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Invited Paper

A DISCUSSION ON FINITE ELEMENTAL ANALYSIS OF SOUND FIELD IN ROOMS WITH SOUND ABSORBING MATERIALS

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ABSTRACT In the analysis of architectural acoustics, the transient response in the three dimensional room with absorbent walls is essentially important. Then, several approaches are presented here to treat the dissipation and to obtain the transient response efficiently. Although the finite element method requires more memory in a computer compared to that of the boundary element method, the finite element method has stronger points in some cases. However, some physical assumptions and some mathematical considerations help to reduce the required memory, which makes a way to such an analysis as is on an auditorium. So, the choice of the combination of them is essential, especially in the analysis on a huge system. To make the point clearer, several computations are carried out. With the results, the characteristics of them are discussed.

1. INTRODUCTION

Numerical methods, *e.g.* finite element method and boundary element method, are broadly used in the research and engineering of acoustics recently. In the field of architectural engineering, the complexity of a room's geometry, or the boundary conditions, have made acoustical investigations hard to be carried out. The computational method like finite element method would be a help to overcome such difficulties and make a way to perform acoustic studies of rooms with complex boundary conditions.

A major part of the paper is concerned with the application of the finite element method to study the transient response of rooms with absorbent boundary conditions. At first, basic treatments are summarized in brief. Secondly, several approaches to obtain the sound pressure are presented. Then some discussions including the number of freedoms to be used in the analysis are presented. Finally, some computed results are compared with the measured values to see the accuracy of the methods.

2. THEORETICAL DESCRIPTION

2.1 Basic Formulae The following discrete formula of the sound field with absorption can be obtained by using the energy principle[1) 2)]

$$[M]\{\ddot{p}\} + [C]\{\dot{p}\} + [K]\{p\} = \rho\omega^2 u\,\{W\} \qquad (=\{f\}) \tag{1}$$

Or, using velocity potential and velocity of driving force, equation (1) can be in the form of

$$[M]\{\ddot{\Phi}\} + [C]\{\dot{\Phi}\} + [K]\{\Phi\} = - v\{W\}. \qquad (2)$$

Here, according to the ordinary finite elemental procedure, sound pressure at an arbitrary point in an element "*e*" can be approximated to be

$$p = \{N\}\{p\}_e \qquad (3)$$

With this shape function, $\{N\}$, element matrices are defined as follows.

$$[K_a]_e = \iiint_e \left[\frac{\partial\{N\}^T}{\partial x}\ \frac{\partial\{N\}^T}{\partial y}\ \frac{\partial\{N\}^T}{\partial z}\right] \begin{bmatrix} \frac{\partial\{N\}}{\partial x} \\ \frac{\partial\{N\}}{\partial y} \\ \frac{\partial\{N\}}{\partial z} \end{bmatrix} dxdydz$$

$$[M_a]_e = \frac{1}{c_a^2} \iiint_e \{N\}^T \{N\}\, dxdydz \qquad (4)$$

While, assuming the locally reactiveness at the wall's surface, the dissipating matrix can be in the form of,

$$[C]_{e^*} = \frac{1}{c_a} \iint_{e^*} \frac{1}{z_n} \{N\}^T \{N\}\, dxdy \qquad (5)$$

Here, z_n in equation (5) denotes the normal acoustic impedance ratio at the wall's surface.

2.2 Absorbent Finite Element In equation (5) the dissipation is modeled using the acoustic impedance. The other way to denote the dissipation in the system is to use some absorbent finite elements. One which represents a rigid porous material was given by Craggs[3] using the generalized Rayleigh model;

$$[M_{ab}]_e = \varepsilon K_s \Omega [M_a]_e, \quad [K_{ab}]_e = \varepsilon [K_a]_e,$$

$$[C_{ab}]_e = \varepsilon \frac{R\,\Omega}{\rho_a}[M_a]_e, \qquad \text{here,} \qquad \varepsilon = \frac{i\,\omega\,\rho_a}{R + i\,\omega\,\rho_{ab}}. \qquad (6)$$

Combining these elements into equation (1) and solving it, the acoustic response in a room with absorbent walls can be obtained.

3. TRANSIENT RESPONSE

In the studies of the architectural acoustics, the investigation about the transient response, like echo time pattern, may usually be important. Several methods can be applied on equation (1) to obtain the transient response. In the former papers, the following methods have been presented: step by step integration method (SSI)[2], modal analysis(Modal)[4][5], and impulse response with inverse Fourier transformation (IR-IFFT)[6].

3.1 Step by Step Integration Method (SSI) If the acceration term in equation (1) can be assumed to change linearly, the sound pressure $\{p\}$ can be in the form of

$$\dot{p}(t+\Delta t) = \dot{p}(t) + \int_t^{t+\Delta t} \ddot{p}(t)\, dt = \dot{p}(t) + \frac{\ddot{p}(t)+\ddot{p}(t+\Delta t)}{2} \tag{7}$$

$$p(t+\Delta t) = p(t) + \int_t^{t+\Delta t} \dot{p}(t)\, dt = p(t) + \dot{p}(t)\Delta t + \frac{\ddot{p}(t)}{2}\Delta t^2 \beta\, (\ddot{p}(t+\Delta t)\text{-}\ddot{p}(t)) \tag{8}$$

In the following analysis, the value of β is substituted by 1/6 [Linear acceleration method], or by 1/4 [Constant acceration method]. SSI can be applied to a system the conditions of which varies as time changes. In the method there is no need to take care about window functions. On the other hand, it is not suitable for the parallel processing because of its algorithmic nature. And the detailed frequency dependency of absorbent conditions are not easy to be involved into a computation. While, if the frequency dependency of the dissipation can be assumed to be constant, such is the case in a narrow band analysis, SSI can be applied successfully and simply enough.

3.2 Modal Analysis (MIR-IFFT) If the modal coupling caused by the dissipation in the system can be assumed to be small enough, the following modal analysis[4)5)] can be applied.

The steady state solution of equation (1) for the sinusoidal excitation of frequency ω can be obtained by

$$\{p\} = \sum_n \alpha_n \{\phi_n\} \tag{9}$$

Here,

$$\alpha_n = \frac{\{\phi_n\}^T\{\bar{f}\}}{(\omega_n^2\text{-}\omega^2 + 2\,i\, h_n\omega_n\omega)\, M_n},\quad h_n = \frac{\{\phi_n\}^T[C_\omega]\{\phi_n\}}{2\,\omega_n\, M_n},\quad \text{and } M_n = \{\phi_n\}^T[M]\{\phi_n\} \tag{10}$$

ω_n and $\{\phi_n\}$ are obtained by solving the following eigen equation:

$$([K] - \omega_n^2[M])\,\{\phi_n\} = 0 \tag{11}$$

Let all the modes excited by an impulse at t=0 begin to dissipate exponentially from t=0, the modal amplitude, α_n can be

$$\alpha_n = \frac{\{\phi_n\}^T\{\bar{I}\}}{\sqrt{1\text{-}h_n^2}\;\omega_n\, M_n} \exp(\text{-}h_n\,\omega_n\, t)\,\sin(\sqrt{1\text{-}h_n^2}\;\omega_n\, t) \tag{12}$$

here,

$$\{\bar{I}\} = \{W\}\,\delta(t) \tag{13}$$

Then, the transient response, after the steady state, *ie.* all the modes have been excited, to an arbitrary external excitation can be derived by applying the convolution to the steady state solutions with frequency ω's. This procedure basically corresponds with the equations derived through different way by Craggs[7)], and by Easwaran and Craggs[8)]. This method can clarify the relationships between acoustic characteristics and modal distribution of a room efficiently when the absorption on the walls are equally distributed and are small enough, or the equation is close to a singular condition.

3.3 IR-IFFT Let the external force term in equation (1) be an impulse, then the response in the frequency domain can be derived by solving the linear matrix equation in case it is not singular. When the external force term is unit velocity, the response sound pressure can also be regarded as the input impedance at the driving point and as the transfer impedance at the other points. Craggs[3)] showed the accuracy of the computed impedances in two dimensional absorbent materials by the method. With the impedance, the transient response to an arbitrary external driving force (sound) can be obtained using the convolution technique. Furthermore, with the impedance, the transfer functions between arbitrary two points in the sound field can be calculated easily.

By the method, the frequency characteristics of absorbent materials can easily be involved in the equation. In addition, as the matrix equations in the frequency domain are independent each other, the method is suitable for the parallel processing. Several example applications are shown below.

4. FREEDOMS IN THREE DIMENSIONAL ANALYSIS

The finite element method is usually said to require huge amount of computer memory especially when it is applied to a three dimensional analysis. Here, the required freedoms in the finite element analysis is discussed using a simple room with rectangular shape and with equaled finite elemental mesh division to show the difference with that of the boundary element method.

Let N be the total amount of the freedoms in the system, then $[M]$, $[C]$ and $[K]$ in equation (1) are symmetric matrices of order $N \times N$. Here, let us denote the total element number, $N \times N$, by T. In the analysis of sound field in actual rooms, $[M]$ and $[K]$ are real sparse-band matrices. Usually the former is positive definite. While, $[C]$ is usually a complex sparse-band matrix. The typical finite elements used in the acoustics field are shown in Fig.1. To make the points in the following discussion simpler and clearer, the 8 node and 27 node isoparametric elements are individually employed. Here, n_x, n_y and n_z are the numbers of node in x, y and z direction respectively. Then N can be

$$N = n_x \, n_y \, n_z \qquad (14)$$

In a simple equal mesh division, the band width N_b, N_{b8} for 8 node element and N_{b27} for 27 node element, on a typical case considering the symmetricity, become

$$N_{b\,8} = n_x \, n_y + (\, n_x + 2) \;, \quad N_{b\,27} = 2\, n_x \, n_y + (2\, n_x + 3) \qquad (15)$$

Then the total numbers of matrix element for a band matrix, T_b, can be

$$T_b = \sum_{n=N-N_b+1}^{N} n = N\, N_b - \frac{N_b(N_b - 1)}{2} \qquad (16)$$

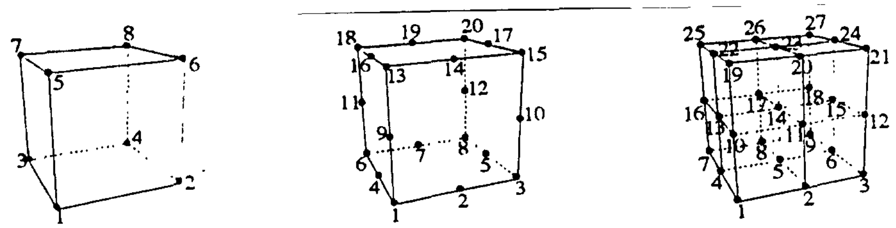

Fig.1 8 node (E8 : Left), 20 node (E20 : Center), and 27 node (SE27 : Right) isoparametric finite elements

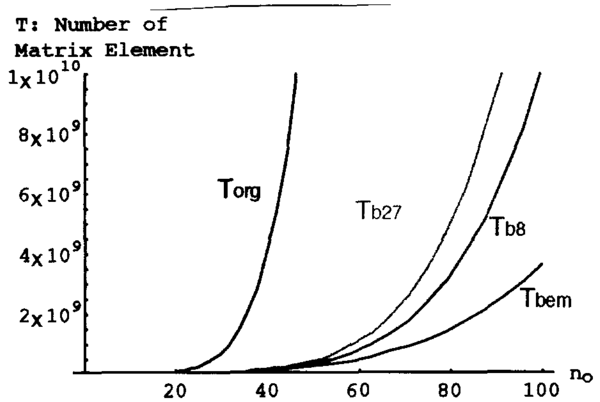

Fig.2 Comparison of total number of matrix elements T.

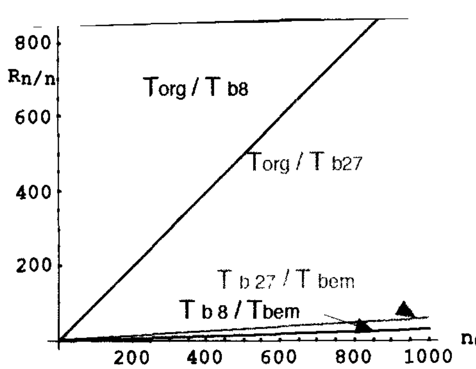

Fig.3 The ratios of T's.

In order to simplify the discussion, we let equal n_x, n_y and n_z to n_0, then the order of T becomes $O(n^6)$ and that of T_b becomes $O(n^5)$. The comparison is shown in Fig.2. There, T_{bem} represents that of a simple boundary element method with the same mesh division. In general, matrices in boundary element method are full and unsymmetric. Fig.2 shows that the ratio, R, of T_b/T_{bem} is 3.0 for E8 and is 5.6 for SE27 at n_z = 100, the ratio is in proportion to the increase of n_z, as is shown in Fig.3.

5. APPLICATIONS AND DISCUSSIONS

5.1 Methods' Combination Using several combinations of the methods, sound pressure response in a tube with a porous material (thickness = 0.1 m) on a wall (Fig.4) are computed to be compared. The combinations are as follows;

(#)	(Solver)	+ (Absorption Treatment)	: (Symbol)
#1.	SSI	+ Absorbent finite element	: SSI(e_{ab})
#2.	SSI	+ Normalized acoustic impedance (at surface)	: SSI(z_n)
#3.	IR-IFFT	+ Absorbent finite element	: IR-IFFT(e_{ab})
#4.	IR-IFFT	+ Normalized acoustic impedance (at surface)	: IR-IFFT(z_n)
#5.	Modal	+ Normalized acoustic impedance (at surface)	: Modal

The normalized acoustic impedance at the surface of the material is computed by the finite elemental procedure to examine the accuracy of itself. The results are shown in Fig.5 compared with the analytic solution[9], which shows good agreements. With the satisfactory result, the impedance values obtained in this way are used in the #2, #3, and #5.

The sound source is assumed to be located on the wall at x=0, and to be a tone burst filtered through Hamming-window to have 12 waves and 1000 Hz centered frequency. Fig.6 shows the comparison of #1,2,3 and 4, on the condition that the absorption coefficient of the wall at x=0 is assumed to be 1 (that of air). Fig.7 shows the comparison of #1~5, when the absorption coefficient to be 0 (hard). As the absorption is not distributed equally, the result by Modal shows some differences. In #2, however, the impedance was given only one value at

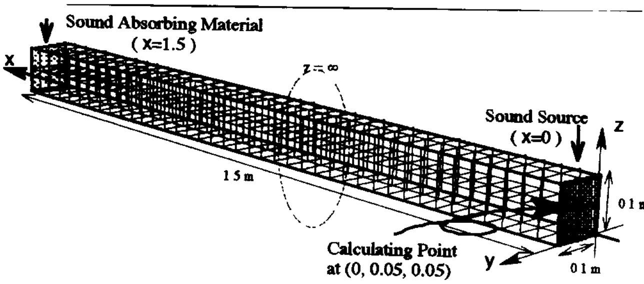

Fig.4 Geometry of Tube

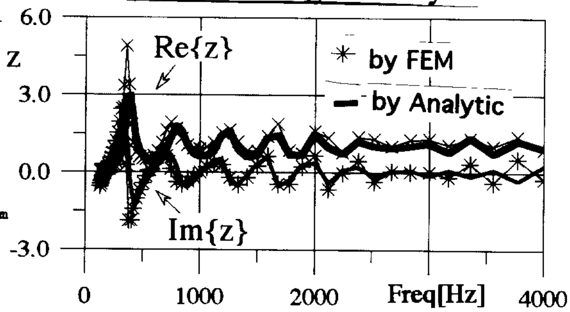

Fig.5 Comparison of computed impedance; FEM (*) vs Analytic solution (-), $R_{material}$=10000 Rayls/m, K_s=1, Ω=1

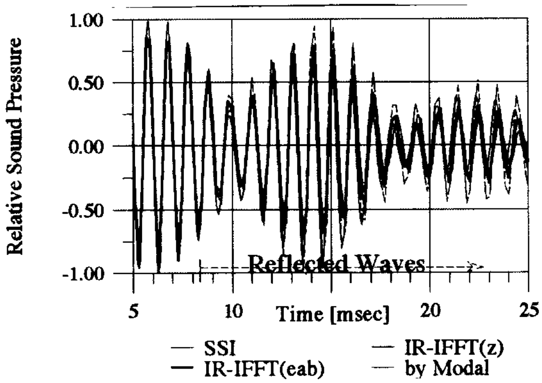

Fig.6 Comparison of computed wave forms.
$z_{n,x=0}$ =1 , $R_{material}$ = 10000 Rayls/m

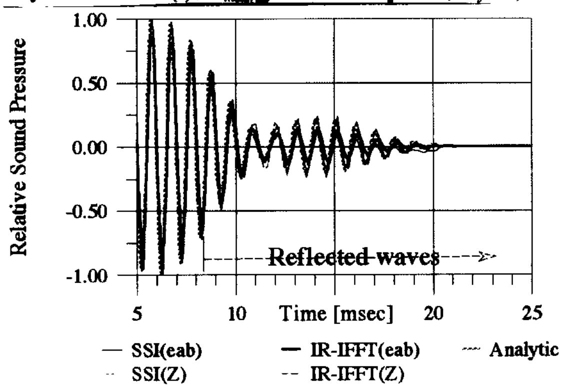

Fig.7 Comparison of computed wave forms.
$z_{n,x=0}$ = infinity, $R_{material}$ = 20000 Rayls/m

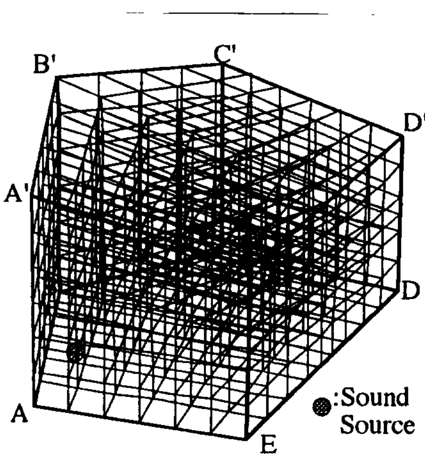

Fig.8 Geometry of a reverberation room (Volume: 188m³)

Table 1 Frequency distribution, $P(\%)$, of κ.

κ	~10^{10}	~0.1	~03	~0.5	0.5~
$P(\%)$	96.18	0.57	1.79	1.21	0.25

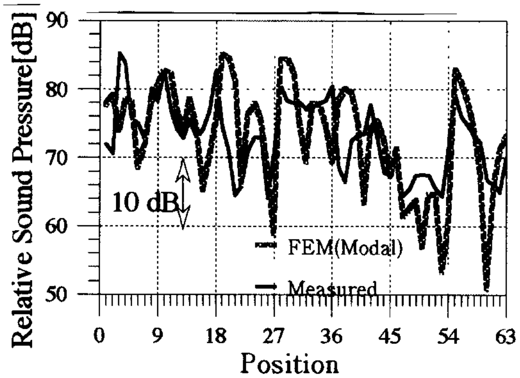

Fig.9 Comparison between computed and measured sound pressure in the reverberation room. (80Hz)

1000 Hz, the results show good agreements, because of the limited frequency components of the sound source.

5.2 Modal Analysis of a Reverberation Room At first, the modal coupling is investigated by calculating the frequency distribution of the ratio of diagonal elements to non-diagonal elements, κ.. The investigation was carried out on the analysis of a reverberation room (Fig.8). The absorbent condition was determined through the following procedure.

1. Averaged sound absorption coefficient, $\bar{\alpha}$, was obtained by the reverberation time measurement.
2. With $\bar{\alpha}$, normal acoustic resistance, γ_n, was obtained assuming the locally reactiveness.
3. Assume z_n equal to γ_n, and apply it to equation (5).

Table.1 shows that more than 96% of non-diagonal elements are less than 0.1 times the diagonal element in $\{\phi_m\}^T[C]\{\phi_n\}$. The computed steady state sound pressure distribution in the reverberation room is compared to measured values in Fig.9, which shows that the tendency of them agree well. Here, the one third octave band with the centered frequency of 80 Hz is assigned to the settings in this computation.

5.3 IR-IFFT Analysis of a Lecture Room To examine the difference between IR-FFT(e_{ab}) and IR-FFT(z_n) in the three dimensional computation, the echo time pattern distributions in a lecture room (7.2 m x 3.6 m x 2.7 m) with a glasswool (thickness = 0.025 m) on a side wall are computed. Here, the sound source is located at (0, 0, 0), and is a Hamming windowed tone burst with the centered frequency of 200 Hz. Fig.10 is the comparison of them on the 1.08 m high section plane. To clarify the similarity, scatter diagram of the echo time patterns at the Node (7, 6, 3) is shown in Fig.11. Its coefficient of correlation, r, is 0.993, and the mean value of r all over the room is 0.997. Then it can be said that the two ways of absorption modeling give almost the same results in this case.

5.4 IR-IFFT Analysis of a Scale Model Room with a Barrier To examine the accuracy of the IR-IFFT, the computed transfer functions in a scale model room (Fig.12) was compared with measured values. The room is the 1/5 scale model of the lecture room used in 5.3. The absorbent condition in the room is as follows.

C-1: Painted plasterboard
C-2: C-1 + a thin carpet on the floor
C-3: C-2+Glasswool(25K, thickness =0.025 m) on the end wall

The impedance value of the C-1 is decided in the same process as 5.2, and those of the other materials are measured by the two microphone method in an impedance tube. The computed and measured transfer functions are compared in Fig.13. As the assigned impedance value for

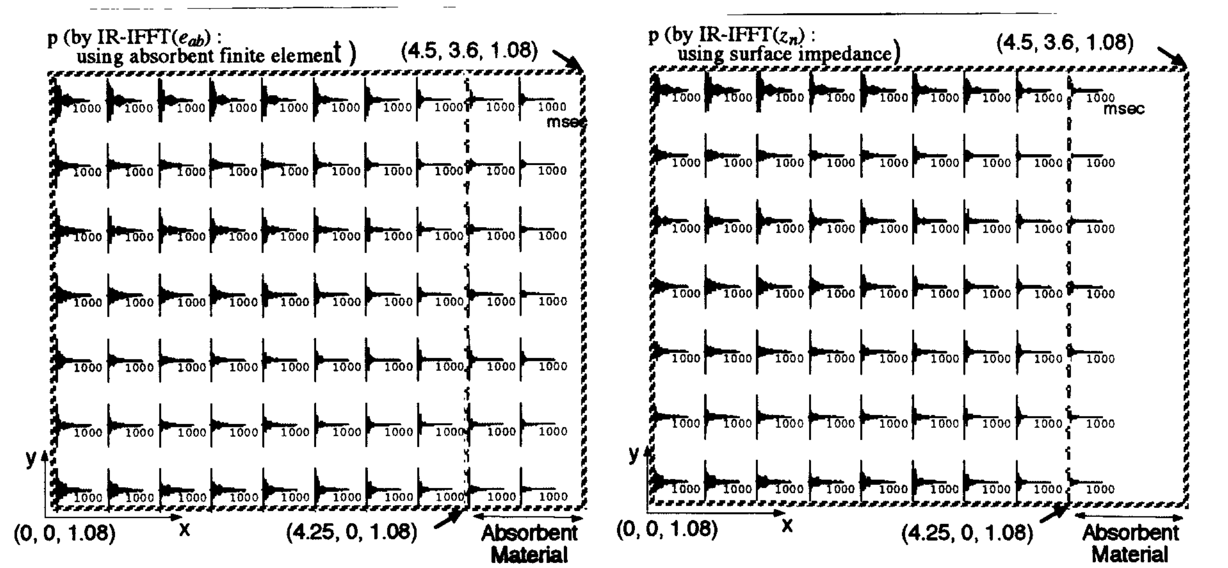

Fig.10 Echo time pattern distribution in a lecture room. (z=1.08 m high section plane)

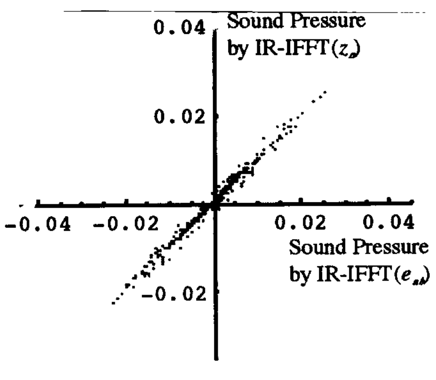

Fig.11 Scatter diagram.
IR-IFFT(e_{ab}) vs. IR-IFFT(z_n)

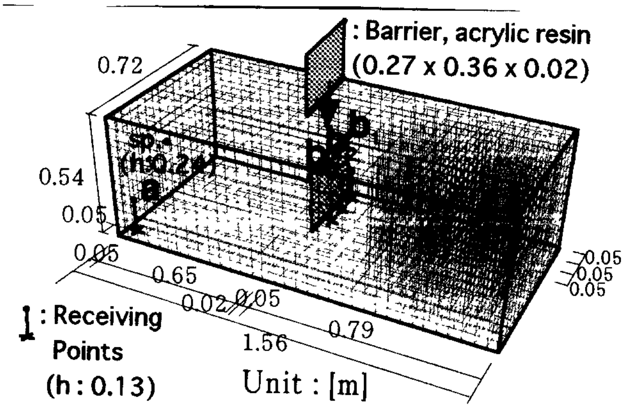

Fig.12 Geometry of a scale model room

the plasterboard was too simple to model the complex absorbent characteristics, the computed transfer functions do not agree very well around the eigen frequencies on the condition C-1. In other cases, however, good agreements can be seen; especially the measured values averaged in the 1/24 octave band by using digital filter agree well with the computed values on C-3. In Fig.14 the sound pressure distribution computed by the method is shown to give an example. It shows the changes caused by the absorbent conditions clearly.

6. CONCLUSIONS

Several combinations of methods to get the transient response of the sound field in rooms with absorbent walls have been presented above to show the following results: the modal analysis can be successfully applied to the analysis of a reverberation room, the modal coupling in the analysis could be regarded small enough, the dissipation matrix using the acoustic impedance gives almost the same result as the absorbent finite element in the cases, and the transfer functions computed by the method showed good agreements with the measured values. As the absorption considered above are simple porous materials only, further research is needed to refine the process and on the treatment of other kinds of absorbent materials.

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REFERENCES

1. Y. Kagawa, Vibration/ Acoustic engineering by FEM, Baifukan(1981) (in Japanese)
2. T. Otsuru, et.al., Finite Elemental Analysis of Sound Field in Rooms with Sound Absorbing Materials, INTER-NOISE 94, 2011-2013(1994)
3. A. Craggs, A finite element model for rigid porous materials, J.Sound Vib. 61, 101-110(1978)
4. T. Otsuru, et.al., Finite elemental analysis on the sound field in a reverberation chamber, Reports of the faculty of engineering Kumamoto Univ., 133-137(1995) (in Japanese)
5. T. Otsuru, Sound transmission analysis by computational mechanics using characteristic impedance derived through a finite elemental procedure, ICSV'2, 477-484(1992)
6. S. Choi, H. Tachibana, Estimation of impulse response in a room by the finite element method, J.A.S.J. 49, 328-333(1993) (in Japanese)
7. A. Craggs, A finite element method for the free vibration of air in ducts and rooms with absorbing walls, J. Sound Vib. 173, 568-576(1994)
8. V. Easwaran and A. Craggs, Transient response of lightly damped rooms: a finite element approach, J.A.S.A 99, 108-113(1996)
9. C. Zwikker, C. Kosten, Sound absorbing materials, Elsvir(1949)

SYMBOLS

c_a : velocity of sound in air
i : $\sqrt{-1}$ v : particle velocity
Ks : stracture factor z : normalized acoustic impedance
p : sound pressure $\delta(t)$: Kronecker's Delta function
R : resistivity Φ : velocity potential
t : time ρ_a, ρ_{ab} : mass density of air, and absorber
u_0 : displacement Ω : porosity

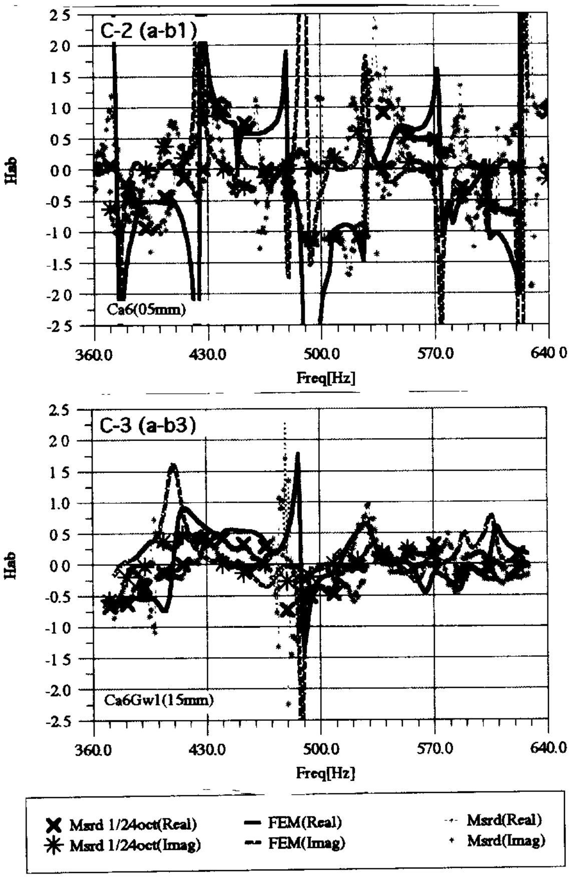

Fig.13 Comparison of transfer functions, (FEM vs Measurement)

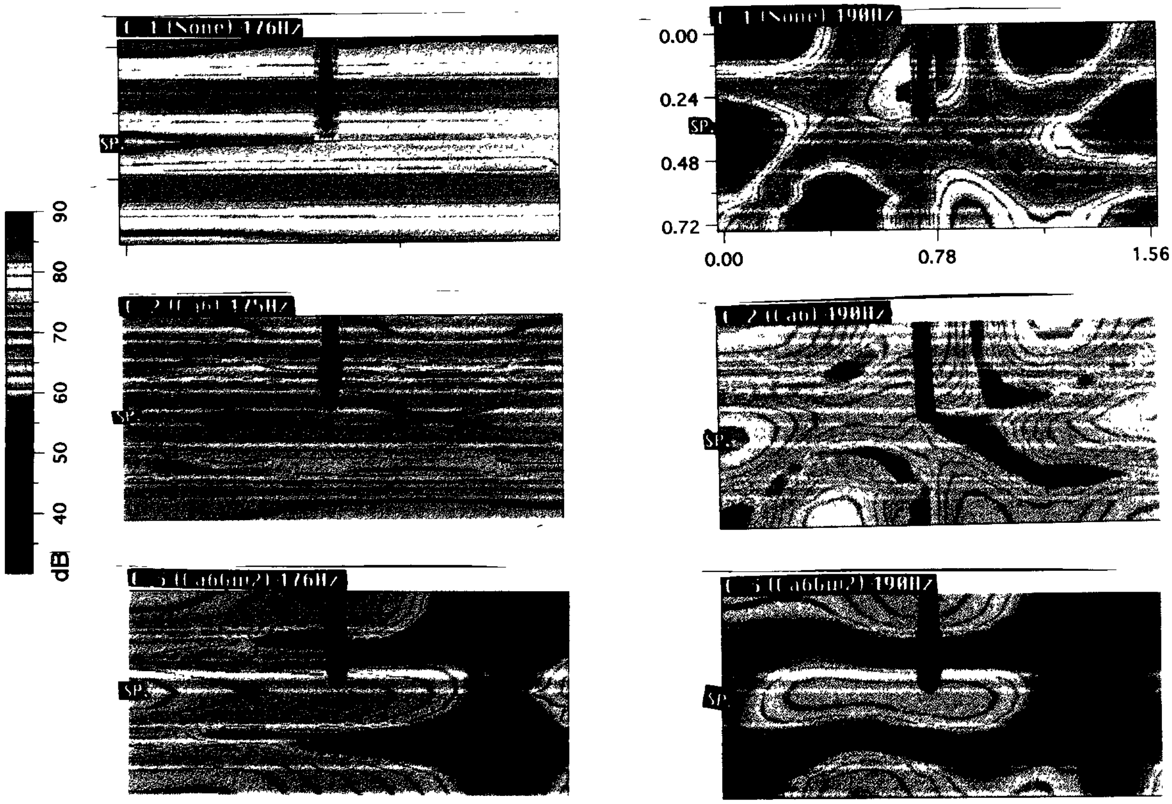

Fig.14 Sound pressure distribution computed by IR-IFFT on the 0.13m high section plane(Absolute values)