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### NEW APPROACHES TO ACTIVE NOISE CONTROL - THEORY AND EXPERIMENTAL RESULTS

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**Abstract:** The objective of the paper is to thoroughly present and examine new concepts of noise cancellation algorithms. They are based on different adaptive control structures: feedforward, feedback, and hybrid as well as different signal processing techniques: simple filtering, filter banks, and multirate signal processing. However, the algorithms have physical / heuristic origin. Their efficiency is confirmed by real-world experiment results presented and discussed in the paper. Stability of FIR filters is also analysed and conclusions are drawn.

#### 1. INTRODUCTION

Active noise control (ANC), in which additional secondary sources are used to cancel noise from original primary sources, has received considerable interest and has shown significant promise especially in fields where passive methods are not adequate for many reasons [1]. Both acoustic and electric paths usually vary in time so ANC algorithms are required to have adaptation features [1], [7]. In the literature there are a lot of solutions based on feedforward, feedback, or hybrid (combined both of these techniques) control. Although the goal is to reduce the levels of unwanted sound via phase cancellation, substantial differences in physical configuration, analog / digital signal processing, adaptive filtering and control techniques, frequency ranges, and attenuation performance exist among different devices. This paper presents some new algorithms based on different concepts aiming at improving cancellation performance.

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To design and test adaptive algorithms a laboratory rig has been assembled. It consists of artificial ear, personal hearing protector (also called ear defender), voltage amplifiers, power amplifiers, loudspeaker, PC 486, DSP board with TMS320C31 signal processor, anti-aliasing low-pass filters, and A/D, D/A converters. 2 [kHz] had been chosen as the basic sampling frequency [3].

On the basis of plant identification it was found a parametric model to be nonminimumphase. From analysis of coherence function follows that only making attempt at cancelling noise in the frequency range of 150 - 840 [Hz] is justified. Test of plant linearity was also performed. It confirmed that all the acoustic and electric paths can be regarded as linear.

Attenuation is evaluated with the aid of Solartron Schlumberger spectral analyser (sampling the signal with 40 [kHz]) in two ways. In the first one the attenuation of only main harmonic  $AF_{1h}$  is assessed, and in the second - the attenuation in the band of 80 - 5000 [Hz]  $AF_{band}$  is assessed. This takes into account the real signal reaching human's ear, including other harmonics due to complex phenomena in the plant as well as additional noise [5].

## 2. FEEDFORWARD CONTROL

Block diagram of feedforward control is depicted on Fig.1. LMS is used for identification.

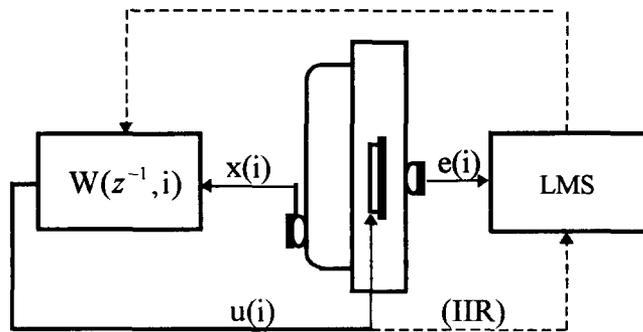


Fig. 1. Personal active hearing protector with feedforward control.

### FIR filter - stability analysis; performance improvement (NWLMS algorithm proposal)

For a FIR filter  $(W(z^{-1}, i) = S(z^{-1}, i))$ , where  $S(z^{-1}, i)$  is a  $z^{-1}$  polynomial of order  $dS=dW$  control value is calculated as a weighted sum of only reference signal  $x$ . It is specific that FIR filter is perfectly adjusted to the frequency of the signal not matter if the signal is cancelled [6]. But for frequencies beyond the attenuation band ( $AB$ ) the filter parameters increase linearly in time. After examining their behaviour, the following relation was noticed:

$$\exists_{n \in \cdot 0, dW-1} \max(|w_n(i)|) > 0.5 \Rightarrow \forall_{n \in \cdot 0, dW-1} |w(i)| \sim i. \quad (1)$$

As the result control values are only constrained by the hardware. This implies that rectangular-shaped signal is send to the secondary source. Cancellation is then impossible. Thus, a solution is to constrain the parameters. Commonly known from the literature *Leaky LMS* [2] failed to cope with this problem. The parameters became bounded but unfortunately the  $AB$  was not extended and even the attenuation factor was worse.

A new modification of the LMS algorithm was proposed. Similarly to normalisation of reference signal, filter parameters are proposed to be normalised. This modification was named *Normalised-W LMS (NWLMS)* and the parameters update equation takes form (2):

$$w(n+1) = w(n) + \frac{\mu x(n)e(n)}{b \text{Max}(|w(n)|) + a}, \quad (2)$$

where  $a$  and  $b$  denote constant coefficients, adjusted experimentally (e.g.  $a=0.05$  and  $b=2$ ), and  $\mu$  is the step size in LMS algorithm. The band was not extended as well, but the speed of convergence was increased about ten times, and the steady state error was diminished, what is extremely important in such application like personal hearing protector. Assuming nullified starting parameters, in the first stage of identification in the denominator only  $a$  exists what reveals as increasing of  $\mu$  20 times. This reflects in speeding up the algorithm but also increasing the steady state error. During the adaptation process, the norm becomes larger and it reveals as decreasing the step size several times. Finally, the steady state error decreases and attenuation improves.

Looking for the reason why such filters diverge for frequencies beyond the attenuation band, an analysis of their roots was carried out [6]. It turned out that they are nonminimumphase outside  $AB$  and minimumphase - inside. Feedforward control with FIR filters should ensure stability of the system unconditionally. But if the filter is adaptive any adaptation algorithm uses error signal to update filter parameters. This introduces "artificial" feedback path to the system and leads to instability if the system is nonminimumphase. Taking into account results of spectral analysis (moduli of frequency responses of the filters and power spectral densities of control signals that confirm proper frequency adjustments) the idea of employing spectral factorisation was put forward. This stabilises the system but still requires a phase matching algorithm.

### PHS - a new physical / heuristic approach

A sinusoid passing through any linear path is changed only in magnitude and phase. To achieve noise cancellation in the real plant at observation point  $e$ , it is not necessary to perform complicated processing over signal  $x$  but only scale it in magnitude and delay in time. The time delay should be of the value that corresponds to phase shift  $\Delta\varphi$  which satisfies the following phase equation (time delay and phase shift are equivalent):

$$\varphi_{x-e} = \varphi_{\text{electr}} + \varphi_{u-e} + \pi + 2n\pi + \Delta\varphi, \quad (3)$$

where:

$\varphi_{x-e}$  - phase shift between points  $x$  and  $e$ ;  $\varphi_{u-e}$  - phase shift between points  $u$  and  $e$ ;  $\varphi_{\text{electr}}$  - phase shift of the electric path;  $\pi$  - symbolises delay of sine-wave of half its period;  $n$  - an integer number;  $\Delta\varphi$  - phase shift performed by the algorithm.

Due to continuous character of the real plant, time delays introduced by all its parts are not integer multiples of the sampling period. To meet Eq. 3, an algorithm able to model the required phase shift should be designed. The operator  $z^{-q}$  allows to roughly delay the signal (noise) with accuracy to half the sampling period. So the possibility of continuous delay correction in the range between zero and half the sampling period is still required. To obtain

that, very simple corrector  $\frac{1}{1 - r_1 z^{-1}}$  is sufficient. Scale factor  $s_1$  added to such a filter ensures

amplitude matching of the two signals to be interfered. As the final result, the phase shifter can be given in form (4):

$$W(z^{-1}, i) = z^{-q} \frac{s_1}{1 - r_1 z^{-1}} = \frac{S(z^{-1}, i)}{R(z^{-1}, i)}. \quad (4)$$

Assuming plant stationarity discrete time delay  $q$  depends only on frequency of the signal (noise) being considered. Its value is suggested to be evaluated on the basis of squared error minimisation [5].

Suggested and presented above phase shifter reveals the following features (see also [5]):

- its concept is based partly on physical and partly on heuristic - not automatic - approaches to noise cancellation problems;
- is suitable only for narrowband sounds with spectrum concentrated around one frequency but can be easily extended to any sounds;
- having minimum order of parameters - an order less than for the other solutions - guarantees great attenuation effects:  $AF_{1h} = 60$  [dB], and  $AF_{band} = 40$  [dB];
- extends attenuation band: 250 - 500 [Hz] with  $f_s = 2$  [kHz];
- convergence speed is almost independent of exciting signal;
- does not accumulate quantization errors;
- perfectly copes with real nonstationarity of amplitude of the noise to be cancelled and with nonstationarity of its frequency up to 30 [Hz].

### PHS2 - sensitivity / computational burden trade off

This algorithm has had the same origin as PHS and is based on similar concept. The number 2 in its name comes from *two* parameters to be identified (5):

$$W(z^{-1}) = z^{-1} \frac{g_1(b_1 + z^{-1})}{1 + b_1 z^{-1}} = \frac{S(z^{-1})}{R(z^{-1})}. \quad (5)$$

The modulus of the frequency response is uniform and the phase can be changed in the range  $\langle -\pi; 0 \rangle$ . For any phase changes only parameter  $b_1$  is responsible and for amplitude matching - parameter  $g_1$ . It is specific that only two parameters are to be identified and such a filter meets all the requirements to actively cancel any pure tones. Its features are very similar to features of PHS presented above. Attenuation factor reaches values like those obtained by PHS but the  $AB$  is wider:  $\langle 180; 680 \rangle$  [Hz]. Besides, it does not need discrete time delay identification what takes majority of the time. Concluding the results presented above and the analysis of computational burden, PHS2 algorithm seems to be better than the others and even PHS. But on the other hand, in PHS2 phase adjustment is performed only via one parameter and the parameter is responsible for correction of  $\pi$  while in PHS algorithm parameter  $r_1$  adjusts the phase only of  $\frac{f}{f_s} \pi$  (e.g. for  $f = 250$  [Hz] and  $f_s = 2000$  [Hz], the adjustment is of  $\frac{1}{8} \pi$ ). So the sensitivity of PHS2 is very high (at least four times higher than of PHS) and finally its robustness to nonstationarities is poorer.

### Complex tones cancellation - PHS-Banks

The idea of PHS (as well as PHS2) was extended to broadband noise and was named PHS Banks. Each bank consists of a band-pass filter and a PHS (see: Fig. 2). A PHS can cope with signal having spectrum not wider than about 40 [Hz], so the filters should be properly designed. They have to have very high selectivity and moduli of the frequency response of neighbouring filters do not have to cross each other in resonance peaks [6]. They are suggested to be designed using a least-squares method. Assuring such constraints, described algorithm is able to attenuate any sound in the whole band up to 40 [dB]. The frequency limit is imposed only by the speed of the signal processor employed. It is very important that all the PHS filters are destined for bands known beforehand. Thus, the discrete time delays can be fixed in

advance and do not have to be identified. Finally, for  $n$  banks only  $2n$  parameters:  $s_1, \dots, s_n$ , and  $r_1, \dots, r_n$  have to be identified (e.g. by LMS) what constitutes the same number of parameters as for PHS2.

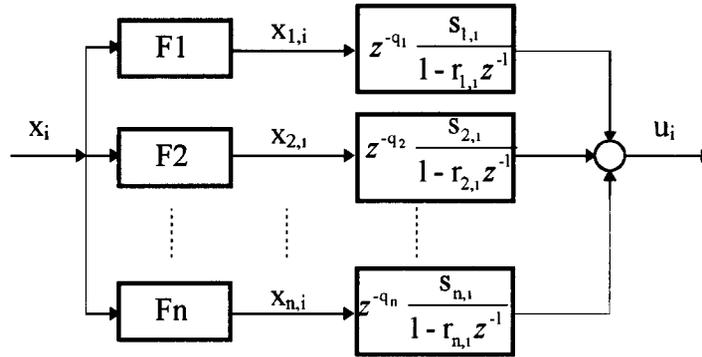


Fig. 2. The block diagram of PHS-Banks algorithm.

### Extension of the attenuation band - multirate signal processing

Experiments performed with various sampling frequencies (2, 1, 0.5 [kHz]) and feedforward control with PHS or FIR adaptive filters show that for each sampling frequency attenuation bands obtained are adjacent or slightly overlapped, and usually octave (e.g. for PHS:  $f_s=2$ [kHz]  $\Rightarrow AB \in <250;500>$  [Hz];  $f_s=1$  [kHz]  $\Rightarrow AB \in <150;300>$  [Hz];  $f_s=0.5$  [kHz]  $\Rightarrow AB \in <100;125>$  [Hz]) [5]. On the basis of these results it was found that varying sampling rate, it is possible to move noise cancellation range along frequency axis. An algorithm converting signal sampled with an arbitrary chosen frequency to signals as if they were sampled with other frequencies is termed *Multirate Signal Processing (MSP)* [4]. So the idea is to sample signals with one frequency and process them in different channels with different rates covering very wide band. For the problem under consideration the MSP system consists of band-pass anti-aliasing filters, down-samplers, adaptive FIR or PHS filters, up-samplers, and low-pass anti-imaging filters (see Fig. 3). It is noteworthy that adaptive filters implemented as FIR filters are identical with exactly the same parameters at each channel [4] what makes the band-pass anti-aliasing filters very efficient if they are properly designed (e.g. for 17 parameters only 4 multiplications are required) [4].

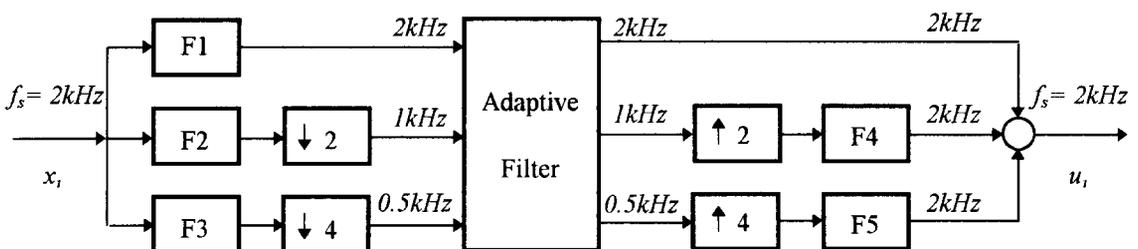


Fig. 3. Multirate signal processing structure with adaptive filtering.

It was experimentally proved that employing the idea of MSP combined with FIR [4] or PHS [5], it is possible to cancel any noise in any band. The limits are imposed only by the hardware equipment used (the lower limit is constrained by the pass-band of loudspeakers and the upper limit - by the speed of the signal processor used).

### 3. FEEDBACK CONTROL

Another algorithm used for noise cancellation was direct adaptive feedback control (Fig. 4).

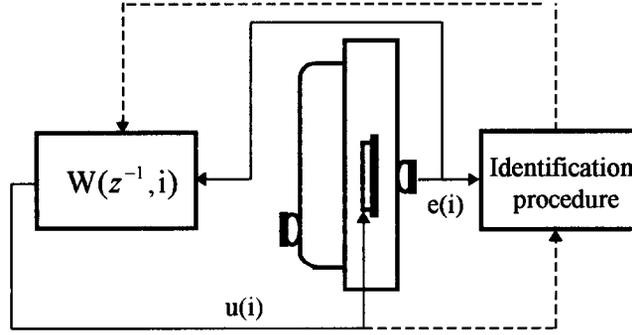


Fig. 4. Personal active hearing protector with feedback control.

Having taken into account that variance is directly related to the amount of energy in a signal and ANC aims at minimising sound energy reaching the human's ear minimum variance controller was employed. The results of off-line identification paid attention to the fact that obtained plant was nonminimumphase. This required special modification of Simple Minimum Variance Control (SMV) to have stable system. The literature suggests weighting of the control value (Weighted Minimum Variance Control - WMV). Control value is calculated according to the following formula:

$$u(i) = -\frac{1}{\bar{b}_0} \varphi^T(i-k) \hat{\theta}(i), \quad (6)$$

where:  $\bar{b}_0$  - the coefficient (assumed beforehand) greater than the free term of the numerator of the plant control path,  $\hat{\theta}(i)$  - controller parameter vector,  $\varphi(i-K)$  - the vector of regressors:

$$\varphi^T(i-k) = [e(i-k), \dots, e(i-p-k+1), u(i-k-1), \dots, u(i-2k-p+1)], \quad (7)$$

where  $p$  is the order of polynomials of the parametric model.

The parameter vector update equation, with gain vector  $\mathbf{k}(i)$  identified by *Weighted Recursive Least Square* algorithm [3], shows Eq. (8) :

$$\hat{\theta}(i) = \hat{\theta}(i-1) + \mathbf{k}(i) \left[ e(i) + \frac{q}{\bar{b}_0} u(i-k) - \bar{b}_0 u(i-k) - \varphi^T(i-k) \hat{\theta}(i-1) \right]. \quad (8)$$

Penalty imposed on the control value influence the behaviour of the whole system. It allows the system to be stable even for nonminimumphase plant, decreases control variance but unfortunately increases output variance. There is no theory to chose optimum  $q$ . Real-world experience showed that to assure stable performance with adequate stability margin it was necessary to agree with noticeable deterioration of noise cancellation.

Carried out experiments proved that stable system based on direct minimum variance control can be achieved without control weighting even for nonminimumphase plant [3]. It turns out

that extra delaying (of discrete time  $\tau$ ) the control value in the parameter update equation (8) (see Eq. (9)) stabilises the system.

$$\hat{\theta}(i) = \hat{\theta}(i-1) + \mathbf{k}(i) \left[ e(i) - \bar{b}_0 u(i-k-\tau) - \phi^T(i-k) \hat{\theta}(i-1) \right]. \quad (9)$$

This method has been termed as "Modified Minimum Variance Control" - MMV. Summarising obtained results one can state that MMV works quite well (better than WMV) for low frequencies: from 100 to 350 [Hz]. Attenuation factor reaches less than 15[dB] but with very high rate of convergence (about one fifth of second). Beyond the range of 100 - 600 [Hz] the system loses stability.

#### 4. HYBRID CONTROL

On the basis of the results presented thus far the hypothesis was advanced that integration of two different techniques: feedforward and feedback can assure extension of the cancellation band with satisfactory attenuation [3]. Unfortunately, known from the literature the way of common identification of the controller and feedback parameters failed for the investigated plant. These two adaptive techniques disturbed each other. So another approach has been proposed. The controller and filter parameters are identified separately but for their adjusting the same output error is used (see Fig. 5).

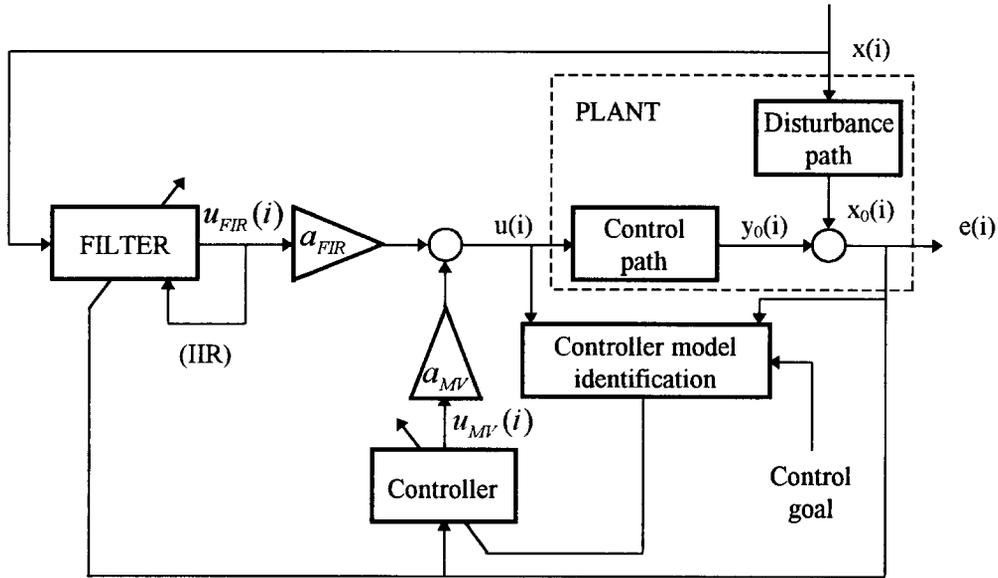


Fig. 5. Adaptive hybrid control system

Control value is calculated as a weighted sum of components derived from both parts (10):

$$u(i) = u_{FIR}(i) \alpha_{FIR} + u_{MV}(i) \alpha_{MV}(i). \quad (10)$$

Weighting coefficient:  $\alpha_{FIR}$  and  $\alpha_{MV}$  are introduced to increase the number of freedom. For good co-operation of these two techniques an arbitrary system has been extra employed. Convergence of the parameters serves as the criterion of proper choice.

$$\Theta_{FIR}(i) = \sum_{j=1}^N \left| \hat{\theta}_{FIR,j}(i) \right|; \quad \Theta_{MV}(i) = \sum_{j=1}^N \left| \hat{\theta}_{MV,j}(i) \right|. \quad (11)$$

At each step the sums of absolute values of parameters are calculated (11) and compared with the number of the filter and controller parameters, respectively (12):

$$\Theta_{FIR}(i) \leq \zeta_{FIR} N; \quad \Theta_{MV}(i) \leq \zeta_{MV} M. \quad (12)$$

$\zeta_{FIR}$  and  $\zeta_{MV}$  are adjusted experimentally. If any of the inequalities (12) is not fulfilled the respective channel is rejected by nullifying either  $a_{FIR}$  or  $a_{MV}$ , or even both. The last possibility expresses a situation when the noise is of such character (spectrum) that making any effort at controlling the system reveals in sound reinforcement.

Obtained results have confirmed theoretical predictions. Suggested method assures quite good cancellation throughout almost the whole frequency range justified by analysis of plant coherence function, keeping stability of the system [3].

## 5. CONCLUSIONS

In the paper new active noise control concepts were proposed. They are based on feedforward (FIR with Normalised-W LMS, PHS, PHS2), feedback (Modified Minimum Variance), and hybrid (feedforward and feedback combined by the arbitrary system) adaptive control. Adoption of filter-banks processing as well multirate signal processing techniques to broadband noise cancellation problems was also developed. Besides, stability of FIR filters was analysed and conclusions were drawn. Although all the algorithms described above were tested on personal active hearing protector, it is obvious that they can be employed to other active noise control appliances.

## 6. LITERATURE

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