

FIFTH INTERNATIONAL CONGRESS ON SOUND AND VIBRATION DECEMBER 15-18, 1997 ADELAIDE, SOUTH AUSTRALIA

# A PREDICTION METHOD FOR THE STOCHASTIC RESPONSE OF COMPLICATED SOUND WALL SYSTEMS ON AN INTENSITY SCALE WITHOUT INTRODUCING ANY ARTIFICIAL ERROR CRITERION

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## ABSTRCT

In the actual situation of measuring an environmental noise, it is very often that only the resultant phenomenon fluctuation contaminated by an additional noise of arbitrary distribution type can be observed. Therefore, for the purpose of predicting the output response probability of acoustic system with an arbitrary stochastic input in the presence of the additional noise, it is necessary to find some new stochastic signal processing method reflecting the effect of the above additional noise fluctuation.

In this paper, first, a relationship between the system output excited by a specific stochastic input of reference type and an arbitrary random input without the additional noise for an arbitrary acoustic systems is introduced in the form on an intensity scale. Next, a relationship between the system output excited by an arbitrary stochastic input in the absence and that in the presence of the additional noise is also introduced in the form on an intensity scale. Then, based on these relationships, a new prediction method of the system output for the arbitrary acoustic systems with the additional noise is proposed especially by use of the observed data excited by the specific stochastic input of reference type with the additional noise. Finally, the effectiveness of the proposed method is confirmed experimentally too by applying it to the actual type sound wall systems.

## **1.INTRODUCTION**

In the actual measurement of environmental noise, the desired signal is very often contaminated by an additional noise of arbitrary distribution type and it is only the resultant signal that can be observed[1-3]. Therefore, for the purpose of predicting the output response probability excited by an arbitrary stochastic input with the additional noise, it is important to find some new signal processing method of reflecting the effect of the above additional noise.

In this paper, first, a model of arbitrary sound system output is introduced in More concretely, a relationship between the form on an intensity scale[4,5]. the output response probability of an arbitrary acoustic system excited by a stochastic input of reference type and the arbitrary random input without the additional noise is introduced in the form on the intensity scale. Next, a relationship between the system output probability excited by an arbitrary stochastic input without and that with the additional noise is also introduced in the form on an intensity scale. Then, based on these relationships, a new prediction method of the output probability density function for the arbitrary acoustic systems with an additional noise is proposed especially by employing the observed data excited by the specific stochastic input of reference type with Finally, the practical effectiveness of our new proposed the additional noise. method is confirmed experimentally too by applying it to the observed response data of various types of sound wall systems.

## 2. TEHORETICAL CONSIDERATION

First, we derive, both on the intensity scale and in the parameter differential form, the relationship between two system outputs excited by a reference stochastic input and an arbitrary input without the existence of an additional noise. Next, the relationship between two kinds of system output emitted by the arbitrary stochastic input with and without the additional noise is also introduced by the expression of parameter differential form with an intensity scale. Based on these relationships, we derive some new prediction method to be able to evaluate the acoustic system output excited by the arbitrary stochastic input in the presence of the additional noise.

## 2.1 RELATIONSHIP BETWEEN SYSTEM OUTPUTS EXCITED BY SPECIFIC INPUT OF REFERENCE TYPE AND ARBITRARY INPUT WITHOUT ADDITIONAL NOISE FOR ARBITRARY ACOUSTIC SYSTEM

Let a system output without an additional noise change from  $y_0$  to y:

$$y = y_0 (1 + \varepsilon / s_0) \tag{1}$$

where  $y_0$  and y denote two system outputs emitted by a specific stochastic input of reference type and an arbitrary random input without the additional noise in

the form on an intensity scale, respectively. The  $\epsilon / s_0$  shows a dimensionless deviation from a standard distribution type and is statistically independent with y. We can express a relationship between two acoustic system responses excited by a specific input of reference type and an arbitrary input without the additional noise in the expression form of the probability density function (abbr. , p.d.f.) as follows[6]:

$$p_{y}(y) = \sum_{l=0}^{\infty} (-1)^{l} / l! \cdot (d/dy)^{l} \left[ \left( \varepsilon \cdot y/s_{0} \right)^{l} \middle| y \right\rangle \cdot p_{y_{0}}(y) \right].$$

$$\tag{2}$$

The notation  $<^*>$  denotes the averaging operation with respect to \*. Here, we must notice the fact that a random variable is changed from original  $y_0$  to y in the p.d.f. expression  $p_{y_0}()$  of  $y_0$ . Also, the conditional moment can be directly obtained as :

$$\langle (\boldsymbol{\varepsilon} \cdot \boldsymbol{y}/s_0)^l | \boldsymbol{y} \rangle = \boldsymbol{y}^l / s_0^l \cdot \langle \boldsymbol{\varepsilon}^l \rangle.$$
 (3)

Accordingly, after substituting Eq.(3) into Eq.(2), Eq.(2) can be easily rewritten as follows :

$$p_{y(y)} = \sum_{l=0}^{\infty} (-1)^l / l! \cdot (d/dy)^l \left[ y^l / s_0^l \cdot \left\langle \varepsilon^l \right\rangle \cdot p_{y_0}(y) \right]$$
$$= \sum_{l=0}^{\infty} (-1)^l / l! \cdot \left\langle \varepsilon^l \right\rangle (d/dy)^l \left[ p_{y_0}(y) \cdot y^l / s_0^l \right].$$
(4)

Paying our attention to the fact that the system output on an intensity scale, y fluctuates always in a non-negative region. The probability density function for the system output, that is, can be expressed in advance especially in the general form of a statistical Laguerre expansion series[7] as :

$$p_{y_0}(y) = \left\{ 1 + \sum_{n=1}^{\infty} B_n \cdot L_n^{(m_0 - 1)} (y/s_0^l) \right\} p_{\Gamma}(y; m_0, s_0)$$
(5)

with

$$m_0 = \langle y \rangle^2 / \langle (y - \langle y \rangle)^2 \rangle, \ s_0 = \langle (y - \langle y \rangle)^2 \rangle / \langle y \rangle, \tag{6}$$

$$p_{\Gamma}(y;m_0,s_0) = y^{m_0-1} \cdot e^{-y/s_0} / \left( \Gamma(m_0) \cdot s_0^{m_0} \right)$$
(7)

and

$$B_n = \Gamma(m_0) \cdot n! / \Gamma(m_0 + n) \cdot \left\langle L_n^{(m_0 - 1)}(y/s_0) \right\rangle , \qquad (8)$$

where  $L_n^{(m_0-1)}(*)$  is a Laguerre polynomial of the n-th order, and  $B_n$  is the expansion coefficient reflecting hierarchically the lower and higher order statistics of the output intensity fluctuation.

Furthermore, Eq.(5) can be transformed into a parameter differential type series expansion expression taking a gamma distribution function as the first expansion term :

$$p_{y_0}(y) = \left\{ 1 + \sum_{n=1}^{\infty} B'_n (\partial/\partial s_0)^n \right\} p_{\Gamma}(y; m_0, s_0)$$
(9)

with

.. .

$$B'_{n} = \Gamma(m_{0}) \cdot (-s_{0})^{n} / \Gamma(m_{0}+n) \cdot \left\langle L_{n}^{(m_{0}-1)}(y_{0}/s_{0}) \right\rangle$$
(10)

After some complicated calculation procedures, the following relationship between the variable differential and the parameter differential can be derived as :

$$(\partial/\partial y)^{l} \left[ p_{y_{0}}(y) \cdot y^{l} / s_{0}^{l} \right] = (-1)^{l} (\partial/\partial s_{0})^{l} p_{y_{0}}(y).$$
<sup>(11)</sup>

Consequently, by employing Eq.(11), Eq.(4) can be rewritten as Eq.(12).

$$p_{y}(y) = \sum_{l=0}^{\infty} 1/l! \cdot \left\langle \varepsilon^{l} \right\rangle \cdot \left( \partial/\partial s_{0} \right)^{l} \cdot p_{y_{0}}(y).$$
(12)

Here, the *l*-th moment  $\langle \varepsilon^{l} \rangle$  can be concretely evaluated after applying the well-known binomial theorem to Eq.(1), as follows :

$$\left\langle \varepsilon^{l} \right\rangle = \left[ \left\langle y^{l} \right\rangle / \left\langle y^{l}_{0} \right\rangle - \sum_{j=0}^{l-1} l! / \{j! (l-j)\} \left\langle \varepsilon^{j} \right\rangle / s^{j}_{0} \right] \cdot s^{l}_{0} .$$
(13)

2.2 RELATIONSHIP BETWEEN SYSTEM OUTPUTS WITH AND WITHOUT ADDITIONAL NOISE FOR ARBITRARY ACOUSTIC SYSTEMS

The output fluctuation on the intensity scale for the arbitrary acoustic system can be described in the following linear form :

$$z = y + \sum_{i=1}^{N+1} a_i v_i , \qquad (14)$$

where z and y denote the system outputs with and without the additional noise. Here, N and  $a_i$  are the system order and system parameter. Also,  $v_i(i=1,2, \dots, N)$  and  $v_i(i=N+1)$  denote the intensities of additional noises on the input and output sides, respectively. Since the first term and the second term in the right hand of Eq.(14) are statistically independent each other, we can obtain the following expression :

$$p_{z}(z) = \sum_{n=0}^{\infty} (-1)^{n} / n! \cdot \left\langle \left( \sum_{i=1}^{N+1} a_{i} v_{i} \right)^{n} \right\rangle \cdot (d/dz)^{n} p_{y}(z).$$
(15)

After substituting Eq.(12) into Eq.(15) under the above condition, it is possible to rewrite Eq.(15) as :

$$p_{z}(z) = \sum_{n=0}^{\infty} (-1)^{n} / n! \cdot \left\langle \left( \sum_{i=1}^{N+1} a_{i} v_{i} \right)^{n} \right\rangle \cdot (d/dz)^{n} \left\{ \sum_{l=0}^{\infty} 1/l! \cdot \left\langle \varepsilon^{l} \right\rangle \cdot (\partial/\partial s_{0})^{l} p_{y_{0}}(z) \right\}$$
$$= \sum_{i=0}^{\infty} (-1)^{n} / l! \cdot \left\langle \varepsilon^{l} \right\rangle \cdot (\partial/\partial s_{0})^{l} \left\{ \sum_{n=0}^{\infty} (-1)^{n} / n! \cdot \left\langle \left( \sum_{i=1}^{N+1} a_{i} v_{i} \right)^{n} \right\rangle \cdot (d/dz)^{n} p_{y_{0}}(z) \right\}.$$
(16)

Consequently, after taking into consideration a p.d.f.  $p_{z_0}(z)$  of z corresponding to only  $y_0$  (instead of y) expressed in the same form as Eq.(15), we directly have :

$$p_{z}(z) = \sum_{l=0}^{\infty} 1/l! \cdot \left\langle \varepsilon^{l} \right\rangle \cdot \left( \frac{\partial}{\partial s_{0}} \right)^{l} p_{z_{0}}(z) .$$
(17)

Thus, we can predict theoretically the response p.d.f. for an actual sound insulation system with an arbitrary stochastic input in the presence of the additional noise, especially by employing the information on the system output p.d.f. for the same system with a specific reference input in the presence of the same additional noise.

#### **3.EXPERIMENTAL CONSIDERATION**

#### **3.1 EXPERIMENTAL ARRANGEMENT**

Figure 1 shows a block diagram of experimental arrangement in two reverberation rooms. The speaker excites the transmission room and two microphones receive respectively the input and output intensity fluctuations of the sound insulation system. We have employed the actual road traffic noise measured in Hiroshima City and the white noise as the stochastic input and the additional noise, respectively. The transmission room has a volume of 50.2 m<sup>3</sup> and the reception room has a volume of 24.6 m<sup>3</sup>. The aperture of the wall between the transmission and the reception has an area of  $1.74m \times 0.84m$ .



Fig.1 Block diagram of experimental arrangement.

The proposed theory is applied to three types of the sound insulation wall systems, (a) a single wall – an aluminum panel (surface density :  $3.22 \text{ kg/m}^2$ , thickness : 1.2 mm), (b) a non-parallel wall – composed of the aluminum (at an angle 9 degrees each other), and (c) a double wall with sound bridge – composed of the aluminum with sound bridge (air gap thickness : 50 mm).

#### **3.2 EXPERIMENTAL RESULTS**

The results of cumulative distribution function (abbr., c.d.f.) for the prediction of the system output are shown in Figs. 2 and 3 in cases of the single wall and the non-parallel double wall, respectively. Here, the 1st, the 2nd or the 3rd approximations correspond to the cases of employing the 1st, the 2nd or the 3rd terms in the above theoretical expansion expression, respectively. For the purpose of minimizing the error caused by employing only the first finite terms in the above infinite series expansion expression, some averaging evaluation procedure can be taken as follows :

$$Q(z) = Q_0(z) + (b+c)/(a+b+c) \cdot Q_1(z) + c/(a+b+c) \cdot Q_2(z) + 1/(a+b+c) \cdot (a\varepsilon_0 + b\varepsilon_1 + c\varepsilon_2),$$
(18)

where  $Q_i(z)(i=0,1,2)$  are respectively the c.d.f. in the special cases taking the 1st, the 2nd or the 3rd terms in the above infinite series type theoretical p.d.f. expansion expression, and a, b and c are the arbitrary constants. Also, the  $\epsilon_i(i=0,1,2)$  denote the errors caused by use of the finite expansion terms in cases of  $Q_i(z)(i=0,1,2)$ , respectively. From these figures, it seems that the 1st 2nd and 3rd approximation curves don't show an agreement with the experimentally sampled points owing to the above error. The averaging method in Eq.(18), however, show a better agreement with the experimentally sampled points compared with the other curves.



Fig.2 Comparison between theoretically predicted curves and experimentally sampled values for cumulative distribution function (a single wall). Experimentally sampled values in cases of the arbitrary input and the reference input are marked by ( $\bigcirc$ ) and ( $\bigcirc$ ), respectively. Theoretically predicted curves are shown by {( $-\cdots$ ): 1st approximation, ( $-\cdots$ ): 2nd approximation, ( $-\cdots$ ): 3rd approximation, ( $-\cdots$ ): averaging method}.



Fig.3 Comparison between theoretically predicted curves and experimentally sampled values for cumulative distribution function (a non-parallel double wall). Experimentally sampled values in cases of the arbitrary input and the reference input are marked by ( $\bigcirc$ ) and ( $\bigcirc$ ), respectively. Theoretically predicted curves are shown by {(---): 1st approximation, (---): 2rd approximation, (---): 3rd approximation, (---): averaging method}.

In this paper, the relationship between two kinds of output response probabilities for the arbitrary acoustic system transmitted by the stochastic reference input and the arbitrary random input without the additional noise has been first discussed in the parameter differential form with random variables on an intensity scale. Next, we have introduced the relationship between system two output probability expressions excited by the arbitrary stochastic input without and with additional noise. Based on these relationships we have proposed some new stochastic evaluation method, without introducing in advance any artificial error criterion like the well-known least squares error Thus, it is possible to predict theoretically the system output p.d.f. criterion. for arbitrary sound insulation systems emitted by the arbitrary stochastic input under the existence of the additional noise, especially by employing the information on the system output p.d.f. for the same system emitted only by a specific reference input in the presence of the same additional noise.

Finally, the practical effectiveness of the proposed prediction method has been experimentally confirmed too by applying it to the actually observed response data in the reverberation room.

Since the present prediction method is at an earlier stage of study, there still remain some kinds of future problems, for example, to apply it to many other actual systems, to find a more simplified method for practical use through the approximation of the proposed method.

#### ACKNOWLEDGMENTS

The authors would like to express our cordial thanks to Prof. K. Hatakeyama, Prof. A. Ikuta, Mr. K. Nishihara and Mr. N. Hanada for their helpful assistance.

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