(x_1,t)] \tag{2b}$$

where k_s is a sensor constant dependent on piezoceramic properties and y' is the slope of the beam. These two simple expressions are used for formulate the desired closed loop control laws for vibrations suppression as discussed in Inman (1995). These are also used in the formulation of the theoretical and experimental results obtained in Table 1.

A SURVEY OF ACTIVE CONSTRAINED LAYER DAMPING

Recently the properties of active and passive damping treatments were combined to create a hybrid damping techniques. The first attempt at this was to make the constraining layer of a passive constrained layer damping treatment (CLD) active, producing a treatment called active constrained layer damping (ACLD). Several investigations of ACLD treatments have shown it to be an effective method of vibration suppression in a variety of situations.

ACLD treatment has a number of advantages. First, the passive damping is a fail-safe in case of failure of the active controller. Also, active damping of high frequencies can be costly and very difficult because of control electronics. However, passive damping works well at high frequencies by inducing more shear in the VEM. On the other hand, active control with piezoceramics works well at low frequency, which means the active and passive elements complement each other. As shown by Huang et al (1996), ACLD will need less power than a purely active system to reduce unwanted vibration due to the inherent damping in the passive element. In particular if gain and thickness constraints are imposed, ACLD can out perform both active damping treatments and passive treatments by themselves. If high gains are needed to improve performance, ACLD will be more effective. This is due to the fact that damping is added to all modes in the system, which delays the uncontrollable modes from becoming unstable (passive damping improves gain margin). This same damping will help keep unmodeled modes stable.

Agnes and Napolitano (1993) and Baz (1993) first combine active and passive treatments to create ACLD as a more effective method of vibration suppression. Stability and controllability of ACLD treatments is addressed by Shen (1994). Baz and Ro (1995) provide a survey and overview of the first two years of research in the field of ACLD. They also address the issue of optimal placement and size of ACLD treatment. Comparisons between passive, active and ACLD are made by Baz and Ro (1995), Liao and Wang (1996), Azvin et al. (1995) and Huang et al. (1996).

In Agnes and Napolitano (1993), a finite element model of a beam with full-coverage ACLD (active sandwich beam configuration) is compared to an analytical model and shown to effectively damp vibration. Baz (1993) uses the damping model of Mead and Markus (1969) to obtain a sixth-order differential equation for an active sandwich beam configuration.

This principle is extended to a beam with partial coverage in Baz and Ro (1993a). Finite element analysis is used to obtain equations for a beam with partial ACLD coverage in Baz and Ro (1993b). The use of ACLD treatments to control vibration of plates is investigated in Baz and Ro (1993c).

Shen (1994a) uses equilibrium to obtain an eighth-order differential equation governing the bending and axial vibrations of a fully covered beam with ACLD. In Shen (1994b) the principles used in previously are extended to a plate with ACLD. Shen (1995a) again uses the equations governing a plate to obtain a model for ACLD. By assuming the width is much smaller than the length, a set of beam equations is obtained. Torsional vibration control of a shaft through ACLD is described in Shen et al. (1994) Variational methods are used in (1995b) to obtain equations of motion for a beam with ACLD. Yellin and Shen (1997) examine the use of self-sensing actuators to control vibrations of a partially covered beam.

Azvine et al. (1995) show that the use of ACLD produces effective levels of damping in a cantilevered beam. Rongong et al. (1997) obtain a mathematical based on the Rayleigh Ritz approach. Veley and Roa (1996) compare active, passive and hybrid damping. Modal strain energy is used to account for damping of the VEM. Crassidis et al. (1997) uses H* control instead of velocity feedback or LQR in order to control ACLD. This is an important contribution because most of the work uses velocity feedback, which as pointed out in the previous section is not the best use of active control.

The complex modulus is used to model the damping of the viscoelastic material in the majority of the research papers described above. As mentioned in the previous section, this restricts the solution to steady-state analysis. Van Nostrand (1994) and Van Norstrand and Inman (1995), following the work of Lesieutre (199?), were the first to use thermodynamic fields instead of the complex modulus to produce a transient model of an ACLD. This approach allows time domain analysis for any disturbance. Saunders et al. (1994) developed a pole-zero model for a composite beam with ACLD, thereby circumventing the issue of modeling damping parameters. Liao and Wang (1996) allow the piezoelectric treatment to run past the VEM layer and introduce edge elements in an attempt to increase the control authority of the active element. Lesieutre and Lee (1996) propose segmented ACLD treatments motivated by the success of segmented PCLD treatments. Various time domain models of viscoelastic materials are used. Ray and Baz (1998) address the optimal placement of ACLD by maximizing energy dissipation of the treatment and showed an optimally configured ACLD has higher damping than an optimally configured PCLD.

The various topics not yet fully addressed by the literature in ACLD are the mechanics issues of layered media, the modeling of the frequency and temperature dependence of the viscoelastic and allowing the layers of viscoelastic and piezoceamic to become separate. In the following we address the configurations of the damping treatments, called hybrid, and the modeling of the viscoelastic frequency dependence.

HYBRID DAMPING

Various configurations of constrained layer viscoelastics and piezoceramics can be fabricated to produce a variety of different effects. A paradox in the design of ACLD systems is that while a piezoceramic layer can induce more shear into a VEM layer than a passive constraining layer can, mounting a piezoceramic actuator as an active control element through a VEM makes a very weak actuator for active control. Hence, is seeking the optimal combination of VEM and piezoceramic a number of different configurations are examined as illustrated in figure 2.

We focus on allowing the passive constrained layer system to be located seperately from the piezoceramic layer creating hybrid, passive active damping treatment. Before we proceed

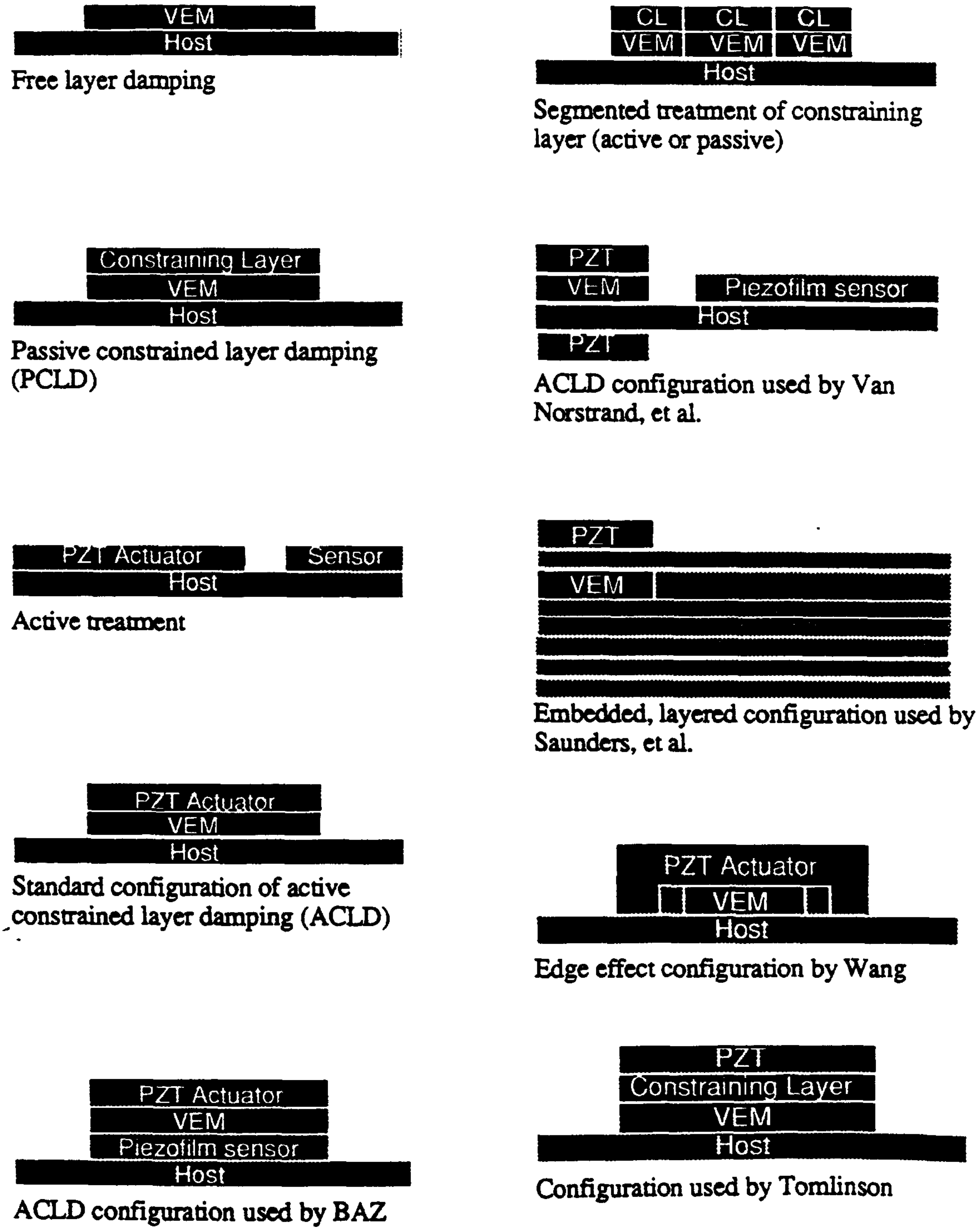

FIGURE 2: Various configurations of viscoelastic materials (VEM), constraining layer (CL), sensors and piezoceramic actuators (PZT) used for vibration suppression.

with results from this configuration some modeling of the VEM is introduced.

VEM MODELING

There are several interesting methods available to model the effects of the frequency dependence of a viscoelastic material rather then using the complex modulus approach. Each of the methods starts with the fundamental constitutive relation for stress and strain and produces a time domain mass, damping and stiffness model for predicting the transient response of structure containing a VEM. The basic relationship is

$$\sigma(X,t) = E\epsilon(x,t) + \int_0^t g(t-s)ds \tag{3}$$

where $\sigma(x,t)$ is the stress, $x \in (o,l)$ is the distance along the beam, $t > 0$ is the time, $\epsilon(x,t)$ is the strain, E is the elastic modulus, and the kernel $g(t-s)$ describes the hysteresis as developed by Christensen (1982), for example.

Bagley and Torvic (1983) advocate using fractional calculus to model the frequency dependence, Johnson (1997) uses an internal variable approach and Segalman (1987) uses a perturbation approach to model slightly viscoelastic structures. Lesieutre (1992) uses an augmented thermodynamic field to model hysteris and the Golla-Hughes-McTavish (GHM) method models the damping of viscoelastic properties (1985, 1993) using additional coordinates. The frequency dependent behavior of the VEM is described through the addition of extra coordinates, called dissipation coordinates. The VEM material properties are therefore introduced through the mass, damping and stiffness matrices. A major advantage of using this method is that symmetry of the mass, damping and stiffness matrices are retained.

Lam et al. (1995) was the first to propose using the Golla-Hughes-McTavish damping model to account for the damping in the viscoelastic layer for structures with ACLD. Liao and Wang (1995) also use GHM along with energy methods to model the behavior of partial ACLD treatment. This results are able to predict transient time responses and hence they form suitable models for closed loop vibration control designs suitable for vibration suppression.

The GHM model adds coordinates to the analytical model of the system and this is sometimes udesirable for computational reasons. Hence a model reduction technique is derived to reduce the dynamic model to its original order.

GHM MODELING

The Golla-Hughes-McTavish modeling approach models hysteretic damping by adding additional "dissipation coordinates" to the system to achieve a linear non hysteretic model providing the same damping values over a wide range of frequencies. The dissipation coordinates are used with the finite element model of the system. Linear matrix-second-order form is maintained as well as symmetry and definiteness of the augmented system matrices. The time domain stress relaxation, eq. (3), is modeled by a modulus function in the Laplace domain.

Equation (3) can be written in Laplace domain as

$$G^*(s) = G_0(1 + h(s)) = G_0\left(1 + \sum_{n=1}^{k} \hat{\alpha}_n \frac{s^2 + 2\hat{\zeta}_n\hat{\omega}_n s}{s^2 + s\hat{\zeta}_n\hat{\omega}_n s + \hat{\omega}_n^2}\right) \tag{4}$$

where G_0 is the equilibrium value of the modulus, i.e. the final value of the relaxation function $G(t)$, and s is the Laplace domain operator. The hatted terms are obtained from a curve fit to the complex modulus data for a particular VEM. The expansion of $h(s)$ represents the material modulus as a series of second order equations. The number of terms kept in the expansion will be determined by the high or low frequency dependence of the complex modulus.

The equation of motion in the Laplace domain is

$$Ms^2q(s) + K(s)q(s) = F(s) \tag{5}$$

where M is the mass matrix, K the complex stiffness matrix, F the forcing function, and $q(s)$ the transform of the generalized coordinate. The complex stiffness matrix can be written as the summation of the contributions of the n complex moduli to the stiffness matrix such that

$$K(s) = (G_1^*(s)\overline{K}_1 + G_2^*(s)\overline{K}_2 ... G_n(s)\overline{K}_n) \tag{6}$$

where $G_n^*(s)$ refers to the nth complex modulus and $\overline{K}_n$ to the contribution of the nth modulus to the stiffness matrix. For simplicity, assume a complex modulus model with a single expansion term and zero initial conditions, so equation (6) can be written as

$$M_s^2q(s) + G_0\left(1 + \hat{\alpha}\frac{s^2 + 2\hat{\zeta}\hat{\omega}s}{s^2 + 2\zeta\hat{\omega}_2 + \hat{\omega}^2}\right)Kq(s) = F(s) \tag{7}$$

The coefficient of equation (7) is determined by the frequency dependent loss factor curves for a particular VEM (determined experimentally or from manufacturer's data). This coefficient in the Laplace Domain contains dynamics that are now associated with a psuedo coordinate $z(s)$ which is essentially a state estimation compensating for the frequency dependence of the VEM. The comparison (see Lam (1995) for a short derivation) and subsequent inverse transform yields a linear viscously damped system of the form

$$M\ddot{x} + C\dot{x} + Kx = f \tag{8}$$

where $x(t)$ is now an expanded coordinate $x(t) = [q^T(t)\ z^T(t)]^T$ containing the additional coordinate $z(t)$. The matrices M, C, and K are all real valued, symmetric and positive (semi) definite and the viscoelastic nature of the system is captured in the viscous damping matrix C combined with a new expanded version of the generalized coordinates $x(t)$. The coefficient matrices M, C and K now take the expanded form

$$M = \begin{bmatrix} M & 0 \\ 0 & \frac{\hat{\alpha}}{\hat{\omega}^2}G_0 I \end{bmatrix},\ C = \begin{bmatrix} \Omega & 0 \\ 0 & \frac{2\hat{\alpha}\hat{\zeta}}{\hat{\omega}}G_0 I \end{bmatrix},\ K = \begin{bmatrix} (1+\hat{\alpha})G_0\overline{K} & -\hat{\alpha}G_0\overline{K} \\ -\hat{\alpha}G_0\overline{K} & \hat{\alpha}G_0\overline{K} \end{bmatrix} \tag{9}$$

where Ω is a diagonal matrix of model damping from the natural damping of the beam.

In order to model the behavior of the VEM which partially covers a beam, the stiffness and mass matrices for the covered area are first assembled. This procedure, outlined in Lam (1995) requires the use of heavyside step functions to locate the various components. The effects of the dissipation modes on the system are calculated. The full mass and stiffness matrices for the whole beam are assembled, using the mass and stiffness matrices obtained from equation (9) to model the effects of the VEM on the whole structure. The order of the system increases as the number of terms in the expansion are kept, which increases the accuracy for modeling the damping effects. An experimental verification is given in figure 3.

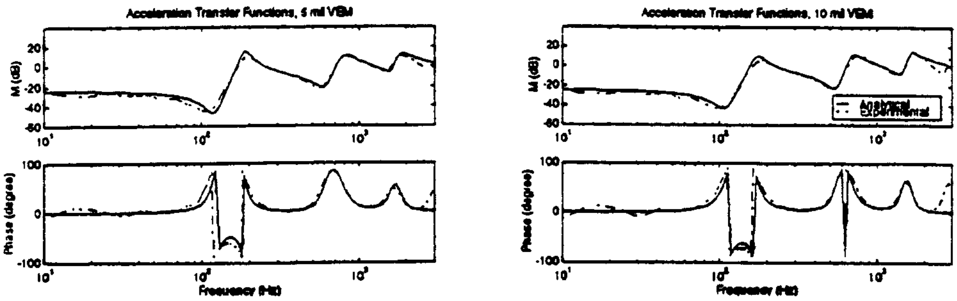

FIGURE 3. A comparison of analytical and experimental transfer functions for a constrained layer beam, verifying the GHM approach used here to model the VEM.

REDUCTION METHODS

Model reduction methods are briefly introduced here as they have been developed in two different disciplines: finite element analysis and control theory. In the case of a condensation process or static reduction, such as Guyan reduction, some of the insignificant physical coordinates are removed such as rotational degrees of freedom at a node point (Guyan, 1965). On the other hand, the internal balancing method of control theory, it is not directly possible to express the reduced model in terms of a subset of the original states. Hence an additional coordinate transformation is introduced and applied (Yae, 1987, 1993). Here the original equations of motion are taken to be the finite element model including the GHM terms and coordinates. Equation (8) is first converted into the state space form such that

$$\dot{x}(t) = AX(t) + Bu(t)$$

$$y(t) = Cx(t) \tag{10}$$

where

$$A = \begin{bmatrix} -M^{-1}D & -M^{-1}K \\ I & 0 \end{bmatrix}, \quad B = \begin{bmatrix} M^{-1}B_1 \\ 0 \end{bmatrix}, \quad C = [C_1 \;\; C_2 \tag{11}$$

and will be denoted by (A, B, C, x). It is assumed that the system (A, B, C, x) is controllable, observable and asymptotically stable. The idea used in this method is to reduce the order of a given model based on deleting those coordinates, or modes, that are the least controllable and observable. To implement this a measure of the degree of controllability and observability is needed. The useful measure is provided for asymptotically stable systems of the form given by Equation (8) by defining the *controllability and observability grammians*, denoted by W_c and W_0 respectively and defined by

$$W_C = \int_0^{\infty} e^{At} BB^T e^{A^T t} dt, \quad W_0 = \int_0^{\infty} e^{A^T t} C^T C e^{At} dt \tag{12}$$

where e^{At} is the state transition matrix of the open-loop system $\dot{x}(t) = Ax(t)$. W_c and W_0 are the unique symmetric positive definite matrices which satisfy the Lyapunov matrix equations:

$$AW_c + W_c A^T = -BB^T, \quad A^T W_0 + W_0 A = -C^T C \tag{13}$$

for asymptotically stable systems. It has been shown that there exists a coordinate system in which two grammians are equal and diagonal. Such a system is then called *balanced*. Let the matrix P denote a linear transformation of the system into the balanced coordinate system, which when applied to Equation (12) yields the equivalent system

$$\dot{\hat{x}}(t) = \hat{A}\hat{x}(t) + \hat{B}u(t)$$

$$y(t) = \hat{C}\hat{x}(t). \tag{14}$$

These two balanced systems are related by

$$\hat{x} = P^{-1}x, \quad \hat{A} = P^{-1}AP, \quad \hat{B} = P^{-1}B, \quad \hat{C} = CP \tag{15}$$

In addition, the two grammians are equal in this coordinate system:

$$\hat{W}_C = \hat{W}_0 = \text{diag}[\sigma_1, \sigma_2, \ldots \sigma_{2n}] \tag{16}$$

where $\hat{W}_c = P^{-1}W_cP$, $\hat{W}_0 = P^{-1}W_0P$ and σ_i's denote the singular values of the grammians. Applying the idea of singular values as a measure of rank deficiency to the controllability and observability grammians yields a systematic model reduction method. The matrix P that transforms the original system (A, B, C, x) into a balanced system $(\hat{A}, \hat{B}, \hat{C}, \hat{x})$ can be obtained using the following algorithm:

a. The reduced order model can be calculated by first calculating an intermediate transformation matrix P_1 based on the controllability grammians. Solving for W_c and find eigenvalues Λ_c and eigenvectors V_c such that $V_c^T W_c V_c = \Lambda_c$. Then define $P_1 = V_c \Lambda_c^{-1/2}$.
b. The coordinate transformation $x = P_1\tilde{x}$ yields an intermediate system $(\tilde{A}, \tilde{B}, \tilde{C}, \tilde{x})$ calculated by $\tilde{A} = P_1^{-1}AP_1$, $\tilde{B} + P_1^{-1}B$, $\tilde{C} = CP_1$.
c. To complete the balancing algorithm, these intermediate equations are balanced with respect to $\tilde{W}_0$. Solving for $\tilde{W}_0$ and find eigenvalues $\tilde{\Lambda}_0$ and eigenvectors $\tilde{V}_0$ such that $\tilde{V}_0^T \tilde{W}_0 \tilde{V}_0 = \tilde{\Lambda}_0$. Let $P_2 = \tilde{V}_0 \tilde{\Lambda}_0^{-1/4}$.
d. Another coordinate transformation $\tilde{x} = P_2\hat{x}$ yields the desired balanced system $(\hat{A}, \hat{B}, \hat{C}, \hat{x})$:

$$\hat{A} = P_2^{-1}\tilde{A}P_2 = P_2^{-1}(P_1^{-1}\ AP_1)P_2. \ \hat{B} = P_2^{-1}\tilde{B} = P_2^{-1}P_1^{-1}B, \ \hat{C} = \tilde{C}P_2 = CP_1P_2 \tag{17}$$

The transformation P is given by P_1 and P_1 as $P = P_1\ P_2$. Using Equation (17), the balanced system $(\hat{A}, \hat{B}, \hat{C}, \hat{x})$ can be partitioned as

$$\begin{bmatrix} \dot{\hat{x}}_r \\ \dot{\hat{x}}_d \end{bmatrix} = \begin{bmatrix} \hat{A}_r & \hat{A}_{12} \\ \hat{A}_{21} & \hat{A}_{22} \end{bmatrix} \begin{bmatrix} \hat{x}_r \\ \hat{x}_d \end{bmatrix} + \begin{bmatrix} \hat{B}_r \\ \hat{B}_d \end{bmatrix} u, \quad y = \hat{C}_r \quad \hat{C}_d] \begin{bmatrix} \hat{x}_r \\ \hat{x}_d \end{bmatrix} \tag{18}$$

Deleting the k least controllable and observable states, i.e., $\hat{x}_d = 0$, yields

$$\dot{\hat{x}}_r(t) = \hat{A}_r\hat{x}_r(t) + \hat{B}_r u(t), \quad y_r(t) = \hat{C}_r\hat{x}_r(t) \tag{19}$$

a reduced model of order $(n - k)$. This produces the balanced system which can now be reduced by looking at the singular values of the balanced system and throwing away those coordinates which have relatively small singular values. This leaves a smaller order system with essentially the same dynamics as the full order system.

Unfortunately the coordinates left after a balanced reduction are not a subset of the finite element nodal coordinates. Thus this is not simple to relate back to the original finite element model as is the case in Guyan reduction. This problem is solved by Yae (1987) who introduced an additional coordinate transformation to produce a reduced order model in a coordinate system consisting of a subset of the original finite element coordinate system. For structural control and measurement applications, it is desirable to provide the designer with a clear, physical relationship between the original vector q in Equation (9) and the reduced state vector $\hat{x}_r$. Such a relationship is found by using the fact that the balanced states are linear combinations of the original states. Symbolically this is written as:

$$\begin{array}{ll} \hat{x}_1 = \sum_{j=1}^{2n} c_{ij} x_j, & \ldots, \hat{x}_{2n-l} = \sum_{j=1}^{2n} c_{(2n-k)j} x_j, \\ \hat{x}_{2n-(k-1)} = \sum_{j=1}^{2n} c_{(2n-k+1)j} x_j \to 0, & \ldots, \hat{x}_{2n} = \sum_{j=1}^{2n} c_{2nj} x_j \to 0, \end{array} \tag{20}$$

where c_{ij}'s are the coefficients in the linear combinations $\{x_1, x_2, \ldots, x_{2n}\}$. Here the last k states are set to zero because they represent the least significant states in the balanced system. Setting each of these summations equal to zero is equivalent to imposing k constraints on the original $2n$ states, which means that the modal reduction imposes dependencies on k number of the original states. In other words, one can construct a reduced order model by selecting $(2n-k)$ states out of the original $2n$ states. If the $(2n-k)$ selected states from the original system are denoted by $x_r = \left[x_{j1} x_{j2} \ldots x_{j2n-k}\right]^T$ and the $(2n-k)$ states of the balanced system by $\hat{x}_r = [\hat{x}_1 \; \hat{x}_2 \ldots \hat{x}_{2n-k}]^T$, then the states in $\hat{x}_r$ are linear combinations of the states in x_r. Thus there exists a new transformation matrix P_r of order $(2n-k) \times (2n-k)$ such that $x_r = P_r \hat{x}_r$. The above constraints and the resulting transformation allow the designer to specify which nodes of the model to be retained in the model reduction. In the following it is shown that the matrix P_r consists of certain rows and columns of the original transformation matrix P, and that there is a systematic way of constructing P_r from P.

a. Select the state variables to be retained from $\{x_1, x_2, \ldots, x_{2n-k}\}$. Let the indices of those selected by $\{j_1, \ldots, j_{2n-k}\}$ rows from P.
b. The transformation matrix P_r can be obtained by selecting first $2n-k$ columns and $\{j_1, \ldots, j_{2n-k}\}$ rows from P.
c. The reduced order system (A_r, B_r, C_r, x_r)

$$\dot{x}_r(t) = A_r x_r(t) + B_r u(t), \quad y_r(t) = C_r x_r(t) \tag{21}$$

is now expressed in terms of a subset x_r of the original state vector x, where

$$A_r = P_r \hat{A}_r P_r^{-1}, \quad B_r = P_r \hat{B}_r, \quad C_r = \hat{C}_r P_r^{-1}. \tag{22}$$

Thus we have provided a scheme that has the best feature of each reduction method: Here we are able to specify which coordinate to keep and provide a dynamically based reduction scheme. This will allow to remove the internal coordinates, $z(t)$, added to the system to build a damping matrix. A compariosn is given in Figure 4.

SUMMARY

The various methods of adding damping to structures have been reviewed and the modeling of viscoelastic materials has been discussed. Results show that damping added by viscoelastic layers can be well modeled with out having to use the primitive concept of a complex modulus. Furthermore, the coordinates added by these modeling techniques can be effectively removed by using balanced model reduction methods.

The best way to reduce damping is to use a passive constrained layer method as it removes a substantial amount of energy yet is uncomplicated and robust. However if temperature deviations are present the viscoelastic properties may change drastically rendering the treatment ineffective. In addition physical constraints of weight, geometry may prevent

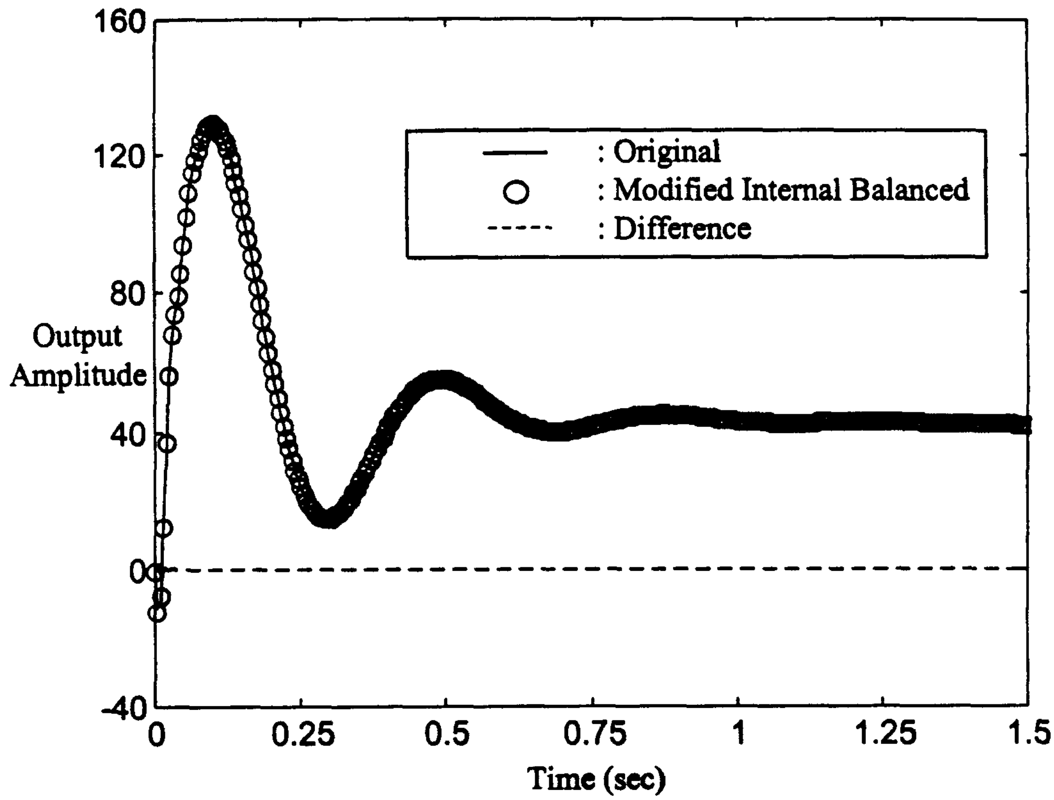

FIGURE 4 Time response of output in original and modified internally balanced GHM Model

a good design from being implemented in which case shunts or active constrained layer damping treatments are useful.

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