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Simulation of Low Shaft Speed Bearing Faults under a Heavy Load

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Abstract

In this paper, a general model of the signal from faulty rolling element bearings under the condition of a heavy load is given. The envelope-autocorrelation of this proposed model in the case of very low shaft speed is given with a mathematical description. The simulated signals of rolling element bearings under the condition of a heavy load with inner race fault, outer race fault, and roller fault are generated using the model. In the power spectrum of signals, the characteristic frequency and its harmonics are submerged in the white noise, but they are obvious in the envelope-autocorrelation and envelope-autocorrelation power spectrum. It is demonstrated that the envelope-autocorrelation and its power spectrum are effective as to a fault detection technique.

1 Introduction

The condition monitoring of rotating machinery is important in terms of system maintenance and process automation, especially for the condition monitoring of rolling element bearings which are the most common wearing components in industral rotating machinery. They span across industries from agriculture to aerospace, in equipment as diverse as descaler pinch rolls to the Space Shuttle main engine turbopumps. For this reason a variety of bearing fault detection techniques have been proposed. But, as far as we know, only a very few articles have reported the theoretical modeling of bearing faults with mathematical descriptions.

Braun [1, 2] has given a theoretical model of the rolling element bearing with multipoint bearing defects. In his model, the repetitious impulse responses induced by bearing defects and the modulation by the bearing structural vibration were introduced.

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McFadden and Smith [3] have improved the model by taking into account the impulse series, the modulation of the periodic signal caused by non-uniform load distribution and of the vibration transmission of rolling element bearing, as well as the exponential decay of vibration.

Wang and Harrap [6] have improved the model further by considering the impulse series, the modulation of the periodic signal from non-uniform load distribution and of the first bearing vibration mode with mathematical descriptions. They do some envelope-autocorrelation analyses for the simulated signals and experimental data. The envelope autocorrelation technique is a viable alternative to the envelope spectral technique was given in [6].

Both [3] and [6] consider the modulation of cage frequency caused by non-uniform load distribution. Actually, the periodical signal of non-uniform load distribution imposes a modulation on the basic impulse series only under a heavy load [10]. Its period is different, depending on where the fault occurs[4, 5].

In this paper, a general model of fault from rolling element bearing under a heavy load is given. The basic impulse series induced by bearing faults, modulation of a periodic signal due to non-uniform load distribution and of bearing induced vibration, the machinery induced vibration, and a Gaussian white noise sequence are considered in the model. The period T_f of vibrational signal due to the non-uniform loading depends on where the fault occurs. Its envelope-autocorrelation is given with mathematical description. Then the simulated signals of inner race, outer race, and roller faults are generated. Their power spectrum, envelope-autocorrelation, and envelope-autocorrelation power spectrum are analysed.

2 A Model for Rolling Element Bearing Faults

Suppose $x(t)$ is an original signal from a rolling element bearing with a single point fault, it can be expressed as (see[10])

$$x(t) = x_{bs}(t) \cdot x_q(t) \cdot x_f(t) + x_s(t) + n(t), \tag{1}$$

where

$x_{bs}(t)$ the bearing induced vibration,

$x_q(t)$ the modulation effect of non-uniform load distribution of the bearings,

$x_f(t)$ the basic impulse series produced by the fault which impacts repetitiously with another surface in the bearing,

$x_s(t)$ the machinery induced vibration and

$n(t)$ a Gaussian white noise sequence with variance σ_n^2. In the presence of bearing fault, $n(t)$ is very small compared with the signal $x_{bs}(t)x_q(t)x_f(t)$.

Impulses Due to Fault: The component due to the bearing fault is

$$x_f(t) = d_0 \sum_{m=1}^{\infty} \delta(t - mT_f) \approx A_0 \sum_{l=1}^{N_f} \cos(2\pi l f_f t + \phi_f^l), \tag{2a}$$

Causes of Periodicities	Outer Race Faults	Inner race Faults	Rolling Element Faults
Stationary loading	No effect	f_r	f_c

Table 1: Periodic characteristics of bearings with faults under non-uniform load distribution[4, 5]. Here f_r is the shaft rotational frequency, and f_c is cage frequency.

where $A_0 = d_0 f_f$ is the amplitude, ϕ_f^l is the initial phase of l-th harmonic, N_f is the number of harmonics induced by impulse series. The characteristic frequency of bearing fault is $f_f = 1/T_f$. This can be the characteristic frequencies of inner race fault(f_{bpfi}), of outer race fault (f_{bpfo}), and of rolling element fault (f_{bsf}), depending on where the fault occurs.

Non-uniform Loading: The component of the vibration signal due to non-uniform loading is

$$x_q(t) = \sum_{k=1}^{N_q} A_q^k \cos(2\pi k f_q t + \phi_q^k), \tag{2b}$$

which is a periodic function with the period of T_q ($f_q = 1/T_q$). The amplitude and initial phase of k-th harmonic are A_q^k and ϕ_q^k respectively and N_q is the number of harmonics. The periodicity f_q depends on the elements of bearing fault as shown in Table 1 [4, 5].

Bearing Induced Vibration: The vibration induced by the bearing is

$$x_{bs}(t) = \sum_{j=1}^{N_{bs}} A_{bs}^j e^{-\alpha_{bs}^j t} \cos(2\pi f_{bs}^j t + \phi_{bs}^j), \tag{2c}$$

where f_{bs}^j, A_{bs}^j, ϕ_{bs}^j, and α_{bs}^j are the resonance frequency, amplitude, initial phase, and damping factor of j-th vibrational mode of the bearing respectively, N_{bs} is the modal order of bearing vibration signal.

Machinery Induced Vibration: The vibration due to machinery other than the bearing in question is

$$x_s(t) = \sum_{n=1}^{N_s} A_s^n e^{-\alpha_s^n t} \cos(2\pi f_s^n t + \phi_s^n), \tag{2d}$$

where f_s^n, A_s^n, ϕ_s^n, and α_s^n are the resonance frequency, amplitude, initial phase, and damping factor of n-th vibrational mode of the machinery system respectively, N_s is the modal order of machinery vibration signal. Theoretically, N_q, N_f, N_{bs}, and N_s are infinite. The band limited nature of most measurements means that it is reasonable to assume finite values for these orders in practice.

Envelope-Autocorrelation: The envelope of a real-valued signal $x(t)$ is $|z(t)|$, where

$$z(t) \triangleq x(t) + i\ \tilde{x}(t), \tag{3}$$

and $z(t)$ is the analytic signal [7] associated with $x(t)$, i is the complex operator, $\tilde{x}(t)$ is the Hilbert transform [8] of $x(t)$. If $|z(t)|$ can be considered ergodic, then the envelope-autocorrelation can be estimated as

$$R_{zz}(\tau) = \frac{1}{T}\int_0^T |z(t)| \cdot |z(t+\tau)| dt, \tag{4}$$

where T is some finite time interval. For estimating convenience, here another function $\Phi_{zz}(\tau)$ which is related to $R_{zz}(\tau)$ is introduced:

$$\Phi_{zz}(\tau) \triangleq \frac{1}{T}\int_0^T |z(t)|^2 \cdot |z(t+\tau)|^2 dt. \tag{5}$$

It is assumed that damping and initial phase may be neglected in the derivation of the envelope-autocorrelation. Therefore, the function $\Phi_{zz}(\tau)$ for the bearing in the very low shaft speed case can be expressed as (see [10])

$$\begin{aligned}\Phi_{zz}(\tau) &= C + \frac{1}{2}\sum_{j,k,l=1}^{N_{bs},N_q,N_f} (D_{j,k})^2\{2\cos[2\pi(2lf_f)\tau] + 2\cos[2\pi(2kf_q)\tau] \\ &\qquad + \cos[2\pi(2lf_f + 2kf_q)\tau] + \cos[2\pi(2lf_f - 2kf_q)\tau]\} \\ &\quad + \sum_{k,k_1=1}^{N_q} \sum_{l,l_1=1}^{N_f} (E_{k,k_1})^2 \sum_{m_1=0}^{7} H_{m_1}(\tau), \quad (\text{except } k = k_1 \cap l = l_1)\end{aligned} \tag{6}$$

where

$$H_{0,1,\cdots,7}(\tau) = \cos\{2\pi[(l \pm l_1)f_f \pm (k \pm k_1)f_q]\tau\}, \tag{7a}$$

and

$$\begin{aligned} C &\triangleq C_1^2 + 2\sigma_{nb}^2, \\ C_1 &\triangleq \frac{(A_0)^2}{4}\sum_{j,k,l=1}^{N_{bs},N_q,N_f} (A_{bs}^j)^2 (A_q^k)^2, \\ D_{j,k} &\triangleq \frac{(A_0)^2 (A_{bs}^j)^2 (A_q^k)^2}{8}, \\ E_{k,k_1} &\triangleq \frac{(A_0)^2}{8} A_q^k A_q^{k_1} \sum_{j=1}^{N_{bs}} (A_{bs}^j)^2, \end{aligned} \tag{7b}$$

where C, C_1, $D_{j,k}$, E_{k,k_1} are constants. In the rolling element bearings, usually it is true for $f_q < f_f$. By comparing $R_{zz}(\tau)$ with $\Phi_{zz}(\tau)$, the main frequencies in the envelope-autocorrelation function $R_{zz}(\tau)$ are the fault characteristic frequency f_f and its harmonics lf_f $(l = 2, 3, \cdots, N_f)$. And kf_q $(k = 1, 2, \cdots, N_q)$ are the side frequencies of main frequencies, but their amplitudes are smaller than those of main frequencies. There are also some components of small amplitude with frequencies of $\frac{1}{2}(l \pm l_1)f_f \pm \frac{1}{2}(k \pm k_1)f_q$ (here $(l \pm l_1)$ is odd, $l, l_1 = 1, 2, \cdots, N_f$; $k, k_1 = 1, 2, \cdots, N_q$, except $l = l_1 \cap k = k_1$) in $R_{zz}(\tau)$.

3 Simulation of Rolling Element Bearing Faults

The simulated signals are generated according to a bearing housing which supports two descaler pinch rolls. The mean rotational speed of rolls is 36.06 RPM (0.601 Hz), and the shaft speed varies randomly in a small range about the mean. The fault characteristic frequencies are listed in Table 2.

Fault type	Outer race fault (f_{bpfi})	Inner race fault (f_{bpfo})	Roller fault (f_{bsf})
Characteristic Frequencies (Hz)	6.500	4.936	2.127

Table 2: Characteristic Frequencies of Bearing Faults

3.1 Simulation of Inner Race Fault

For the inner race fault, the modal orders of vibrational signals from bearing itself, machinery system, and impulse series are $N_{bs} = 5$, $N_s = 5$, $N_f = 7$ respectively, with a Gaussian white noise sequence (standard deviation $\sigma_n = 4.0$). The modal order of vibration signal from non-uniform load distribution is $N_q = 2$, the frequency of this vibration signal is the shaft rotational frequency, $f_r = 0.601\ Hz$. Fig. 1a is the simulated signal of inner race fault with the sampling frequency $f_{samp} = 5000Hz$. Fig. 1b, Fig. 1c, and Fig. 1d are the power spectrum, envelope-autocorrelation, and envelope-autocorrelation power spectrum respectively. The fault characteristic frequency and its harmonics are completely submerged in the white noise in the power spectrum (shown in Fig. 1b). The main frequencies are the fundamental characteristic frequency of inner race fault $f'_{bpfi} = 6.500Hz$ and its harmonics (kf'_{bpfi}, $k = 2, 3, \cdots$) from Fig. 1c and Fig. 1d. Some side frequencies kf_r ($k = 1, 2, \cdots$) are found in Fig. 1d.

3.2 Simulation of Outer Race Fault

For the outer race fault, the periodicity of non-uniform load distribution does not impose any modulation on the impulse series from the outer race fault of bearings (see Table 1[4, 5]). The simulated signal (shown in Fig. 2a) consists of impulse series ($N_f = 7$), bearing itself ($N_{bs} = 5$), machinery system ($N_s = 5$), and a Gaussian white noise sequence (standard deviation is the same as it in Section 3.1). Fig. 2b, Fig. 2c and Fig. 2d are the power spectrum, envelope-autocorrelation and envelope-autocorrelation power spectrum. The fundamental characteristic frequency of outer race fault $f'_{bpfo} = 4.900Hz$ and its harmonics (kf'_{bpfo}, $k = 2, 3, \cdots$) are notable in Fig. 2c and Fig. 2d. No side frequency exists in Fig. 2d.

3.3 Simulation of Roller Fault

For the roller fault, the periodicity of non-uniform loading distribution is $f_c = 0.260$ (Hz). The modal orders and the standard deviation of the Gaussian white noise sequence are the same as Section 3.1. The simulated signal is shown in Fig. 3a. And Fig. 3b, Fig. 3c and Fig. 3d are the power spctrum, envelope-autocorrelation, envelope-autocorrelation power spectrum of Fig. 3a. The main frequencies are the fundamental characteristic frequency of roller fault $f'_{bsf} = 2.100\ Hz$ and its harmonics (kf'_{bsf}, $k = 2, 3, \cdots$) from Fig. 3c and Fig. 3d. Side frequencies (kf_c, $k = 1, 2, \cdots$) do exist in Fig. 3d, but they are difficult to find because of the small value of f_c.

4 Conclusion

In this paper, a general model of bearing faults is given. The envelope-autocorrelation is given with mathematical description. By simulating signals of different bearing faults with the given model, the following results are obtained:

1. The envelope-autocorrelation of simulated signals from the rolling element bearing with different element faults in the very low shaft speed confirms its theoretical envelope-autocorrelation.

2. The envelope-autocorrelation and envelope-autocorrelation power spectrum are effective as a fault detection technique for bearings in the low shaft speed bearing, even under a heavy load.

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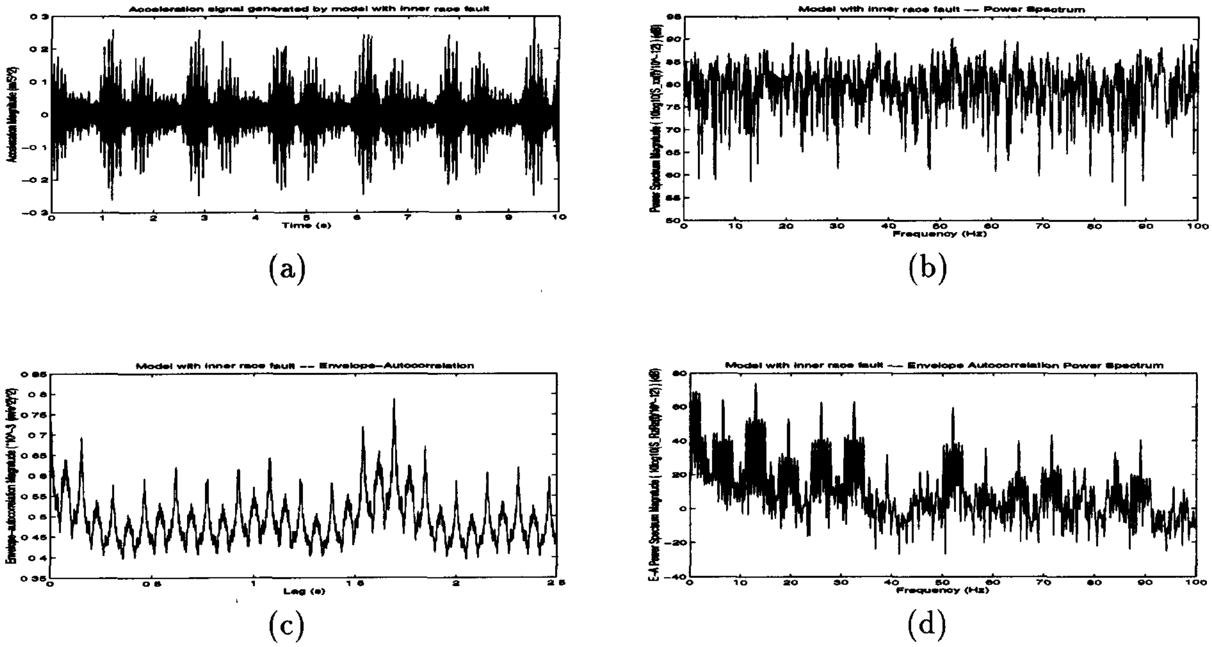

Figure 1: Simulated signal of bearing with inner race fault ($f_{samp} = 5000Hz$): (a) - simulated signal generated by model, (b) - power spectrum of (a), (c) - envelope-autocorrelation of (a), (d) - envelope-autocorrelation power spectrum of (a)

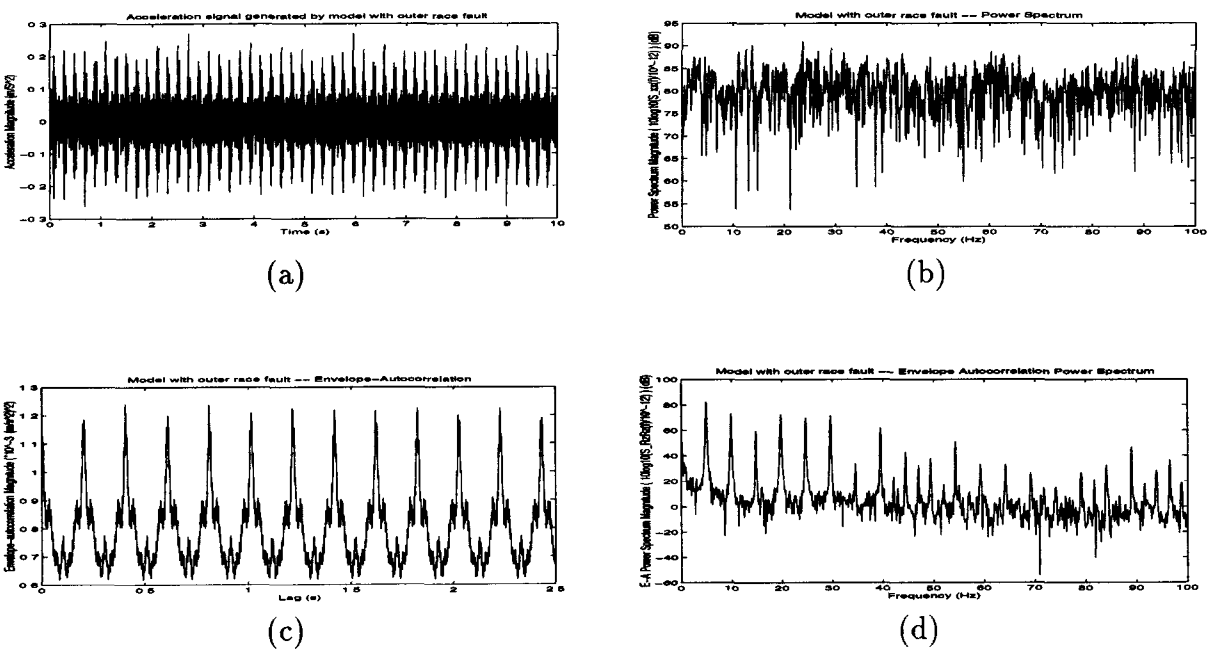

Figure 2: Simulated signal of bearing with outer race fault ($f_{samp} = 5000Hz$): (a) - simulated signal generated by model, (b) - power spectrum of (a), (c) - envelope-autocorrelation of (a), (d) - envelope-autocorrelation power spectrum of (a)

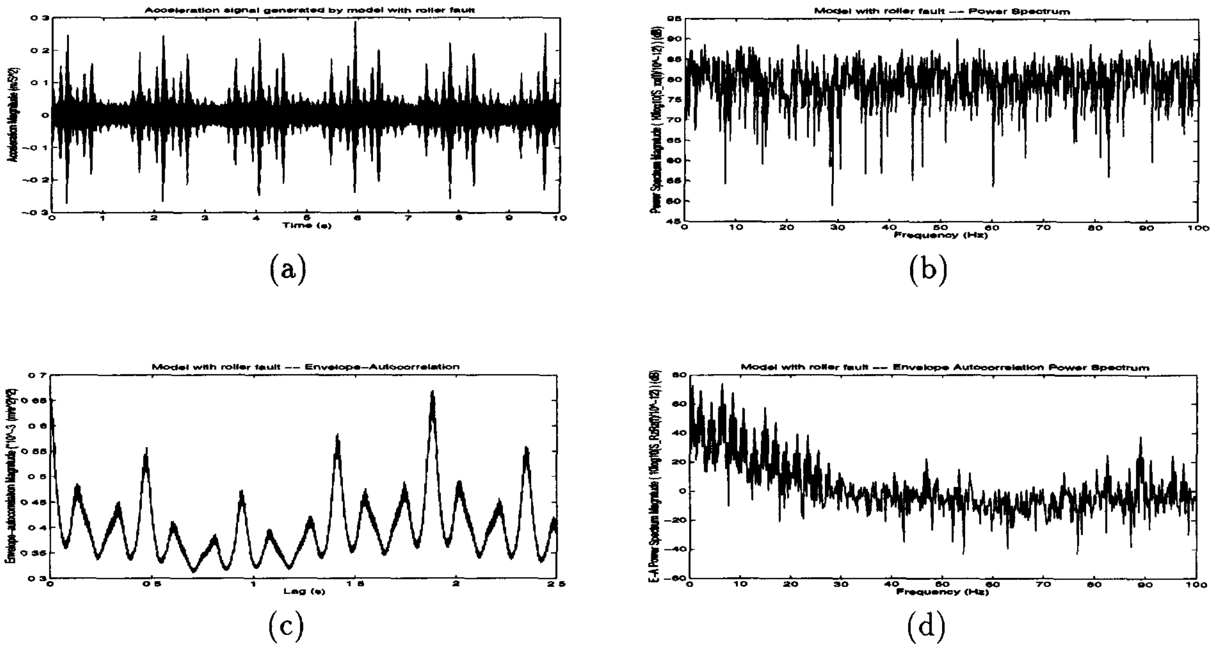

Figure 3: Simulated signal of bearing with roller fault($f_{samp} = 5000Hz$: (a) - simulated signal generated by model, (b) - power spectrum of (a), (c) - envelope-autocorrelation of (a), (d) - envelope-autocorrelation power spectrum of (a)