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DIAGNOSTICS OF GEARS USING HIGHER ORDER SPECTRAL ANALYSIS TECHNIQUES

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ABSTRACT
Assuming that the basic role in transmission of such diagnostic information is played by the phenomena of amplitude and phase modulation of the vibroacoustic signal, the author points to the significance of the signal's non-linear components. Utilizing the higher - order spectral analysis method and time-frequency reprezentations methods, a procedure to determine the changes in the spectrum structure caused by system nonlinear effects and modulation phenomena of input signal was developed.

INTRODUCTION
Relative ease of installation of detectors and respectively simple organization of transmission and storage of data open up increasing application possibilities for vibroacoustic diagnosis methods. Particular place is occupied by early defect's detection problems, which essentially concerns non-linear effects, including amplitude and phase modulation of a signal.
Let us note that additionally the very process of defect formation is non-linear. If the intensity of defects is small in the early stages, the pre-failure stages of operation are accompanied by general growth of the amount of dissipated energy and by the often related to it growth of vibration and noise. The observations mentioned here became on the one hand an impulse to elaborate many norms of diagnostic evaluation, for example referring to the value of effective vibration velocity or to the peak values of the registered vibroacoustic signals, and on the other the basis for elaboration and development of methods to evaluate and signal the pre-failure states of the diagnosed objects [1].
A tool that is fully useful for implementation of this task turns out to be the analysis of the power of signal's spectrum, which in its essence draws on the Fourier's analysis of stationary signals. Thus the frequency characteristics achieved by with the use of Fourier's transform on the basis of a sample with defined dimensions can generally be interpreted as averaged frequency structure of this signal for the whole period of analysis. Apparent controversy between assumption of the condition stationary character and observation of the growth of signal's power results for the fact

that comparative observations refer to consecutive measurements during the operation period (observations of the so-called slow-changing process), while the frequency analysis is conducted for each measurement result separately and concerns dynamic phenomena (observation of the so-called quick-changing process). However, such procedure, following the models of relations between the stages of defect development and the generated vibroacoustic signal as assumed in literature on this topic, enables detection of defects in their final development stages leading to the growth of the overall level of signal power. Broader description of the problem is found in [2]. At this place let us just note the appearance of a defect and the low-energy stages of its development are accompanied most often by local disturbances of the signal's run, and the resultant growth of power is not essential from the point of view of its measurement. However, local defects, while maintaining the general level of vibroacoustic signal's power, can cause measurable changes both in the run of the signal in time and its frequency structure. Such state of things inclines one to formulate the diagnosis of defect origin on the basis of diagnostic information carried by the low-energy components of a vibroacoustic signal.

MODELLING OF THE PROCESS OF SIGNAL GENERATION

The dynamic properties of a vibrating system such as a pair of toothed wheels are to a great extent shaped by the periodically variable rigidity of meshing. Both the period of variability, as well as dependence of rigidity of meshing on the location on the path of contact are on the one hand a function of the main structural parameters of a toothed gear, and on the other a function indicating precision of manufacturing and assembly, and of wear and tear. In general we can list a number of external and internal factors, shaping the mechanism of vibroacoustic signal generation. In the first case among the most important we have: load changes, occurrence of impact, imbalance of elements and sub-assemblies, and errors connected with assembly and wear and tear of the units of a kinematic chain.

Respectively, among internal factors we above all point to the already mentioned rigidity of meshing. Additionally we should include the effect of the change of direction of the sliding velocity at the rolling point of contact, errors related the involute errors connected with coming into and out of contact, errors of eccentricity, in particular of base circles and tip circles.

The basic specification shows that the majority of listed factors are characterized by a defined variability which is determined respectively by a rotating motion of gear shafts or by the parameters of the wear and tear process of individual elements, including the profiles of directly mating teeth. As a result, when constructing and analyzing the models of vibroacoustic signal generation, attention should be chiefly paid to the necessity of restoring the variable conditions of contact. One of the possible approaches is introduction of variable geometry of meshing by accounting in the model for momentary apparent base circles (Fig. 1). By simultaneously introducing the notion of apparent interference we can thus examine the variability of the force working between the teeth in the function of a summary error manifesting

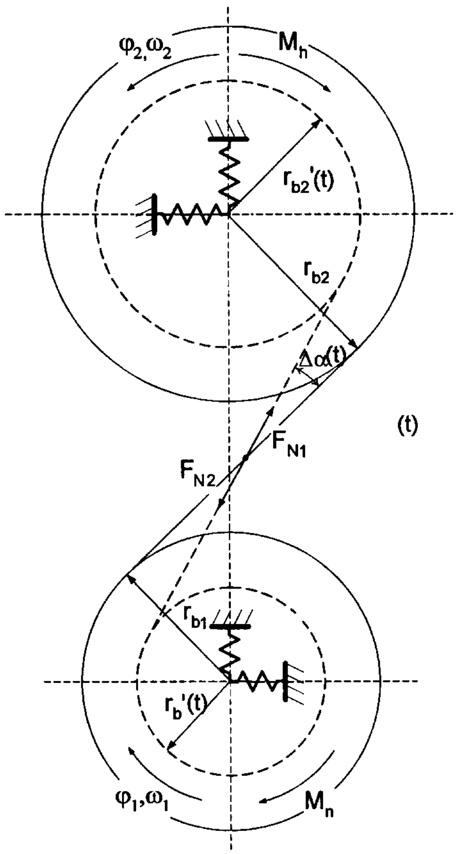

Fig. 1

itself with difference of location on the path of contact between the actual and theoretical point of contact. Taking advantage of the fact that the first frequency mode, in comparison to the wheel's free vibration and the frequency of inputs, caused by the periodic entry of a tooth into contact, is essentially bigger, we can make an assumption about linear relation between the tooth vibration speed and the force working between the teeth. Thus it results that in this case we can treat a tooth as a plate excited by low frequency vibration, and so as a system which is analogous to a system with one degree of freedom.

Occurrence of errors can lead to change of the conditions of contact, including: the qualitative change of the process of coming into / out of contact, disturbance of the linearity of the path of contact, and others which as a result lead to non-linear changes of the system's parameters, in particular of rigidity and damping. This has essential impact upon the frequency structure of generated vibration. For example, occurrence of non-linearity which may be defined by means of a bilinear operator influencing harmonic functions

$$L_2(x_1+x_2) = L_2(x_1) + 2L_2(x_1,x_2) + L_2(x_2) \tag{1}$$

where: $L_2\,(x_1,x_2) \neq 0$ will lead to appearance in the spectrum of additional components which will be shaped by the sums and differences of frequencies occurring in x_1 and x_2.

Terms $L_2(x_1)$ and $L_2(x_2)$ will respectively generate components having frequencies corresponding to the double values of the input frequencies. The work [3] additionally points to the fact that a square element of a system generates components in which initial frequency and phase are defined by the same relationship (square phase coupling) This means that in order to identify coupled phase-wise it is necessary to preserve the data about the run of the phase.

Such information, in contrast with power spectrum, is preserved by higher order spectra, particularly by bispectrum. Additionally occurrence of non-linearity (as pointed out by Bendatt [4]) leads to occurrence of relevant feedbacks. Thus we can expect that relevant disturbance of the input signal will appear, manifesting itself by amplitude and phase modulation of its parameters (Fig.2).

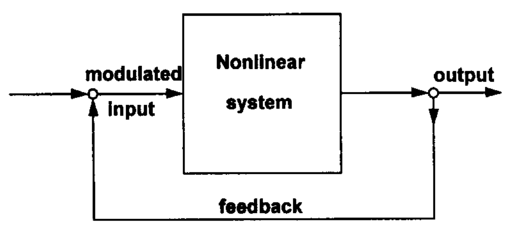

Fig. 2

Thus the problem of disturbed signal analysis can be analyzed in two aspects: analysis of signal's frequency structure changes in its modulated bands, and search for components and bands caused by the system's non-linearity.

4. DETECTION OF NON-LINEAR DISTURBANCE

Let us note that low-energy impulse disturbances cause broad-band response with small amplitude and that is why typical spectral analysis of averaged power spectrum in the domain of frequency, as well as the correlation function in the domain of time can contain information of second order process. For example a sufficient characteristic of a Gauss process with an average value equal to zero is its covariance function. However, in the case of process with non-Gauss distribution of probability, respectively the correlation function or the power spectrum supply only partial information about the process. Analysis of higher-order spectra, properly defining the non-linear effects, is required to obtain a more precise description. This observation pointed to the need for the analysis of the frequency structure of a signal in the plane defined by time and frequency [3]. Some of these, and in particular the spectrogram, multi-dimension Fourier transform, Wigner-Ville distribution, and bispectral analysis were discussed in [3,5], in the context of their usability to examine the modulated vibroacoustic signals. Let us note at this place that as a result of signal modulation additional components will appear in the spectrum, respectively in the form of sums and differences of carrier frequencies and modulating frequencies in the case of modulation by a harmonic function. This points to the significance of the bi-spectrum, which is responsible for "third-order information". If the generally accepted interpretation of the spectrum of vibroacoustic signal's power does not raise any objections, the attempts to interpret the information found in the higher-order spectrum are not so clear. From the interesting to us point of view of using the time and frequency representation to analyze the phenomena of modulation of a signal generated by disturbance of the meshing and contact conditions, the most interesting proposal is presented in [3]. Referring to Cohen's classification, it was assumed that squared time and frequency distribution of:

$$T_x(t,f) = |a_1|^2 T_{x_1}(t,f) + |a_2|^2 T_{x_2}(t,f) + a_1 a_2 T_{x_1 x_2}(t,f) + a_1 a_2 T_{x_2 x_1}(t,f) \quad \textbf{(2)}$$

with: $x(t) = a_1 x_1(t) + a_2 x_2(t)$, representing the dependence of the signal energy from these coordinates, or the momentary power spectrum.

Let us note that in a this way, by using $c_{3,x}(t,\tau_1,\tau_2)$ - a third order cumulant it is possible to present a relationship defining the bispectral Wigner distribution:

$$W_{3,x}(t,f_1,f_2) = \int\int c_{3,x}(t,\tau_1,\tau_2)\, e^{-j2\pi(f_1\tau_1 + f_2\tau_2)} d\tau_1 d\tau_2 \quad \textbf{(3)}$$

With the assumption of relevant stationary character of a signal in accordance with Gerr's proposal [5], the relationship is in force for a bispectrum and for a bispectral Wigner's distribution:

$$E\{W_{3,x}(t,f_1,f_2)\} = \int W_{3,x}(t,f_1,f_2)\, dt = S_{3,x}(f_1,f_2) \quad \textbf{(4)}$$

Thus a question appears, can more data be also obtained, both quality and quantity related data concerning the type and the extent of signal's modulation, by using the bispectral Wigner's distribution. Taking into account the fact that multiparameter modulation phenomenon, and the related additional complication of the spectrum's structure appear in a toothed gear, obtaining of a positive answer can have essential application significance.

EXAMPLE OF ANALYSIS OF A MODULATED SIGNAL

To present the possibilities and limitations of the discussed manner of signal analysis we performed a computer simulation of the phenomenon of time and frequency representation of phase modulation of a signal:

$$x(t) = A_0 \cos[2\pi f_i (1 + \frac{dz}{f_r}) \cdot \cos(2\pi f_l) t + m \sin(2\pi f_r (1 + \frac{dz}{f_r}) \cdot \cos(2\pi f_l) t + \varphi) + \theta_i]$$

$$when \begin{cases} dz = 0{,}1 \rightarrow f_l = f_r/4 \\ dz = 0 \rightarrow f_l = f_r \end{cases} \quad \textbf{(5)}$$

where: m - modulation index, f_i - signal's carrier frequency, f_r - frequency of the modulating function φ, θ_i - relevant initial phase.

The simulation was performed with the assumption tha one carrier frequency appears in the signal, respectively: $f_i = f_m = 200\ Hz$, phase-modulated with the modulating frequency of $f_r = 10\ Hz$.

Additionally it was assumed that the value of modulation index is $m=2$.
What captures attention is the effect of frequency deviation noticeable both in the contour and in the spatial layout. This effect conforms to the period of the modulating function. The thus obtained distribution of signal power in the time and frequency system is relatively complex while the effects of disturbance can be observed far beyond the theoretically possible bandwidth of the modulated band.

Obtained this way signal's power function in the time and frequency system is relatively complex, while the effects of disturbance can also be observed beyond the energetically essential width of the modulated band. The effect of the "modulating function's echo" is particularly visible. It constitutes an essential disturbance, making correct identification of the modulated band's and modulating function's parameters impossible. The theoretical considerations devoted to Wigner's distribution [3,5], in accordance with the common terms found in formula (5) point to the possibility of appearance of additional components, the so-called "artefacts". We would like to point to the fact that the number of additional components grows together with the number of harmonic signal components, and that as is shown by experience, together with the increase of the order of Wigner's distribution. To decrease the disturbance we propose that Choi-William's [3] filtration is used which allows for essential decrease of the amplitudes of the above mentioned artefacts. Illustrating the above, for a signal presented by formula (5), we obtained a Wigner's distribution with a reduced interference (Choi-William's filtration).

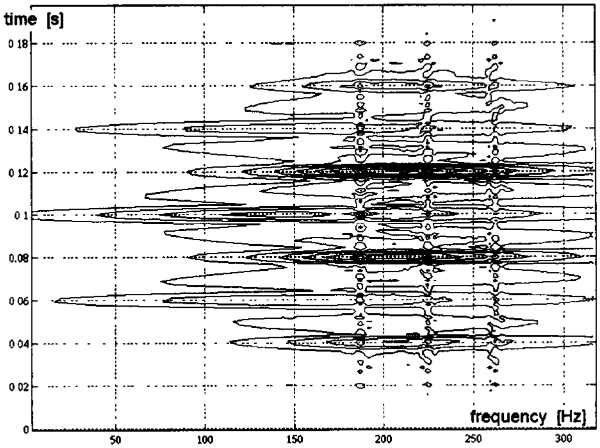

Fig. 3

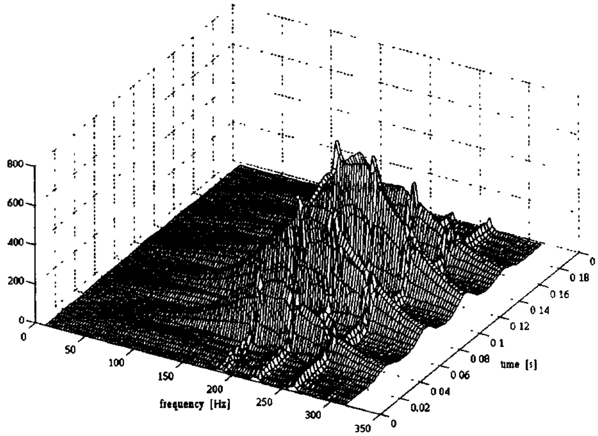

Fig. 4

Similar results for the case $\Delta f \neq 0$ were obtained for Wigner's bispectrum. The results are presented in figures 3 and 4. Both in the contour system, as well as in the case of presentation in the form of spatial image we obtained significant improvement of the legibility of obtained results as compared to the non-filtered Wigner's bispectrum, however in this case the additional problem is the appearance of disturbances connected with the length of a sample (the so-called time aliasing). The problems of minimizing this disturbance in analysis of modulated signals, constituting a noticeable obstacle, is one of the problems conditioning broader application of the discussed method.

Starting the analysis of non-linear effects of system, let us study a system in which apart from the linear part there is also a square element:

$$z(t) = x(t) + \varepsilon x^2(t) \tag{6}$$

As the input signal let us consider an example of an input signal which was earlier subjected to frequency modulation:

$$x(t) = A_0 \cos[2\pi f_i t + m \sin(2\pi f_r t + \varphi) + \Theta] \tag{7}$$

where: f_r - modulation frequency, m - modulation index, φ, Θ - initial phase

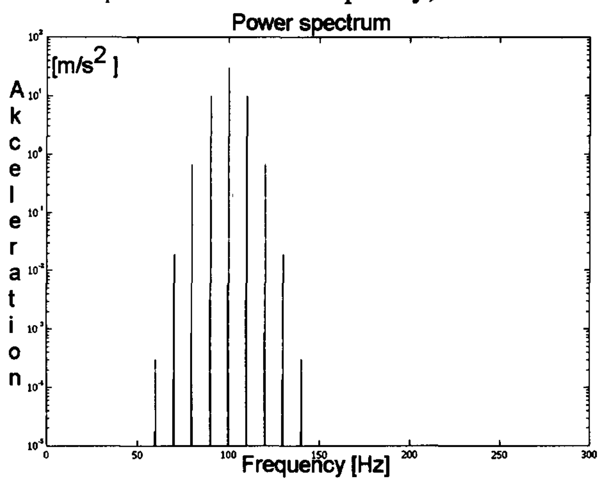

Fig. 5

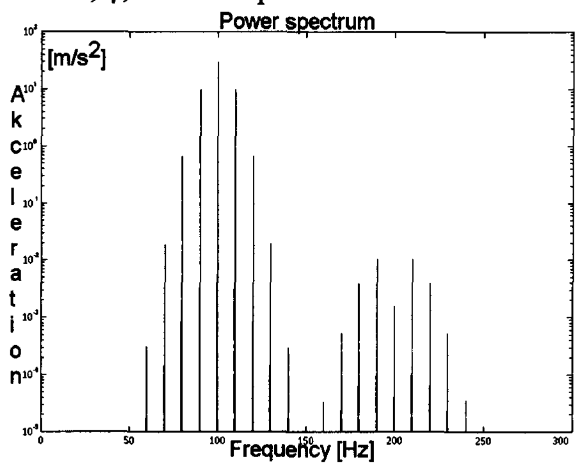

Fig. 6

Figure 5 presents the spectrum of the input signal (*f_i=100 [Hz], f_r=10 [Hz]*), while figure 6 presents the spectrum of the output signal. What captures attention is the fact that as a result of influence of the square element there appeared additional modulated band, whereas the carrier frequency of this band corresponds to the double value of the input carrier frequency.

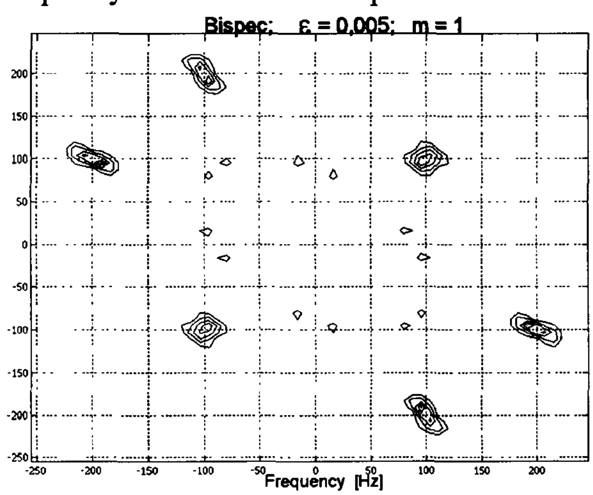

Fig. 7

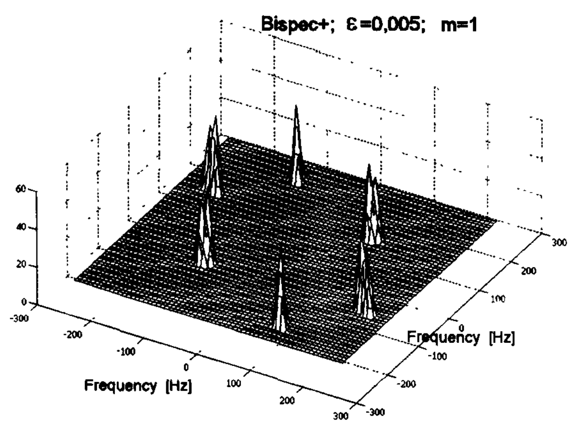

Fig. 8

Figures 7 and 8 present the results of bispectral analysis. Let us note that marked as essential are the frequencies which were caused by occurrence of non-linearity in the system (there respectively appears phase coupling between the frequencies of 100 Hz and 200 Hz). Passing of a signal which is not modulated frequency-wise through a non-linear system points to the possibility of tracking the influence that each of those factors has in isolation upon the frequency structure of the output.

Let us assume that together with the growth wear and tear the disturbance of contact conditions increases (for example growth of play in slide bearings a toothed gear shafts).

This is accompanied both by the growth of the size of influence of the square element (ε = 0.05 instead of ε = 0.005), as well as growth of the modulation index (m=2, instead of m=1).

Respectively, the results of the numerical experiment conducted for this data are presented on Fig. 9-12. What attracts our attention is the fact that together with the growth of the non-linear disturbance the value of phase-coupled components appearing in the bispectrum is essentially higher. The growth of the modulation index is respectively accompanied by extension of the band independently of the size of the square element. Let us particularly note that it is a particularly important observation for a toothed gear, since it points to the possibility of isolating the effects of disturbance at higher harmonic frequencies of meshing which are the energetically essential components of the spectrum of a vibroacoustic signal generated by a toothed gear.

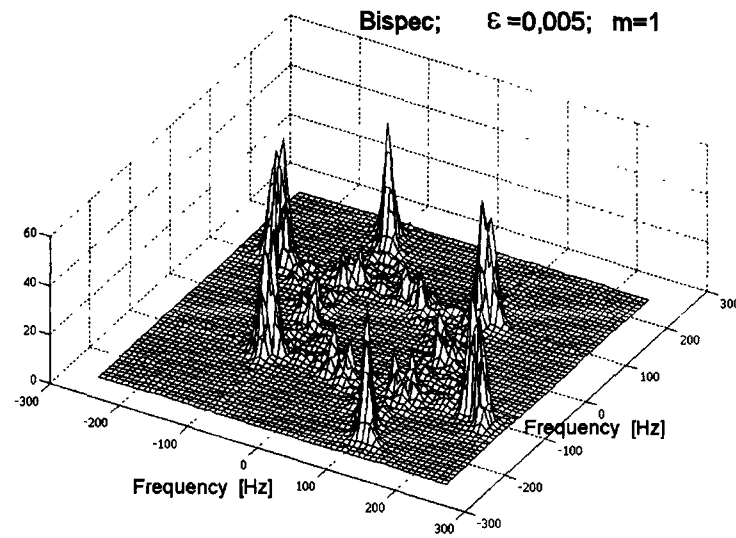

Fig. 9

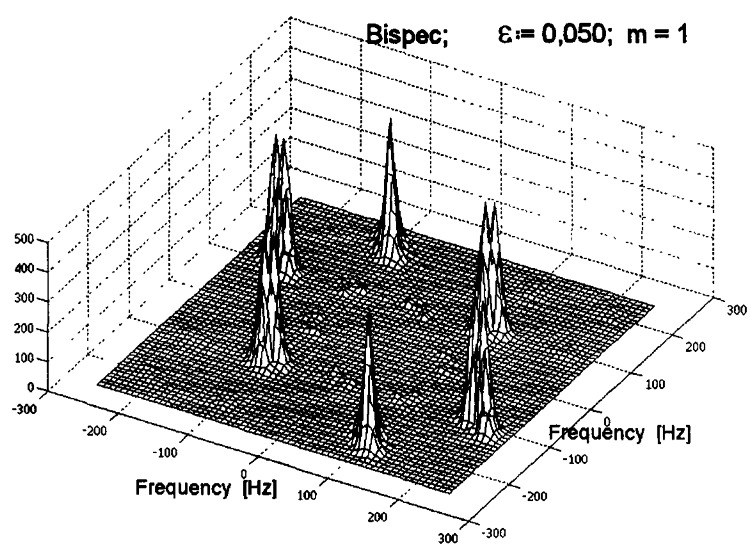

Fig. 10

SUMMARY AND CONCLUSIONS

Occurrence of disturbances of contact conditions, causing non-stationary effects in a signal and responsible for non-linear disturbances of the signal's frequency structure requires application for analysis of appropriate tools coming from the temporal and frequency, and multi-dimensional representation of a signal.

For example, Wigner's bispectrum enables location of modulated bands, and above all enables examination of the modulation phenomenon in terms quality and quantity. This may turn out to be particularly useful in the analysis of non-linear modulation phenomena. Appropriate application of the bispectrum allows analysis of the influence that non-linearity has on the spectrum structure, including detection of phase-coupled components. What captures our attention is the fact of existence of the possibility to define the additionally generated bands and their input into the power of the analyzed frequency band. It opens up possibilities of further development of non-linear diagnostic models enabling the analysis of the links between the growth of disturbance and change of vibroacoustic signal's spectrum.

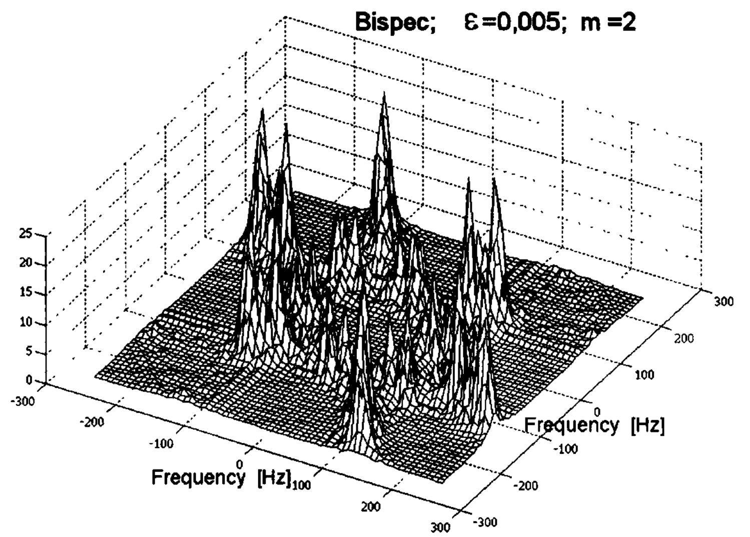

Fig. 11

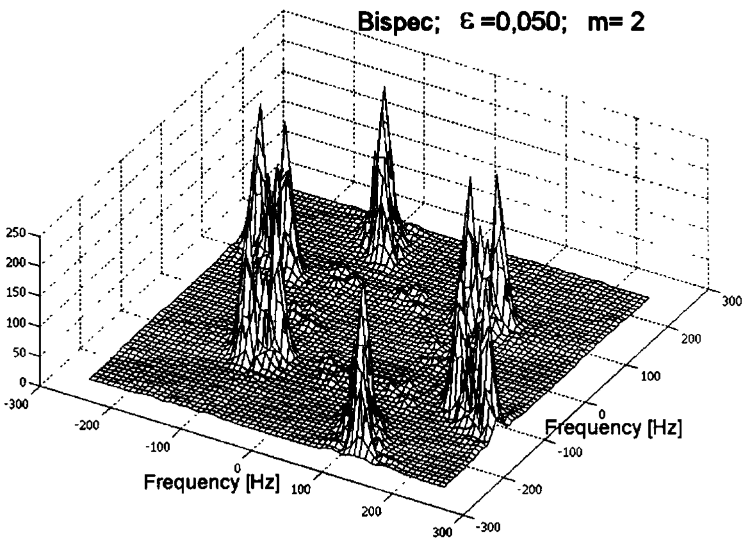

Fig. 12

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