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STUDY ON GENERATION MECHANISM OF SELF-EXCITED VIBRATION IN CYLINDRICAL GRINDING

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ABSTRACT

Self-excited vibration in cylindrical grinding is always experienced if the work speed is high. In this paper, the effect of the work speed on the occurrence of self-excited vibration is investigated analytically and numerically, and the amplitude and phase shift of the work vibration in the steady state is also determined, using the averaging method with the nonlinearity of the damping force taken into consideration. If the work speed is high, self-excited vibration always occurs. It is caused by that the phase of the work displacement is delayed by $\pi/2$ to that of previous grinding for all work speed. Consequently, in the steady state, if the grinding rate factor is large, the amplitude of the self-excited vibration increases with increase of the work speed.

1. INTRODUCTION

It is generally accepted that self-excited vibration in machine tools is caused by the regenerative effect[1,2] and it occurs if the work speed is high. It has been discussed the approach to the early prediction of cutting chatter, considering cutting force[3]. In the steady state, the stability and complicated dynamics of cutting is also discussed, considering the nonlinear stiffness of the machine tool and regenerative terms[4,5]. The self-excited vibrations under cutting are due to the time lag of the restoring force.

Self-excited vibration in cylindrical grinding is also caused by the time lag in the restoring force term and has been studied about the work speed range when it occurs analytically and experimentally[6,7]. The self-excited vibration with large amplitude is experienced and often brings about troubles with unevenness on the surface of the work. However, the general discussion about the reason why the self-excited vibration in cylindrical grinding always occurs in the high speed range has not been done widely.

In this paper, we obtain the obvious approximate critical speed of the work, using Nyquist's stability criterion[8]. It is also shown that the delay of the phase of the work displacement influences on the work speed range where self-excited vibration occurs.

Moreover, the amplitude and the phase shift from the previous grinding in the steady state is investigated, considering the nonlinearity of the damping force.

2. ANALYTICAL MODEL AND GOVERNING EQUATION

The system under consideration(Fig.1) consists of a grinding wheel and a work, where m is the mass of the work; k is the rate of the horizontal spring; c is the viscous damping coefficients of the work vibration; r is the radius of the work; v and V are the constant rotation speeds of the work and the grinding wheel respectively. The work in its equilibrium state has the displacement x_{st}. The motion of the work is confined to move only horizontally.

Under the above assumptions, the equation governing the motion of the work is derived. The following dimensionless variables will be used,

$$\xi = x / x_{st}, \quad t^* = t / T$$

where $T = \sqrt{m / k}$.

Considering the dynamic grinding force which is proportional to the work volume removed in a unit time, the dimensionless equation governing the vibration of the work is obtained as follows:

$$\frac{d^2\xi}{dt^2} + 2(\gamma + \beta\xi^2)\frac{d\xi}{dt} + \omega_n{}^2\xi = \kappa\xi(t - \tau) \tag{1}$$

where $\omega_n{}^2 = 1 + \kappa$. $\xi(t - \tau)$ indicates the work displacement of previous grinding. The asterisks indicating the dimensionless time are omitted in the equation (1) and henceforth. The dimensionless parameters involved in the equation (1) are expressed as follows:

$$\gamma = \frac{c + c_{ep}}{2m}T, \quad \kappa = \frac{k_{ep}}{k}, \quad \beta = \frac{\alpha x_{st}{}^2}{2m}T, \quad \tau = \frac{2\pi r}{v} / T$$

where c_{ep} and k_{ep} which depend on the work speed are the equivalent viscous damping coefficients and rate of the spring, considering the dynamic cutting force. Fig.2 shows the values of these dimensionless parameters versus the work speed. In this system, κ and τ are large and vary in the wide range.

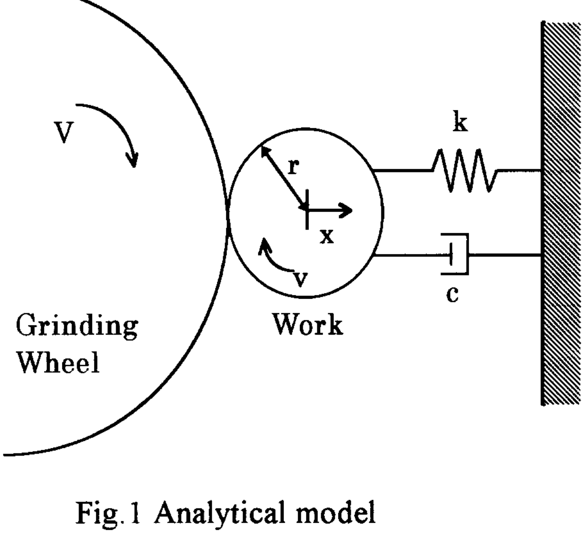

Fig.1 Analytical model

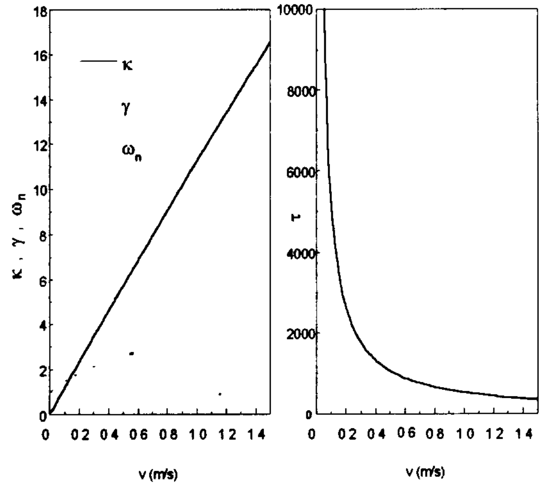

Fig.2 Values of dimensionless parameters versus the work speed

3. WORK SPEED RANGE WHERE SELF-EXCITED VIBRATION OCCURS

The loop transfer function of the linear system($\beta = 0$) is obtained as follows:

$$G_l(i\omega) = \frac{\frac{\kappa}{\omega_n^2} e^{-i(\omega\tau+\varphi)}}{\sqrt{(1-\frac{\omega^2}{\omega_n^2})^2 + (2\gamma\frac{\omega}{\omega_n^2})^2}} \tag{2}$$

where

$$\tan\varphi = 2\gamma\frac{\omega}{\omega_n^2} \Big/ (1-\frac{\omega^2}{\omega_n^2}). \tag{3}$$

The system will be stable if $G_l(i\omega)$ does not enclose the point $(i,0)$. Setting the amplitude of the equation (2) equal to 1 and solving, we obtain

$$\omega^2 = \omega_n^2 - 2\gamma^2 \pm \sqrt{(\omega_n^2 - 2\gamma^2)^2 - (\omega_n^4 - \kappa^2)} \tag{4}$$

The magnitude of the radius vector of $G_l(i\omega)$ attains the value of 1 twice at ω_1 and ω_2. It follows that the magnitude of the radius will be greater than 1 between ω_1 and ω_2, and will be less than 1 everywhere else. In the case that the work speed is not so small, it is shown that $\gamma \ll \kappa$ in Fig.2. Then we get the approximate solutions as follows:

$$\omega_1 \approx \sqrt{1+2\kappa-2\gamma^2} \approx \sqrt{1+2\kappa} + o(\gamma^2)$$

$$\omega_2 \approx \sqrt{1+2\gamma^2} \approx 1 + o(\gamma^2).$$

Here, we set $\beta_1 = \omega_1\tau + \varphi_1$, $\beta_2 = \omega_2\tau + \varphi_2$. If $|\beta_1 - \beta_2| > 2\pi$, the system becomes unstable because this condition is equivalent to that the $G_l(i\omega)$ encloses $(i,0)$. Using the relation $\omega_1\tau, \omega_2\tau \gg \varphi_1, \varphi_2$, this condition is represented approximately as follows:

$$\tau(\omega_1 - \omega_2) > 2\pi. \tag{5}$$

We find the following relations when the work speed is not so low(See Fig.2).

$$\kappa > 1, \quad \tau \approx o(1000) \tag{6}$$

These equations denote respectively that the equivalent stiffness on grinding is larger than the flexural rigidity of the work and the rotation period of the work is longer than the natural period of the flexural vibration of the work. When the equations (6) are satisfied, the equation (5) is always satisfied. In other words, if the equation $|G_l(i\omega)| = 1$ has two $\omega^2 > 0$, the system will be unstable. Therefore, we get the following approximate unstable condition.

$$h = \frac{\kappa^2}{4\gamma^2(\omega_n^2 - \gamma^2)} > 1 \tag{7}$$

The maximum amplitude during the 10^{th} revolution of the work is shown in Fig.3. It is shown that the self-excited vibration always occurs if the work speed is high. Fig.4 shows the unstable condition from the equation (7). It is in good agreement with the numerical result about the work speed range when the self-excited vibration occurs.

Fig.5 shows values of $\omega_1, \omega_2, \varphi_1, \varphi_2$ and $|\beta_1 - \beta_2|$. At the critical speed of the work, $|\omega_1 - \omega_2|$ and $|\beta_1 - \beta_2|$ increase rapidly in the small range of the work speed. These properties are due to the values of the parameters in cylindrical grinding system.

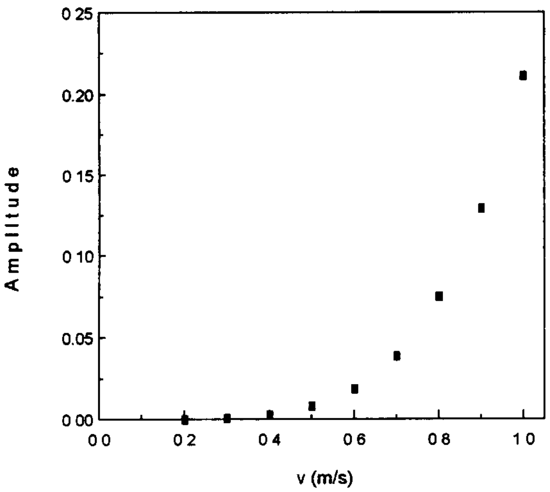

Fig.3 Maximum amplitude of ξ during 10^{th} revolution (v=0.8m/s)

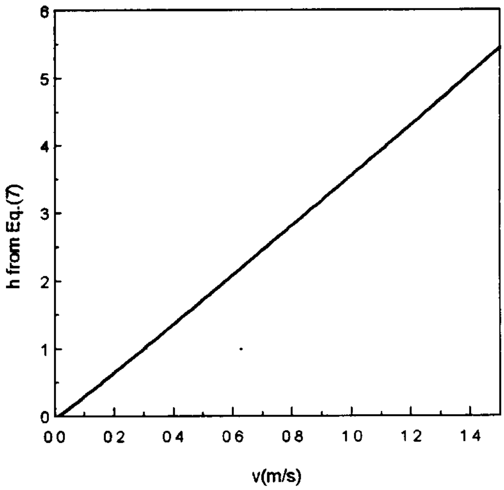

Fig.4 Critical work speed for stability

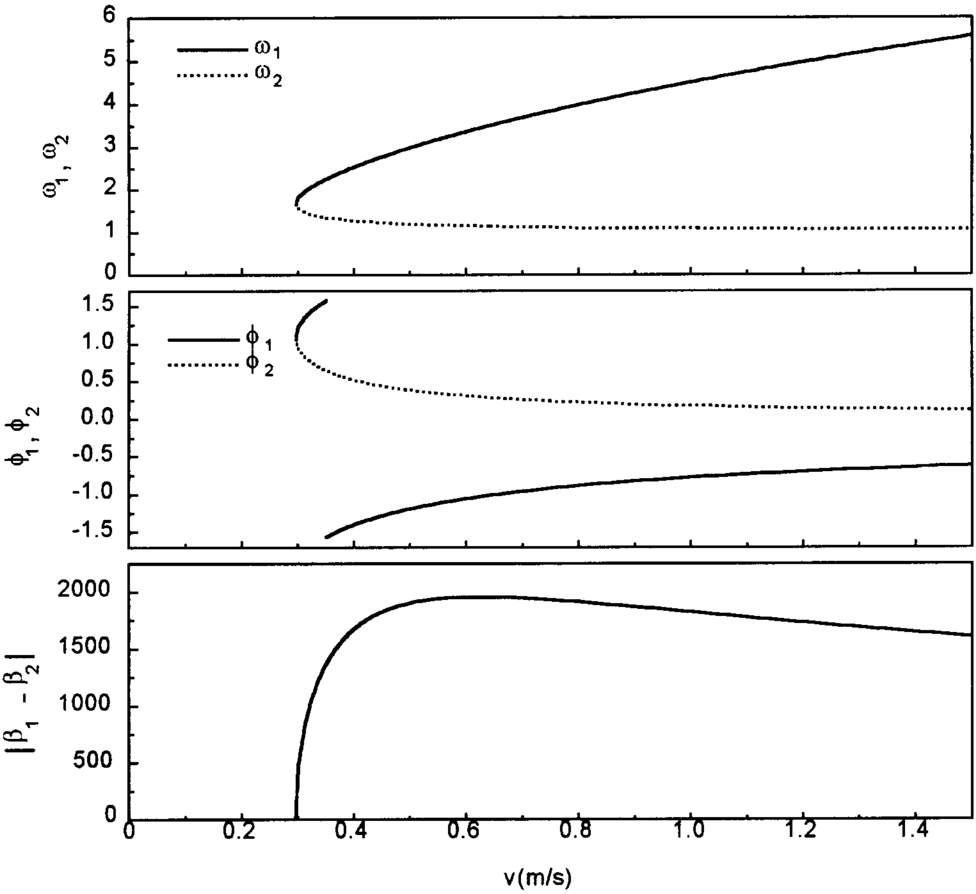

Fig.5 $\omega_{1,2}$, $\varphi_{1,2}$ from Eq.(3),(4) and $|\beta_1 - \beta_2|$

4. AMPLITUDE AND PHASE SHIFT IN THE STEADY STATE

In this section, we consider the response of the nonlinear system($\beta = 4$) in the steady state. The solution of the equation (1) is assumed to have the forms

$$\xi = a(t)\sin\omega t + b(t)\cos\omega t \tag{8}$$

$$\xi(t-\tau) = a(t)\sin[\omega(t-\tau)] + b(t)\cos[\omega(t-\tau)] \tag{9}$$

where are assumed $a(t-\tau) \approx a(t)$, $b(t-\tau) \approx b(t)$. Substituting the equations (8) and (9) into the equation (1) and using the method of averaging, we obtain

$$\frac{da}{dt} = (-\gamma - \frac{\kappa}{2\omega}\sin\omega\tau)a + \frac{1}{2\omega}(\omega^2 - \omega_n{}^2 + \kappa\cos\omega\tau)b - \frac{\beta a}{4}(a^2 + b^2) \tag{10}$$

$$\frac{db}{dt} = -\frac{1}{2\omega}(\omega^2 - \omega_n{}^2 + \kappa\cos\omega\tau)a + (-\gamma - \frac{\kappa}{2\omega}\sin\omega\tau)b - \frac{\beta b}{4}(a^2 + b^2) \tag{11}$$

In the steady state, we obtain

$$(\tilde{a}^2 + \tilde{b}^2)(\omega^2 - \omega_n{}^2 + \kappa\cos\omega\tau) = 0 \tag{12}$$

$$(\tilde{a}^2 + \tilde{b}^2)\left[-\gamma - \frac{\kappa}{2\omega}\sin\omega\tau - \frac{\beta}{4}(\tilde{a}^2 + \tilde{b}^2)\right] = 0 \tag{13}$$

where the tilde indicates to the stationary solution. From the equations (12) and (13), the amplitude in the steady state $\tilde{A}$ ($\tilde{A}^2 = \tilde{a}^2 + \tilde{b}^2$) is obtained.

$$\tilde{A} = 0, \tag{14}$$

$$\tilde{A} = \sqrt{\frac{4}{\beta}(-\gamma - \frac{\kappa}{2\omega}\sin\omega\tau)} \tag{15}$$

Here, if we consider the equation (15),

$$\omega^2 - \omega_n{}^2 + \kappa\cos\omega\tau = 0 \tag{16}$$

The stability of the these vibrations can be obtained by letting,

$$a = \tilde{a} + \Delta a, \quad b = \tilde{b} + \Delta b \tag{17}$$

Developing the right-hand sides of the equations (10) and (11) in powers of Δa and Δb and keeping only linear terms, we have

$$\frac{d\Delta a}{dt} = \left[-\gamma - \frac{\kappa}{2\omega}\sin\omega\tau - \frac{\beta}{4}(2\tilde{a}^2 + \tilde{A}^2)\right]\Delta a + \left[\frac{\omega^2 - \omega_n{}^2}{2\omega} - \frac{\beta\tilde{a}\tilde{b}}{2} + \frac{\kappa}{2\omega}\cos\omega\tau\right]\Delta b \tag{18}$$

$$\frac{d\Delta b}{dt} = \left[-\frac{\omega^2 - \omega_n{}^2}{2\omega} - \frac{\beta\tilde{a}\tilde{b}}{2} - \frac{\kappa}{2\omega}\cos\omega\tau\right]\Delta a + \left[-\gamma - \frac{\kappa}{2\omega}\sin\omega\tau - \frac{\beta}{4}(2\tilde{b}^2 + \tilde{A}^2)\right]\Delta b \tag{19}$$

If we let $\Delta a = c_a e^{\lambda t}$ and $\Delta b = c_b e^{\lambda t}$, we obtain the following the unstable condition.

$$\tilde{A}^2 > \frac{2}{\beta}\left(-\gamma - \frac{\kappa}{2\omega}\sin\omega\tau\right) \tag{20}$$

The equations (15) and(20) show that the occurrence of the self-excited vibration depends on the phase $\omega\tau$.

It is known that the frequency of the self-excited vibration will be close to one of the natural vibration. Then we let

$$\omega = \omega_n + \delta \tag{21}$$

In numerical results, it is shown $\delta << 1$. Substituting the equation (21) into the equation (16), we obtain

$$\cos\omega\tau = \frac{(\omega-\delta)^2-\omega^2}{\kappa} = \frac{-2\delta\sqrt{1+\kappa}+o(\delta^2)}{\kappa} \le o(\delta) \tag{22}$$

Considering the grinding rate κ is larger than 1, we obtain

$$\omega\tau = \frac{2n-1}{2}\pi + o(\delta) \tag{23}$$

If $A \neq 0$, $\sin\omega\tau < 0$ from the equation (15). Consequently, we obtain

$$\omega\tau \approx -(\frac{1}{2}\pi + 2m\pi) \tag{24}$$

Hence it is presented that the phase of the work displacement is approximately delayed by $\pi/2$ to that of previous grinding for all work speed.

If we assume the solutions (8) and (9), we obtain the following condition that the supplied energy is larger than dissipated one.

$$\kappa\sin\omega\tau < -2\gamma\omega \tag{25}$$

Using the relation (24), it is also shown that the condition (7) is approximately equivalent to the condition (25).

Consequently, we obtain the following the solutions in the steady state.

$$\tilde{\xi} = \tilde{A}\sin[\omega t - \tilde{\theta}] \quad , \quad \tilde{\xi}(t-\tau) = \tilde{A}\sin\left[\omega t + \frac{\pi}{2} - \tilde{\theta}\right] \tag{26}$$

where

$$\tilde{A} = \sqrt{\frac{4}{\beta}(-\gamma + \frac{\kappa}{2\omega_n})} \tag{27}$$

The solutions (14) and (15) in the steady state and the border line of the stability from the equation (20) are shown in Fig.6. The comparison between the analytical solutions and the numerical ones is also shown in Fig.7.

Fig.8 shows the time history of ξ and $\xi(t-\tau)$ in cases of $v = 0.8$ m/s and $v = 1.2$ m/s. In these cases, the phase of the work displacement is delayed by $\pi/2$ to that of previous

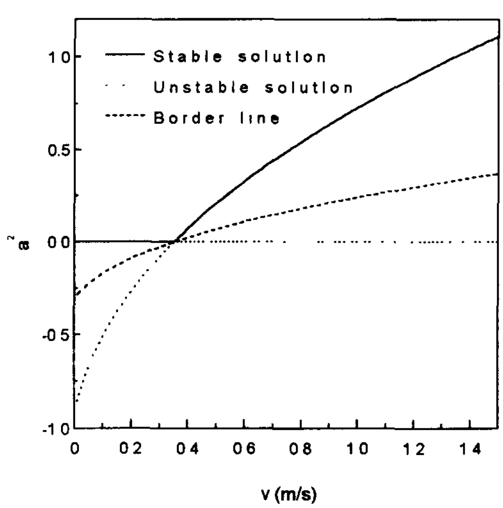

Fig.6 Stability with the work speed

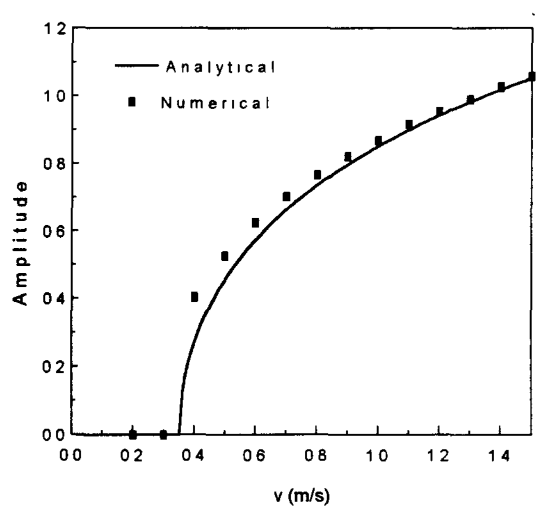

Fig.7 Steady-state amplitude

grinding. In Fig.9, as expected, it is shown that the phase shift is approximately $\pi/2$ for all work speed.

Fig.10 shows that the frequency of the self-excited vibration is close to the one of the natural vibration. It is shown that the assumption $\omega = \omega_n + \delta$ which we used in the previous discussion was accurate.

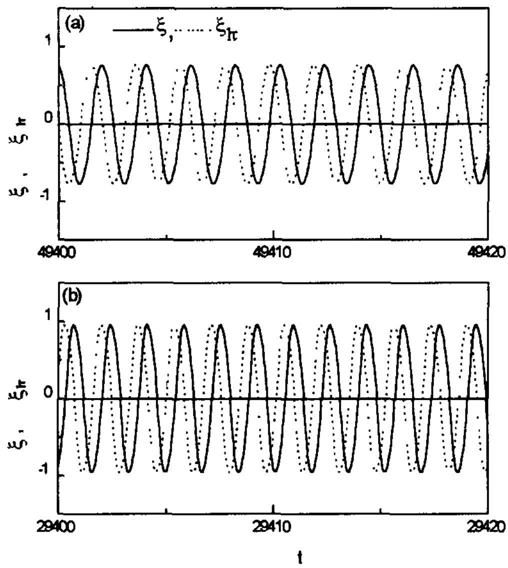

Fig.8 Time history of ξ and $\xi(t-\tau)$ in the steady state((a) v=0.8m/s,(b) v=1.2m/s)

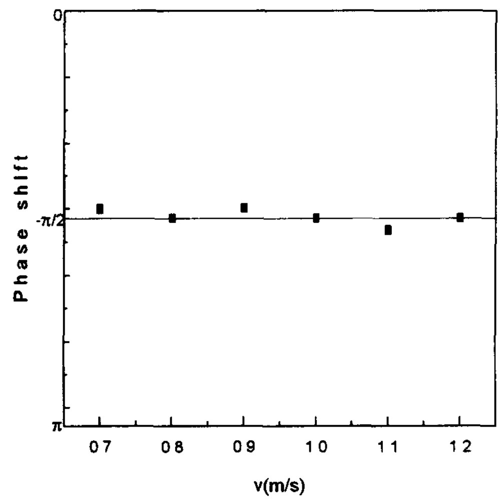

Fig.9 Phase shift of the work vibration with the work speed(numerical)

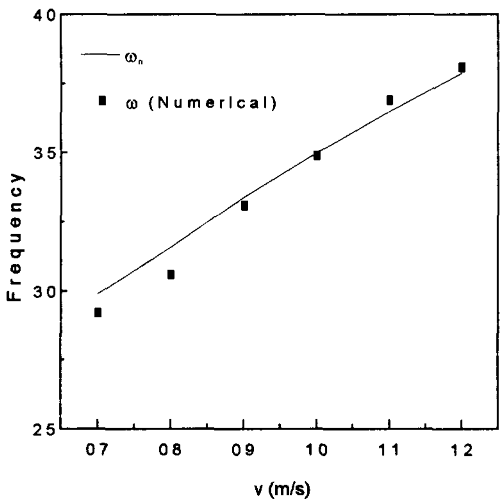

Fig.10 Frequency of the work vibration

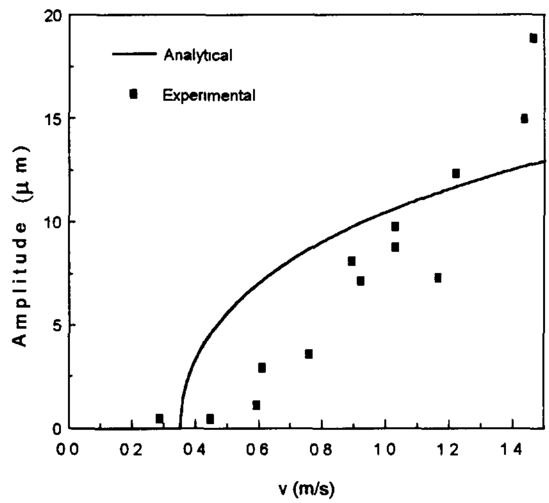

Fig.11 Comparison between analytical amplitude and numerical one

Fig.11 shows the comparison between the theoretical results and the experimental ones from Ohno's previous work[6]. The values of the dimensionless parameters in this paper correspond to the principal data of the grinding machine which is used in the experiment. Works are made of the low carbon steel and are supported by centers at both ends. Coolant is not used because the vibrometer has to be kept dry. The feed of the grinding wheel is continuously given. The theoretical results about the critical work speed and the amplitude in the steady state are in qualitative agreement with the experimental ones.

5. CONCLUSION

A self-excited vibration in cylindrical plunge grinding is investigated, considering time lag and the nonlinear damping force term. The system which consists of the grinding wheel and the work is regarded as the single-degree-of-freedom system. The effect of the work speed on the occurrence of self-excited vibration and the amplitude and the phase shift in the steady state are presented analytically and numerically. The main results are as follows:

First, if the work speed is low, self-excited vibration does not occur. On the other hand, if the work speed is high, self-excited vibration always occurs. The critical speed of the work revolution is approximately given, using Nyquist's stability criterion.

Second, if the grinding rate is large, the amplitude of the self-excited vibration in the steady-state increases with increase of the work speed. It is caused by that the phase of the work displacement is delayed by $\pi/2$ to that of previous grinding for all work speed.

The above theoretical results about the critical work speed and the amplitude of the work vibration in the steady state are in good agreement with the experimental one.

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