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DAMPING OF BELLS USING EXPERIMENTAL AND NUMERICAL METHODS

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ABSTRACT

The decay rates of the different partials of a vibrating bell are an important characteristic of the bell's sound spectrum. These decay rates are caused by material damping of the bell structure and acoustic damping due to the surrounding air. Designing new bells, bell founders would like to know the damping of the different partials before they cast a bell since casting a new bell is expensive and time consuming. However, the damping can only be determined accurately experimentally after a bell is cast. Numerical methods (FEM/BEM) can give an estimation of the damping values.

To gain a better understanding of the damping behavior of bells, modal measurements were carried out on four different bells to obtain the eigenfrequencies and the material and acoustic damping values of the partials. The results were compared with the results from fast numerical analyses methods utilizing axisymmetric Fourier elements in circumferential direction for both the structural and acoustic analyses. Some characteristic features of the damping values of bells are obtained and the comparison of experimental and numerical results shows that the damping values for most of the seven lowest partials can be predicted fast within 30% of the measured values.

INTRODUCTION

Bells have been cast by bell founders in Northern Europe since the Middle Ages. In those days casting a pleasant sounding bell or even a pure ringing bell was merely a question of luck. The global shape of the traditional minor third bell resulted from experiments over many years. With today's computer power and design tools, bell founders are able to design bells meeting specific requirements of the sound spectrum without the need to first cast bells, reducing time and costs.

Least than a decade ago, a new type of carillon bell was developed using structural optimization techniques (Roozen-Kroon [1]). The optimization problem formulation involved the definition of objective and constraint functions for which characteristics of the sound spectrum were taken. One important aspect of the sound spectrum is the decay rate of each partial which determines the time the partial can be heard. This decay rate is caused by damping phenomena always present in vibrating structures. For a vibrating bell, both material and acoustic damping exist. In the optimization problem, the damping values were specified to match certain target values in order to achieve a distinct sound spectrum.

Van Heuven [2] presented a thorough study on the measurements of acoustical properties of bells. He found that acoustical damping is sensitive to small changes in the bell shape and that there is no simple relation between frequency or shape and the damping of bells.

For a better understanding of the damping behavior of bells, both experimental and numerical investigations were carried out on four differently shaped and sized bells. Goal of this study is to see if it is possible to predict the damping values of the different partials accurately with numerical methods.

First, some characteristics of a vibrating bell are presented and the phenomenon of damping is described. Next, the measurement procedure and numerical approach to obtain the modal properties are outlined. Finally the numerical and experimental results are compared and discussed and some conclusions are drawn.

DAMPING OF BELLS

After a bell is excited with a stroke of a clapper or hammer, the bell starts to vibrate. Assuming the bell to behave in a linear elastic way, the vibration consists of the superposition of an infinite number of excited eigenmodes with distinct eigenfrequencies. Every eigenmode vibrates with its own eigenfrequency in a unique mode shape and decays at its own decay rate due to damping. The eigenfrequency, mode shape, damping and amplitude of an eigenmode are called the modal properties. Every eigenmode is characterized by its typical vibrational pattern in the vertical and horizontal cross-section (figure 1). The vibrational patterns of the eigenmodes in the vertical cross-section are denoted by roman numbers H, F, etc. and the patterns in horizontal direction by numbers 2, 3, ..., etc.. These numbers describe the number of periods in circumferential direction.

Each eigenmode excites the air surrounding the bell and generates a vibration in the air with the same frequency. A sound field is generated in which every pure tone or partial can be heard. Although all partials contribute to the sound spectrum, not all partials are equally important. To be important, the partials have to be loud and of low order since higher tones decay faster. A mode with a lower frequency has a greater initial vibrational amplitude, hence low frequencies appear louder in the sound spectrum. The frequency ratios of the partials form the overtone structure.

Any vibrating structure possesses some kind of damping. The damping of every eigenmode of a bell can be divided into material (η_m) and acoustic (η_a) damping. In each cycle of the periodic deformations, part of the elastic deformation energy is dissipated and passed into heat due to internal friction. This type of damping is also called internal damping. Generally it is assumed that material damping depends only on the material of which the bell is cast and is independent of the modal properties of the structure. So, the material damping is usually taken

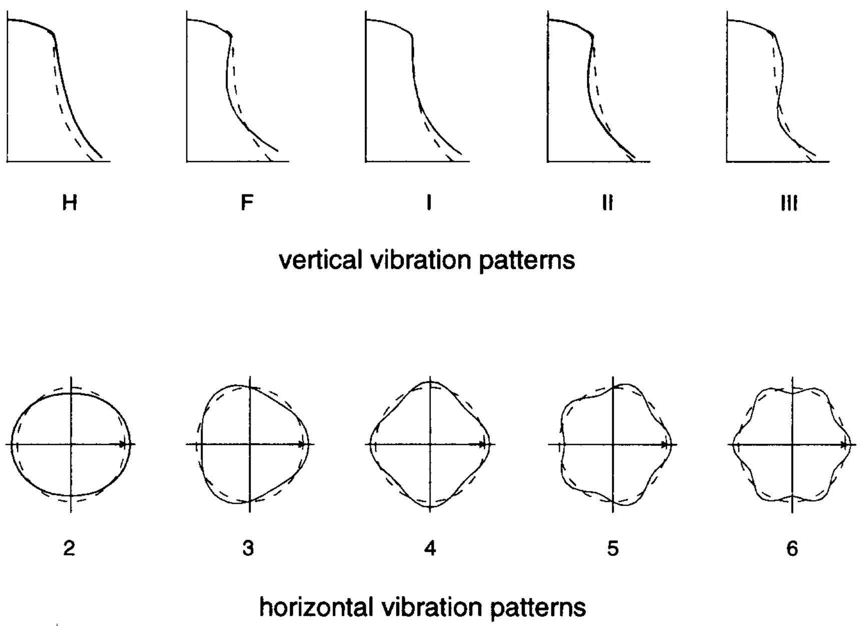


Figure 1: Vibration patterns of a bell.

to be a constant value.

Due to the sound radiated into the surrounding air, energy is transmitted from the vibrating bell to the air. This radiation of sound energy reduces the vibrational energy of the eigenmode and depends besides the shape of the bell, upon the modal properties.

Since the bell is assumed to behave linearly elastic and weakly damped, the damping mechanisms of two different eigenmodes are unrelated. Hence, the damping of one eigenmode is undisturbed by the excitation of other eigenmodes. Therefore the damping of the bell can be modelled as modal damping and every eigenmode possesses its own damping value. The total damping is equal to the sum of the material damping and acoustic damping and is represented dimensionless as a fraction of the critical damping of the eigenmode. The decay rate of a partial is assumed to be equal to the decay rate of the vibrational amplitude of the eigenmode, i.e. halving the amplitude of an eigenmode leads to half the pressure amplitude of the pure tone in air. In addition, the partial frequencies are assumed to remain the same during the slow decay. Other types of damping, like damping of the suspension of the crown, are not considered since they are assumed to be negligible for the vibration modes looked at here.

MEASUREMENT OF MODAL PROPERTIES

A modal analysis was performed to detect the eigenfrequencies and eigenmodes of the bells. If a bell is excited by a clapper, all frequencies are present in the sound spectrum. Using accelerometers as measurement devices, an eigenfrequency appears as a peak in the measured signal. Using measurement points on the bell wall at a fixed distance in the horizontal and vertical planes, the eigenmode belonging to the eigenfrequency can be detected. The bell was fixed at the crown but isolated with rubbers from the surrounding.

Next, for each eigenfrequency the damping of the vibration was measured. The bell was excited in one eigenfrequency with an electro-magnetic shaker coupled to a pure tone generator so the bell would vibrate with this eigenfrequency. Additional measurements showed that the bell indeed vibrated only with this frequency and the effect of other eigenfrequencies, although present in the sound spectrum, were negligible with peaks in the sound power spectrum more than 40dB less than the excitation frequency. The excitation was stopped at time $t = t_0$ by removing the shaker and the decay of the initial amplitude A_0 of the vibration mode was measured. In a linearly elastic structure the free vibrational amplitude $y(t)$ of an eigenmode decays exponentially:

$$y(t) = A_0 e^{-\epsilon\omega(t-t_0)} sin(\omega(t - t_0) + \phi) \quad (1)$$

The power of the exponential function determines the decay rate and contains the product of modal damping ϵ and the angular frequency ω. In order to determine the product $\epsilon\omega$, the acceleration of an eigenmode is measured during a short period of time, in which the acceleration amplitude decays substantially. Plotting the natural logarithm of the absolute acceleration versus time the slope represents $-\epsilon\omega$ which yields the modal damping value of the vibrational amplitude.

In order to obtain the values of the material and acoustic damping, one set of measurements was carried out in free air for the total damping. The other set was done in a vacuum compartment able to hold bells of diameters up to 1 meter, yielding the material damping. The acoustic damping results from the subtraction of total and material damping. Both material and acoustic damping values are of the same order of magnitude (typically 10^{-4}, a dimensionless quantity).

NUMERICAL ANALYSIS OF MODAL PROPERTIES

Since there is no simple analytical relation between the shape of the bell and the frequency and damping values, numerical analysis tools must be used to derive these quantities. Assuming bells to be axisymmetric, as they usually are, a substantial reduction in CPU time can be achieved. In circumferential direction the non-axisymmetric boundary conditions are expressed as Fourier series, reducing the dimensionality of the problem from three to two. The structural-acoustic problem is treated as an uncoupled one.

The mesh used for the numerical analyses is derived from geometry measurements of the different bells. The effect of ornaments present on the bell surface was not accounted for in the mesh. Previous investigations (Roozen-Kroon [1]) showed an axisymmetric mesh with 20 quadratic (8-node) elements to give sufficient accurate results for the structural analyses. For the acoustic analyses quadratic (3-node) line boundary elements were used for mesh compatibility.

For the structural vibration analysis, the axisymmetric finite element method (FEM) code solves the eigenvalue problem. The analysis results in the eigenfrequency ω (in radians), mode shape with normalized velocity u and kinetic energy E_k of a partial:

$$E_k = \frac{1}{2}\rho\pi \int_A u^2 r dr dz \quad (2)$$

where ρ is the specific mass of the bell material and A is the bell surface in rz-plane where the bell is modelled in. The kinetic energy is related to the maximum amplitude of the vibration mode and hence is not a time-average value.

Using the results of the structural vibration analysis one is able to calculate with the axisymmetric boundary element method (BEM) code (Kuijpers *et al.* [3]) the radiated sound power P_o of the vibrating bell:

$$P_o = \frac{1}{2} \oint_S \mathrm{Re}(p^* v_n) dS \tag{3}$$

where p^* is the complex conjugate of the sound pressure and v_n is the normal velocity on the bell surface S. The normal velocity is related to the mode shape velocity as $v_n = u \cdot n$ where n is the outward unit normal of the bell surface. The value of P_o is a time average value.

Computation times are typically 10 seconds for a FEM-axisymmetric analysis of selected partials and 20 seconds for a BEM-axisymmetric analysis of one partial. The results of the acoustic analyses were checked with SYSNOISE [4]. For an accurate 3D BEM analysis of the lowest eigenfrequency a quarter bell with at least 20 elements in circumferential direction was needed to get accurate results but lead to computation times of 25 minutes for one partial.

Considering the bell as a simple one degree of freedom mass-spring system, the acoustic damping η_a is defined as the loss of vibrational energy due to sound radiation into the air surrounding the bell (Lesueur [5]):

$$\eta_a = \frac{P_o}{\omega E_k} \tag{4}$$

and is independent of the amplitude of the vibration. Also for similar shaped but differently sized bells, the acoustic damping value is the same (van Heuven [2]).

In literature often confusion arises on how to define the expression for the damping. Here the acoustic and material damping are defined as loss of vibrational energy and is twice the modal damping of the vibrational amplitude: $\eta_a = 2\epsilon$. A value of $\epsilon = 1$ means critical damping.

DISCUSSION OF RESULTS

The modal properties of four differently shaped and sized bells were measured and numerically analyzed: a minor-third bell (hereafter called bell A), a frequency optimized major-third bell (B), a damping optimized major-third bell (C) and a prototype major-third bell (D). These bells were cast at the Royal Eijsbouts bell foundry in The Netherlands and the measurements were carried out at the laboratory of the Department of Mechanical Engineering.

The frequency of the hum for each of the bells A to D is resp. 261, 220, 440 and 417 Hertz and the radius of the lip of each bell, i.e. the largest radius, is resp. 38.8, 44.9, 22.4 and 27.4 cm. For a qualitatively good bell the product of frequency and diameter is important and is approximately 200 m/s for the four bells.

Inherent to every practical experiment are measurement errors. For some modes the intensity was not high enough for detection due to the higher background noise of the measurement equipment. Also, results for frequencies above 3000 Hz are less accurate due to the filter frequencies of the measurement equipment in this area. Above these frequencies a drastic reduction of intensity of the response occurred.

In table 1 the experimentally measured eigenfrequencies of the most important modes are summarized along with the numerically calculated values. For a (idealized) minor-third bell the eigenmodes, name and frequency ratios of the lowest and loudest partials are also given. Note that a major-third bell has a frequency ratio 2.5 instead of 2.4.

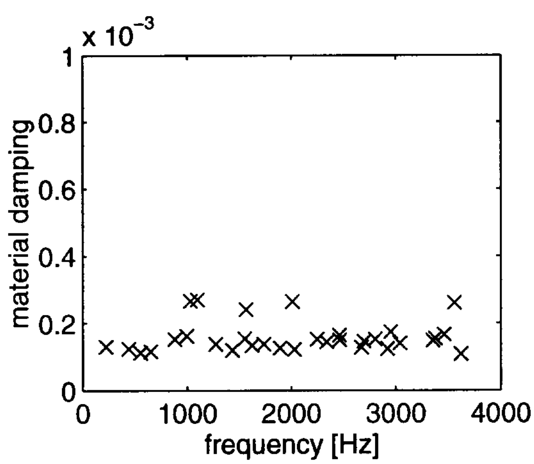
mode	name	freq. ratio	bell A		bell B		bell C		bell D	
			E	N	E	N	E	N	E	N
H-2	hum	1	261	258	220	220	440	440	417	416
F-2	fundamental	2	521	520	440	440	876	880	835	834
I-3	third	2.4	623	621	554	552	1100	1107	1052	1039
II-3	fifth	3	783	770	652	654	1318	1320	1248	1244
I-4	nominal	4	1040	1032	881	876	1764	1765	1664	1655
I-5	twelfth	6	1556	1543	1274	1277	2574	2577	2414	2416
I-6	double octave	8	2140	2126	1735	1748	3508	3537	3264	3289

Table 1: Most important eigenfrequencies in Hz (E = experimental, N = numerical).

Numerical results are in accordance with the measured frequencies and the maximum deviation for the modes shown is less than 2%, indicating the finite element model to be sufficiently accurate. For eigenfrequency ratios higher than 8 the deviation between measurement and analyzed frequency increases. The frequency ratios for the twelfth and double octave are not exactly matched. In general it appears to be difficult to satisfy the exact ratios for these two modes and therefor bell founders usually tune a bell to match only the five lowest important frequencies accurately and this deviation is accepted.

Figure 2 shows the measured material and acoustic damping values for bell B. To reduce errors in the damping values three measurements were done for each frequency and the average value was taken. The standard deviation of the damping values ranged from 1% for a good measurement up to 6% for a less accurate measurement. Also keep in mind that the acoustic damping values result from two measurements.

As expected the material damping can be assumed to be independent of the eigenfrequency. Deviations may be caused by measurement errors, non-homogeneous material or material dislocations, causing the bell to consume more or less energy for specific eigenfrequencies. At least no dependence on frequency or mode shape is detected.



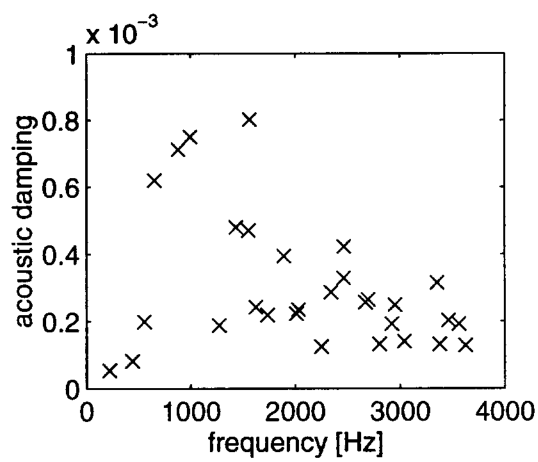


Figure 2: Measured damping values for bell B.

For bell A the mean material damping is $\eta_m = 1.4 * 10^{-4}$ with a standard deviation $\sigma = 0.2 * 10^{-4}$, for bell B: $\eta_m = 1.3 * 10^{-4}$ ($\sigma = 0.1 * 10^{-4}$), for bell C: $\eta_m = 2.6 * 10^{-4}$

($\sigma = 1.0 * 10^{-4}$) and for bell D:, $\eta_m = 1.6 * 10^{-4}$ ($\sigma = 0.2 * 10^{-4}$). Differences between bells exist since they were cast with some months in between so probably the composition of the alloy was different.

For the acoustic damping no direct relation between frequency and damping value is observed. Frequencies between about 600 Hz and 1500 Hz have a high acoustic damping and therefor will disappear quickly from the sound spectrum. For bell B, the material damping is in general higher for low frequencies (approx. 600 Hz), for mid-frequencies (approx. 600-2500 Hz) the acoustic damping is generally higher and for high frequencies (above approx. 2500 Hz), the acoustic and material damping are of the same order of magnitude. This behavior is in analogy with general structural elements (Lesueur [5]) and is observed for all four bells.

In figure 3 the measured (o) and calculated (*) acoustic damping values as function of the frequency ratio for the seven most important modes for each bell are displayed. Note that the points are connected with lines only for a clearer picture and for frequency ratios other than the ones displayed the damping values can not be obtained from these figures.

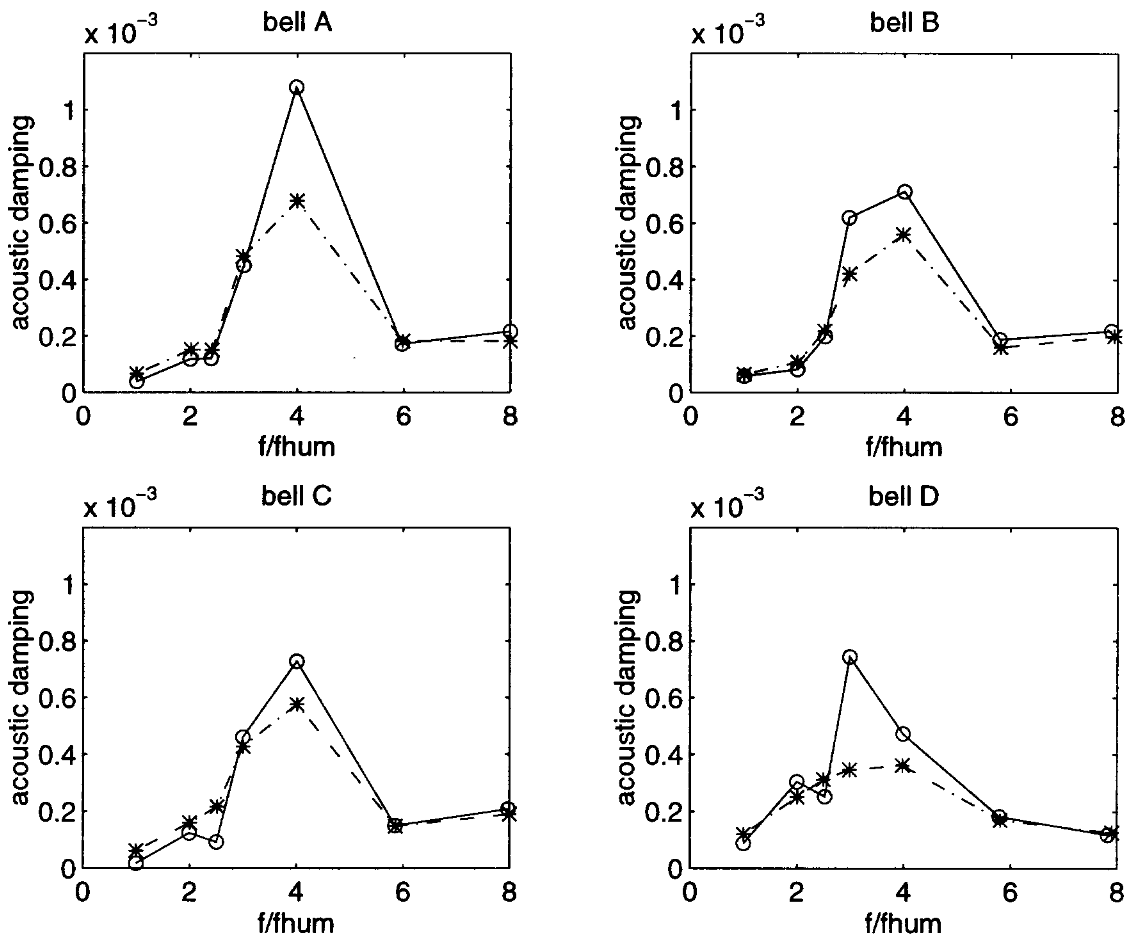


Figure 3: Measured (o) and calculated (*) acoustic damping values.

For all four bells the hum has the lowest acoustic damping and can be heard longest in the sound spectrum. Calculated and measured acoustic damping values show a good agreement for the low and high frequencies considered here. In general the calculated damping value is about 20% higher for low frequency ratios. For the nominal mode the damping value is predicted too low for all bells and the fifth mode for bells B and D. Differences up to 60% for bell A are observed, 30% for bells B and C and more than 100% for bell D. In all these cases the acoustic damping values are higher than about $4 * 10^{-4}$ and may indicate that either measurements for high damping values are less accurate or the numerical analyses are less accurate. The large differences can not be explained from the measurements of total and material damping separately. An additional source of damping, like damping of the crown is unlikely since the mode shape has a node at the crown. The existence of additional modes causing extra damping is also unlikely since it was checked whether only one eigenfrequency was present.

From earlier investigations it is known that acoustic damping is sensitive to small changes in the shape of the bell. Comparing the calculated values of the four bells with each other shows a close resemblance between the behavior, which may indicate that the numerical model can not completely cope with small local changes in shape. The ornaments not taken into account in the mesh may have some influence here. Future investigations have to focus on this topic.

CONCLUSIONS

The modal properties of four bells of different shape and size were compared, both experimentally and numerically. Goal was to predict the frequency and especially the acoustic damping values of the different partials of a vibrating bell with numerical analysis tools. This can provide the bell founder with some insight in the damping of a bell before he starts casting a new bell, hence reducing time and costs of traditional bell founding.

It is found that bells of approximately same size show a different acoustic damping for the most important partials, indicating that the shape of the bell is essential for the value of the acoustic damping. Hence, if analysing the acoustic damping of a vibrating bell, not only the modal properties and size of the bell are important but also the exact shape.

Despite the accurate prediction of frequency values which are within 2% of the measured values, larger differences are found for the acoustic damping values. The damping values can be predicted within some accuracy and a difference of 20 to 30% between calculated and measured values has to be accepted. Speaking in terms of time a partial can be heard one must think in a variation of seconds or less and can only be distinguished by a skilled listener. The numerical analysis methods provide a fast tool for the bell designer to quickly investigate the acoustic damping of new bell shapes.

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