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RADIATION FROM PARTIALLY EXCITED PLATES

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A residential building with high sound insulation was recently built in Stockholm. This building was designed to have a sound insulation between the apartments, both horizontally and vertically, that was substantially higher then normal buildings in Sweden. Since the building costs are high this new building should at the same time not be more expensive to build. In order to achieve this a special type of supporting structure was used.

In connection to this project some theoretical and experimental studies were made. One of these studies concerned the sound insulation of light-weight double walls, which were used between the apartments, and another concerned the radiation efficiency of finite plates.

In this paper, an evaluation of the influence of the plate size on the radiation efficiency and the sound reduction index is presented. The radiation efficiency is obtained from the Fourier transform of the plate velocity over the excited area of the plate and it is presented for both rectangular and square plates of different sizes.

INTRODUCTION

The evaluation of the radiation efficiency and the sound reduction index is carried out with the help of a model presented by Ljunggren [1], but the radiation efficiency is here directly evaluated for the three dimensional case.

This paper deals with airborne sound radiation of single plates in a special case, as did the paper by Ljunggren, where the plate is excited over a part of its surface only. This case is thought to represent a common building where the walls and especially the floors often are larger than the room where the excitation occurs.

If, for instance, the floor is made from concrete supported by concrete columns and the party walls as well as the walls within the apartments are made in a lightweight construction (gypsum walls with steel studs), then the floor area can be considered to be very large compared with the excited area. For this case Ljunggren [1] showed that the sound insulation in the vertical direction will be appreciably higher than in a conventional building with party walls made from concrete.

For the calculations a single plate is used, but some results (the radiation efficiency for forced vibrations) will also be useful for calculations of the sound reduction index of double walls, see e.g. Ref. 2. The 200 mm thick concrete plate used for the calculated examples weighs 2400 kg/m^3.

The plate is assumed to be of infinite dimensions and excited with airborne sound over a finite area. The radiation is obtained from the Fourier transform of the plate velocity over the excited area of the plate.

The results obtained are compared with those presented by Ljunggren [1] and Sato [3]. It should be noted that the surface integral in Sato's work has no general analytical solution. Instead he presented the results of a numerical solution in the special case of a square plate.

THEORETICAL MODEL

The model used for the calculations is illustrated in Fig. 1. The plate is homogeneous, isotropic and linearly elastic. It is assumed to be surrounded by air and excited over the area $-L$ to $+L$ and $-B$ to $+B$ by an incident, propagating plane sound wave with an incidence angle θ creating a wave in the plate. The plate is assumed to be of infinite size. In this way the influence of the resonant plate field can be ignored. A co-ordinate system is introduced with the plate in the plane $z=0$.

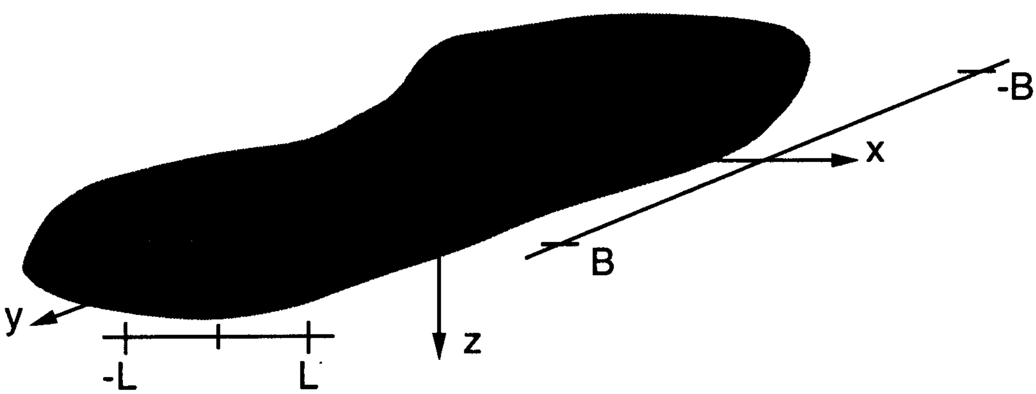

Figure 1.
The three-dimensional model.

Radiation from a plate excited by a plane airborne wave

The radiation is assumed to occur from an area equal to the excited one. The radiation takes place into a semi-infinite space. In this way the influence of the resonances in the room can be ignored.

Since the excitation occurs in all directions in the plate the velocity of the excited part of the plate is

$$v(x,y) = v_1 \exp(-jk\cos(\varphi)x - jk\sin(\varphi)y) \tag{1}$$

where φ is the angle of the wave in the plate and k is the trace wave number along the plate surface according to

$$k = k_a \sin\theta \, . \tag{2}$$

k_a is the wave number of the sound in air and θ the angle of incidence. The time factor $\exp(j\omega t)$ is omitted throughout the paper. The Fourier transform of the plate velocity can be written as

$$\begin{aligned} &v(k_x, k_y) = \\ &\int_{-B}^{B}\int_{-L}^{L} v_1 \exp(-jk\cos(\varphi)x - jk\sin(\varphi)y)\exp(jk_x x)\exp(jk_y y)dxdy = \\ &v_1 4\frac{\sin((k_x - k\cos(\varphi))B)}{k_x - k\cos(\varphi)}\frac{\sin((k_y - k\sin(\varphi))L)}{k_y - k\sin(\varphi)} \end{aligned} \tag{3}$$

The power radiated from the lower face of the plate can then be taken as the mean value of all the φ angles and

$$P = \frac{\rho c}{8\pi^2}\int_{-k_a}^{k_a}\int_{-k_a}^{k_a} k_a \frac{|v(k_x, k_y)|^2}{\sqrt{k_a^2 - k_x^2 - k_y^2}} dk_x dk_y \, , \tag{4}$$

in the same way as was described by Cremer *et al* [4] where ρ is the density of air and c the speed of sound.

The radiation efficiency, introduced by Gösele [5], can then be calculated according to

$$s = \frac{P}{\rho c S \tilde{v}^2} \, . \tag{5}$$

S is the area of one side of the plate and $\tilde{v}$ the root-mean square velocity in time and space of the radiating surface. All results in the following will be presented as 10log$s$.

Radiation from a plate with free vibrations

In order to calculate the radiation efficiency of a plate with free vibrations k in the equations above is changed to k_B. k_B is the wave number of the free propagating bending waves.

RESULTS

Radiation efficiency for forced transmission

The result presented here are calculated with a k_a value equal to 1 i.e. a frequency equal to 54 Hz.

First of all it is seen that the radiation efficiency curves for square plates follows the curve corresponding to $10\log(1/\cos\theta)$, for infinite plates, up to a certain point after which they levels out, Fig. 2. The curves in Fig. 3 showing the radiation efficiency for a rectangular plate do not follow $10\log(1/\cos\theta)$ in the same way as was the case in Fig. 2. The differences between the maximum and minimum values for the curves are smaller than in the previous case, i.e. the curves are flatter. The plate response is assumed to be forced in both cases.

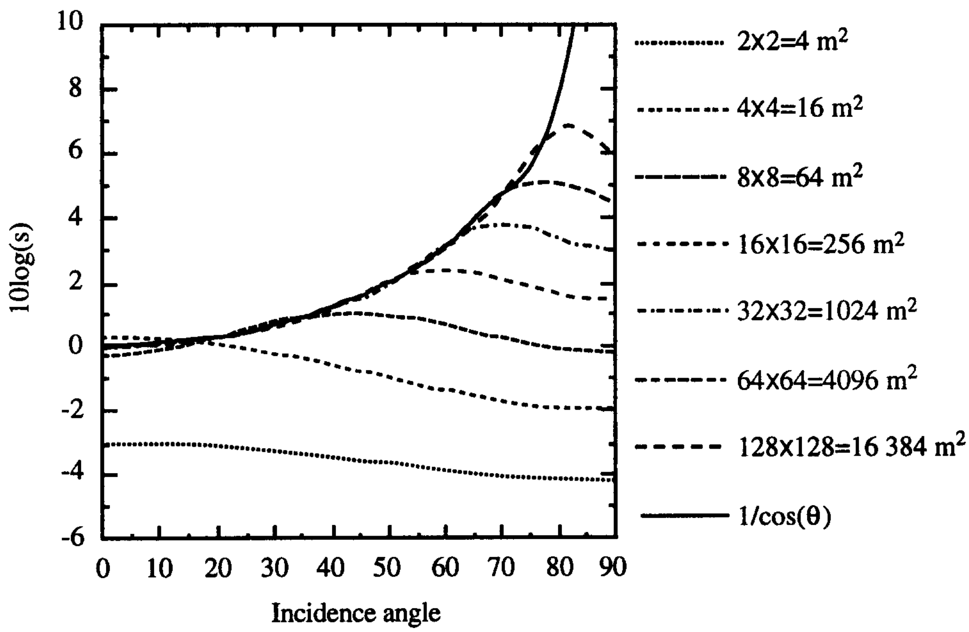

Figure 2.
Radiation efficiency for square plates.

The results in Fig. 2 and 3 show that the radiation efficiencies for a square plate are larger than those for a rectangular plate even if the area of the rectangular plate is larger. The shorter width is the limiting factor in the latter case, which is only to be expected.

The radiation efficiency for plates subjected to a diffuse sound field can, for low frequencies, also be presented as a function of $k_a L_m$. L_m is here equal to

$\pi S / U$, where S is the area of the radiating surface and U is the perimeter, that is Kosten's [6] expression for the mean free path in the two dimensional case. The

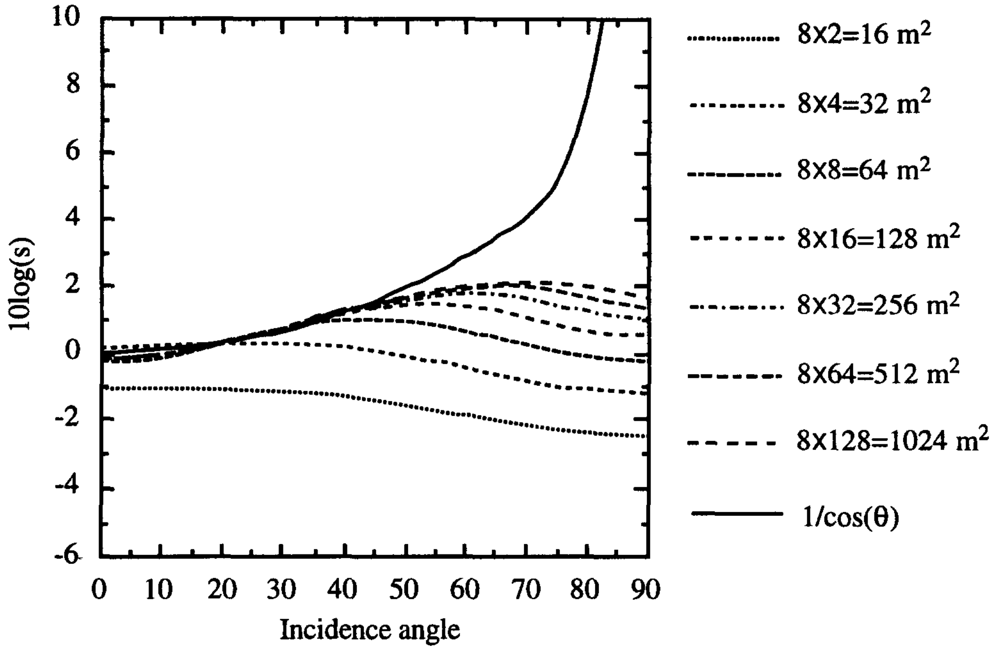

Figure 3.
Radiation efficiencies for rectangular plates .

results, see figure 4, for square and rectangular plates are very close to each other, in agreement with Ljunggren's [1] hypothesis. In the figure the two curves are so close together that it is hard to tell them apart.

<u>Radiation efficiency for free vibrations and diffuse incidence</u>

If the radiation efficiencies for free waves are calculated for different frequencies, Fig. 5, it is evident that the radiation efficiency for plates which are small compared with the wave length in the air decreases with decreasing frequency, which is expected. For larger plates there exists a maximum value for a certain frequency. This value is not reached at the critical frequency but for a somewhat higher frequency.

The same type of result is obtained for rectangular plates, Fig. 6.

CONCLUSIONS

<u>Comparison of the present method with older ones</u>

If s is calculated with the present method or by one of the other methods mentioned in this paper, then the radiation efficiency stays finite even for an incidence angle of 90° which is not the case if $1/\cos\theta$ is used. This means that gracing incidence poses no problem, see also Kihlman [7] who studied the radiation from a standing wave of sinusoidal shape in a room compared with the radiation into a semi-infinite space.

It can be seen from the presented results that θ has little importance if the Helmholtz number is small, which is not the case if the factor $1/\cos\theta$ is used.

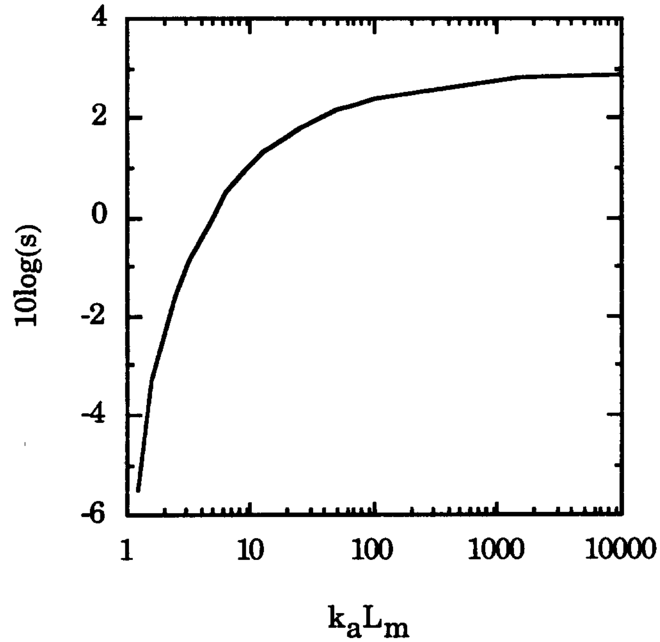

Figure 4.
Radiation efficiencies (in dB) for square and rectangular plates forced by a diffuse airborne sound field, as a function of $k_a L_m$. $L_m = \pi S / U$, where S is the area of the transmitting surface, as seen from the receiving room and U the perimeter. The two curves are place on top of each other.

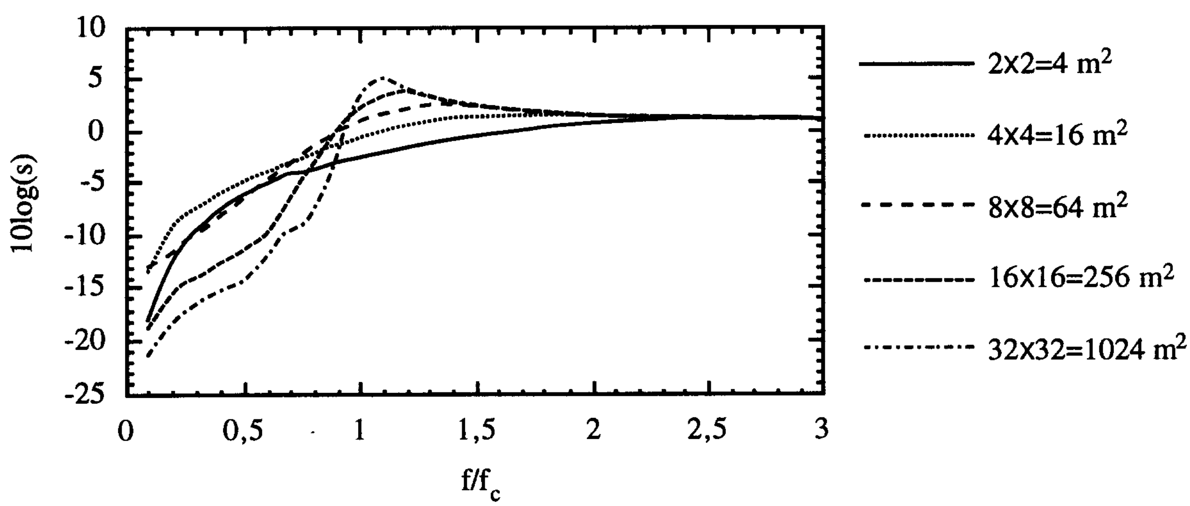

Figure 5.
Radiation efficiencies for square plates with free vibrations.

The calculated radiation efficiency for forced transmission shown in Fig. 2 agrees very well with the result obtained by Ljunggren [1] and Sato [3] (see also Rindel [8]). A comparison between the three different methods for two different plates is shown in Fig. 7.

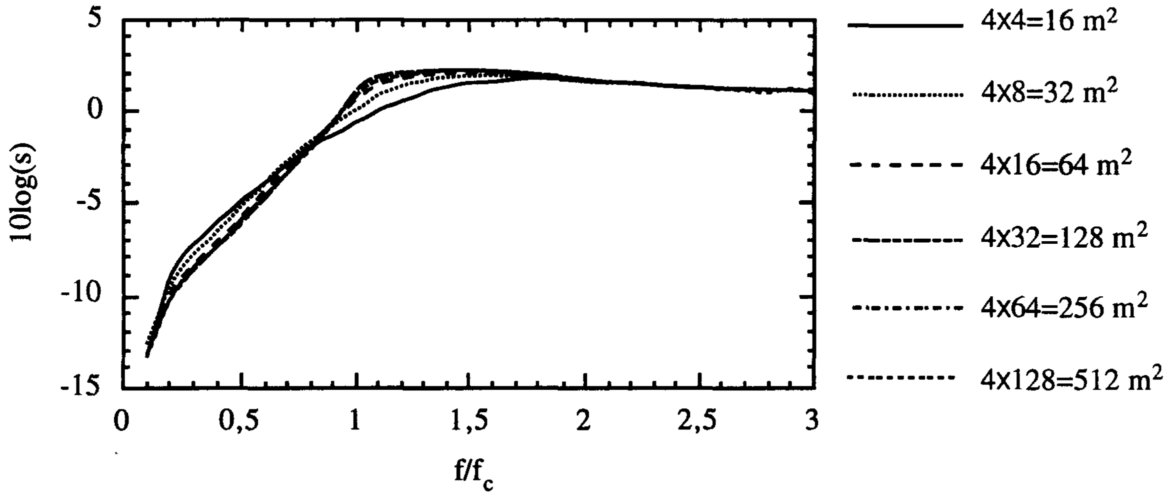

Figure 6.
Radiation efficiencies for rectangular plates with free vibrations.

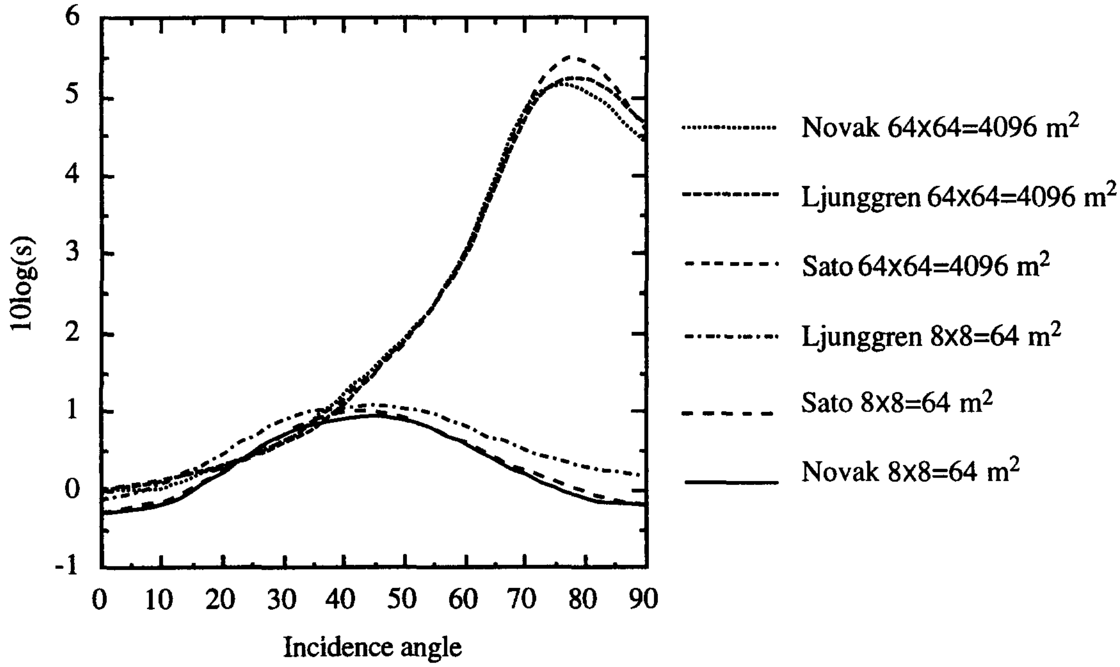

Figure 7.
Radiation efficiency for forced transmission calculated with the presented method compared with the methods by Ljunggren [1] and Sato [3].

It is seen that the difference between the three methods is well within 0.5 dB. Note that the method by Ljunggren only is a two-dimensional one.

Radiation efficiency for free vibrations and diffuse incidence

The results in Fig 5 and 6 show that the radiation efficiency for small plates decrease with decreasing frequencies. This is not the case with larger plates for which there existed a maximum for a certain frequency that is higher than the critical frequency. This means that the dip in the sound reduction curve which earlier was thought to exist at the critical frequency instead will occur at a somewhat higher frequency and it will be somewhat deeper.

Sound reduction

When the radiation efficiency has been calculated the sound reduction (*TL*) can easily be calculated. To simplify matters, Heckl's [9] expression for the *TL* at frequencies well above the critical frequency is used in order to get more compact results.

$$TL_H = 20\log(\omega m / 2\rho c) + 10\log(2\eta / \pi) + 10\log(f / f_c) \tag{6}$$

If for instance $f<f_c$ then the *TL* due to forced transmission can be written as

$$TL = 20\log(\omega m / 2\rho c) - 3 - 10\log s_d \tag{7}$$

where s_d is the radiation efficiency with respect to diffuse incidence. The *TL* must in this case be referred to the area of the part of the plate which is common to both rooms.

The *TL* due to the resonant transmission can be written as

$$TL = TL_H - 10\log(s_R s_S) - 10\log(S_R / S_{tot}), \tag{8}$$

where s_R is the radiation efficiency toward the receiving room and s_S that toward the source room. The *TL* is in this case referred to the source room area S_S. The receiving room and the total plate areas are denoted by S_R and S_{tot}.

ACKNOWLEDGEMENT

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