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ACOUSTIC RADIATION FROM FINITE LENGTH CYLINDRICAL SHELLS USING BOUNDARY ELEMENT METHOD

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ABSTRACT: The radiation efficiency of an acoustically thick circular cylindrical shell has been examined by calculations using the boundary element method for three different boundary conditions, and by conducting experiments using point excitation with the boundary conditions being free at both ends. Both the numerical and experimental results indicate that the radiation efficiency attains a value of unity at frequencies well below the critical frequency. Analysis has been made to explain the behaviour of acoustically thick shells.

1. INTRODUCTION

In acoustics, plane, spherical and cylindrical sound sources are generally treated as basic sound sources because of their simple geometries and their many practical applications. However, up till now a cylindrical source has usually been treated by assuming infinite length because the corresponding solution could be expressed analytically by a series of cylindrical waves[1]. Obviously, in practice, because most circular cylindrical structures are finite in length, and because the end effects are expected to be more important for shells with short length, the results obtained by using an infinite length model could be in severe error.

In studying the acoustic radiation from finite length cylindrical shells analytically, cylindrical shells are normally classified into acoustically thin and acoustically thick shells according to the ratio of the ring frequency f_r to the critical frequency f_c, i.e. $f_r/f_c<1$ for thin shells and $f_r/f_c>1$ for thick shells[2]. In practice, a typical example of acoustically thin shells is airplane fuselage, and most industrial pipes are acoustically thick.

For acoustically thin cylindrical shells, since the modal density is normally high especially at high frequencies, the acoustic properties are lumped together so that individual frequency analysis becomes impractical and statistical analysis has to be employed[3,4]. The most successful work on a thin cylindrical source was carried out by Szechenyi[4] who developed a set of general, but approximate equations for predicting the radiation efficiencies of cylindrical shells having large length to thickness and radius to thickness ratios within a given frequency bandwidth. According to [4], when $f_c > f_r$, the radiation efficiency of an acoustically thin cylindrical shell has three distinguished features corresponding to three frequency ranges. Below the ring frequency, the radiation efficiency increases at a rate of 3 to 6 dB per octave to a maximum at the ring frequency. Above the ring frequency, the curvature effects are no longer important and the cylindrical shell vibrates like flat plates. Therefore in the frequency range between f_r and f_c , the radiation efficiency would first decrease then increase as the frequency approaches f_c in a manner similar to flat-plate radiation. Above the critical frequency, the radiation efficiency then maintains a value of unity. These results have been verified by experiments except for low frequencies[4].

For acoustically thick cylindrical shells, because of the low modal density, the vibration of the shell would strongly depend on the behaviour of each vibration mode. Therefore, the boundary conditions and external excitations could affect the vibration distributions remarkably and thus the acoustic radiation. Furthermore, curvature effects could play an important role in determining the flexural wave speed and the acoustic radiation behaviour. In this case, deterministic analysis in which the individual vibration velocity distributions at each frequency are considered has to be employed. Although various studies associated with cylinder structures using deterministic analysis have been reported[5,6,7,8], the behaviour of the radiation efficiency and the physical significance of f_c for acoustically thick cylindrical shells have not been analysed fully. Previous research[2] only showed that the radiation efficiency of an acoustically thick cylindrical shell normally increases smoothly with frequency until it reaches a constant value of about unity at high frequencies, and is strongly influenced by the nature and type of the excitation[9]. No further and detailed discussion about the effect of curvature on f_c has been reported.

In this paper, the sound radiation of acoustically thick cylindrical shells is discussed. By using the boundary element method, the sound radiation efficiencies of a circular cylindrical shell with three different boundary conditions respectively are calculated. The radiation efficiency is also determined experimentally using point excitation with both ends of the shell being free. An analytical treatment will be introduced to explain the behaviour of acoustically thick circular cylindrical shells.

2. BOUNDARY ELEMENT MODEL

It is difficult to determine analytically the sound radiation efficiencies of finite length cylindrical shells due to the complexity of the vibration behaviour of shells and the difficulties of taking the end effects into account. Usually, an approximate series method is used to approach the exact solution[5], or alternatively a theoretical model, which to some extent reflects the nature of the practical structure, is chosen in order to get an approximate solution[7,8]. For the latter approach which is mostly used in engineering, it can be seen that the accuracy strongly depends on the model selected.

In this study the boundary element method is used to calculate the radiation efficiencies of an acoustically thick circular cylindrical steel shell. The dimensions of the shell are 0.2m long, 0.0016 m thick and 0.0635 m in radius. Hence the critical and ring frequencies of the cylindrical shell are 7392 Hz and 13190 Hz respectively. In order to examine the effects of curvature on the critical frequency, calculations were made from 300Hz to 8 kHz with a step of 100 Hz using a commercial boundary element code SYSNOISE version 5.2 on a SUN SPARC workstation. The number of nodes and elements of the BEM model as shown in Fig.1 is 2480 and 2400 respectively. As a result, the number of elements per acoustic wavelength below 8 kHz is greater than 6. The analysis option used in the calculation was BEM Indirect Coupled Analysis[10]. A force was applied on the surface of the shell in the radial direction at a point 0.066 m away from one end. The radiation efficiencies of the cylindrical shell with three different boundary conditions, namely simply-supported, free and clamped at both ends respectively, have been calculated. The results are shown in Fig.2. It can be seen that the radiation efficiencies for these three cases reach unity at a frequency much lower than the critical frequency f_c. Depending on the boundary conditions, it can be as low as $f_c/3$. These results thus indicate that acoustically thick shells do warrant a more detailed analysis.

3. EXPERIMENT

In order to verify the above numerical results, an experiment was carried out in an anechoic room. The geometry of the cylindrical shell is the same as that used in the calculations. For convenience, only the shell with both ends free was tested. The shell was excited by a shaker, B&K type 4810 driven by B&K 2706 power amplifier. The excitation signal used was random noise from 0 to 6.4 kHz which was provided by an HP3569A dual channel analyser. The acoustic power output from the shell was measured by the sound intensity technique with HP 3569A. The spacer of the sound intensity probe was 12mm, which gives an acceptable frequency range from 250 Hz to 5 kHz. The sound power radiated from the shell was determined by scanning the sound intensity probe over the cylindrical shell. The vibration levels of 70 points which were distributed uniformly on the shell were measured using a B&K type 2635 charge amplifier, and a B&K type 4383 accelerometer. The vibration data recorded by HP3569A were downloaded to a PC for processing to give the averaged mean square vibration velocity over the shell surface. By using the definition of sound radiation efficiency as shown in equation (1)[1], the variation of the radiation efficiency with frequency is plotted in Fig.3. It can be seen that there is good agreement between the experimental results and BEM predictions.

$$\sigma = \frac{W_a}{\rho_o c S \left\langle \overline{v^2} \right\rangle} \tag{1}$$

where$S$ is the radiating surface area of the structure, $\langle v^2 \rangle$ denotes the average mean square velocity over the radiating surface, and $\rho_o c$ is the characteristic impedance of the medium.

4. ANALYSIS

The numerical results presented in section 2 show that the radiation efficiency of an acoustically thick shell possesses characteristics very different from plates and thin shells. In order to explore this phenomenon, the vibration behaviour of a circular cylindrical shell has to be examined.

It has been shown in [11] that the relationships between natural frequencies and wave numbers of finite length cylindrical shells are independent of the boundary conditions. In order to simplify the discussions, an approximate solution to the exact relationship due to neglecting in-plane defections[11,12] is used here:

$$\omega_{mn}^2 = \frac{D}{\rho h} \cdot \left[k_{zm}^2 + k_{\theta n}^2\right]^2 + \frac{K(1-\mu^2)}{\rho h a^2} \cdot \frac{k_{zm}^4}{\left[k_{zm}^2 + k_{\theta n}^2\right]^2} \tag{2}$$

where $D = \frac{Eh^3}{12(1-\mu^2)}$, $K = \frac{Eh}{1-\mu^2}$, E is the Young's modulus, μ is the Poisson ratio, h is the thickness of the shell, a is the radius of the circular cylindrical shell, ρ is the material density of the shell, ω_{mn} is the natural frequency for mode (m,n), k_{zm}, $k_{\theta n}$ are the wave numbers in the axial and circumferential directions respectively. For different boundary conditions, k_{zm} and $k_{\theta n}$ take different forms as discussed in [11].

In Fig.4(a), based on equation (2), a series of wave number curves associated with different frequencies are presented. It can be clearly seen that, unlike flat plates, the vibration modes of shells having the same natural frequencies could have different wave numbers k_s, where $k_s^2 = k_{zm}^2 + k_{\theta n}^2$. Only above the ring frequency of 13 kHz would these wave numbers approach the same value. This is due to the curvature effects which increase the wave speed in the axial direction below the ring frequency.

In order to illustrate the curvature effects, three wavenumber curves and three corresponding acoustic wavenumber curves are plotted in Fig.4(b). From the acoustic point of view[9], the vibration modes could be classified into acoustically fast modes and acoustically slow modes, sometimes called supersonic and subsonic modes. Acoustically fast modes refer to those of which the structural wavenumbers are smaller than the corresponding acoustic wavenumber, and the modal radiation efficiencies of these modes are unity. Acoustically slow modes are inefficient in acoustic radiation because structural wavenumbers are greater than the corresponding acoustic wavenumbers, which would lead to some cancellation in the radiation. For isotropic and flat plates[1,9], the unique demarcation for these two cases is the critical frequency. However, for cylindrical shells, from Fig.4(b), it can be seen that, below the critical frequency, the structural wavenumber curve will always intersect the acoustic wavenumber curve, such as point A, at a given frequency. This point of intersection in the wavenumber domain changes as the frequency changes. When the frequency reaches the critical frequency, the curves are tangent to each other as shown by point B in Fig.4(b). When the frequency is greater than the critical frequency, the two curves will no longer intersect. This result indicates that at any frequencies below the critical frequency, acoustically fast modes and slow modes could exist simultaneously, and the demarcation depends on the frequency. Therefore, for cylindrical shells, it is impossible to define an unique "critical frequency" for describing the acoustic properties as for flat plates. However, Fig.4(b) shows that the critical frequency defined for plates indicates the condition for all possible vibration modes of cylindrical shells to be supersonic. Thus the radiation efficiency of cylindrical shells should be unity above the critical frequency because the modal radiation efficiencies of all modes are unity.

Below the critical frequency, since both supersonic and subsonic modes could exist simultaneously, the overall radiation efficiency would depend on the number and the types of modes dominating the vibration response. For example, if the mode is supersonic, the modal radiation efficiency is unity, and if it is subsonic, the modal radiation efficiency is less than one. Therefore, the modal averaged radiation efficiency could be unity if supersonic modes dominate the response. Moreover, because any changes of boundary conditions could make some supersonic modes subsonic or vice versa, it can be expected that the variation of the modal averaged radiation efficiency of cylindrical shells due to the change of boundary conditions could be larger than that for flat plates.

In order to explain why the radiation efficiency reaches unity well below the critical frequency as shown in Fig.2, let's further examine the model presented above. The natural frequencies of a circular cylindrical shell can be calculated according to the method described in [11]. Corresponding to each natural frequency, there must exist a structural wavenumber curve to which the vibration mode belongs and an acoustic wave number curve of the same frequency, as shown in Fig.4(b). Note that at each point of the intersection of the two curves, the structural wave number is equal to the acoustic wave number. So one can compare the structural wave number of this mode k_S with the acoustic wave number k to determine whether this mode is supersonic or not. For this purpose, an index can be defined as follows,

$$\Delta L(m,n) = k - k_s = \frac{\omega_{mn}}{c} - \sqrt{k_{zm}^2 + k_{\theta n}^2} \tag{3}$$

Obviously, this index depends on the natural frequency and the structural wave numbers of the vibration mode. If $\Delta L \geq 0$, which means that the acoustic wavenumber is greater than structural wave number, the mode is supersonic, and if $\Delta L < 0$, the mode is subsonic. In Fig.5, ΔL of all the vibration modes associated with three different boundary conditions is plotted against the corresponding natural frequencies. It can be seen that the critical frequency f_C of the equivalent flat plate is an indicator showing whether all the vibration modes of the cylindrical shell are supersonic or not. Below the critical frequency, corresponding to each supersonic mode in Fig.5, there is a peak in the modal averaged radiation efficiency in Fig.2, for example, the peaks around 2300 Hz for simply supported and clamped conditions, 2800 Hz for free condition, etc. According to Fig.4, although subsonic modes exist below the critical frequency, the number of supersonic modes increases as the frequency increases so that the modal averaged radiation efficiency could reach unity at a frequency much lower than the critical frequency, as shown in Fig.2.

5. CONCLUSIONS

The results obtained by the boundary element model show that the acoustic radiation behaviour of acoustically thick circular cylindrical shells is quite different from that of plates or thin shells. Each vibration mode has its own demarcation for being acoustically fast, and it is no longer possible to define a unique critical frequency for all vibration modes of a thick shell. The critical frequency defined by an equivalent flat plate becomes the condition for all vibration modes of an acoustically thick shell to be acoustically fast. Therefore, the radiation efficiency of an acoustically thick shell could reach unity at a frequency much lower than the critical frequency. Also, because the acoustic behaviour of each vibration mode would be affected by the boundary conditions at both ends of the shell, the radiation efficiency of the shell has been shown to depend strongly on the boundary conditions.

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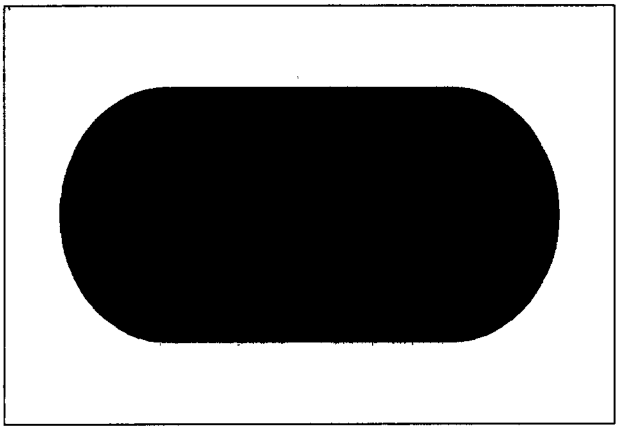


Fig.1The boundary element model of a cylindrical shell

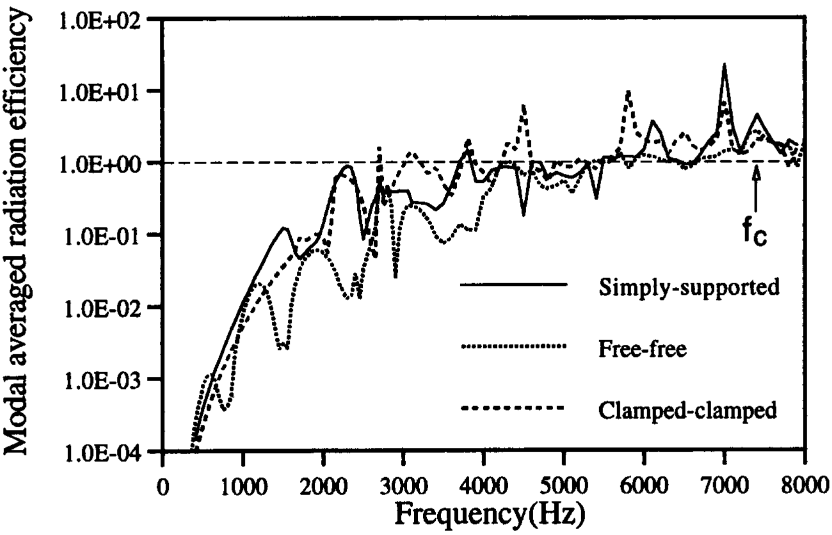


Fig.2. Radiation efficiency of an acoustically thick cylindrical shell

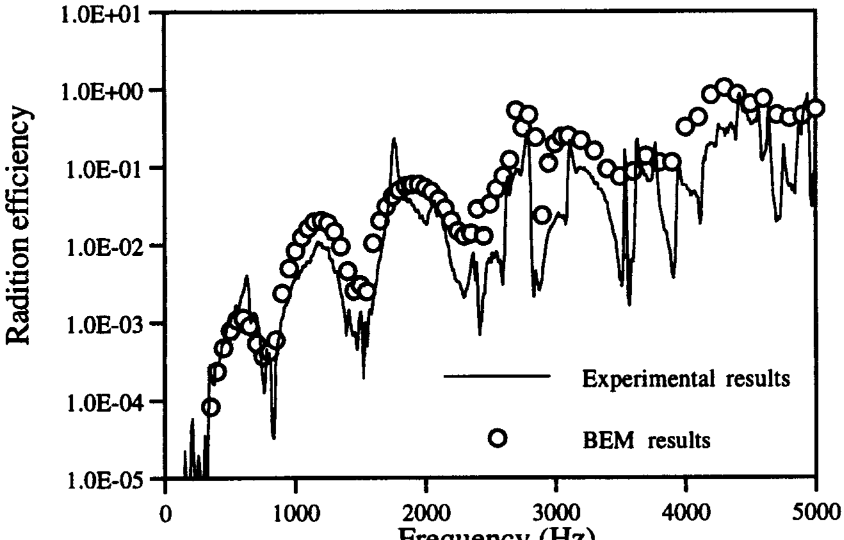
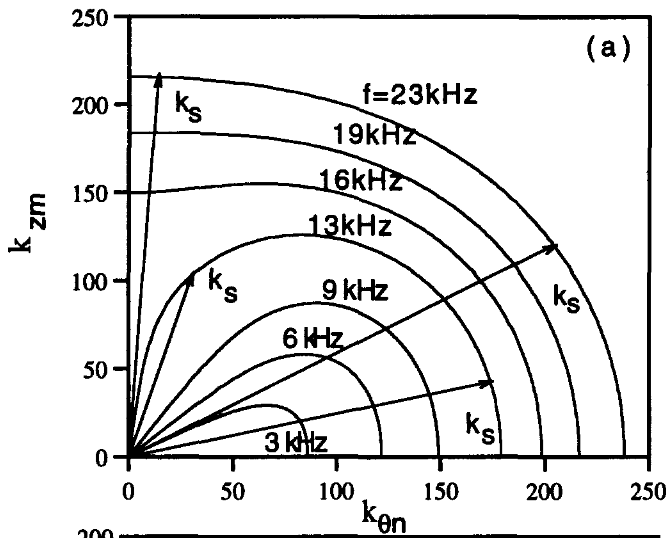


Fig.3 Comparison of the experimental results with BEM calculations



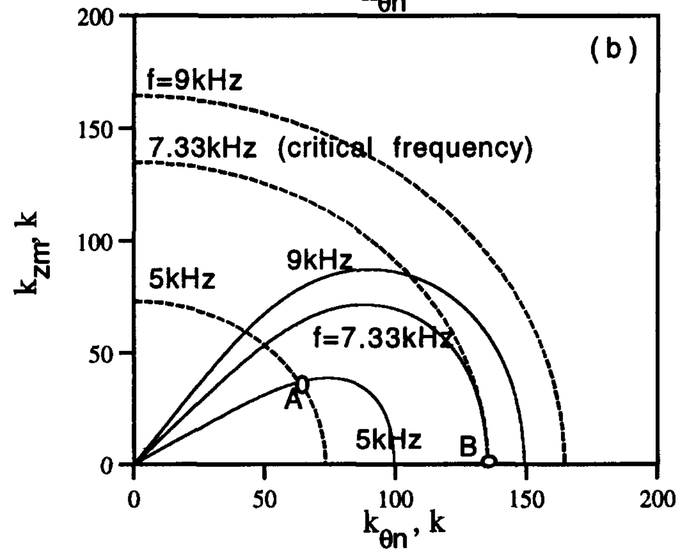


Fig.4. Wavenumber diagram for a cylindrical shell: ----, acoustic wavenumber; ——, structural wavenumber; $k_{\theta n}$, circumferential structural wavenumber; k_{zm}, axial structural wavenumber; k, acoustic wavenumber.

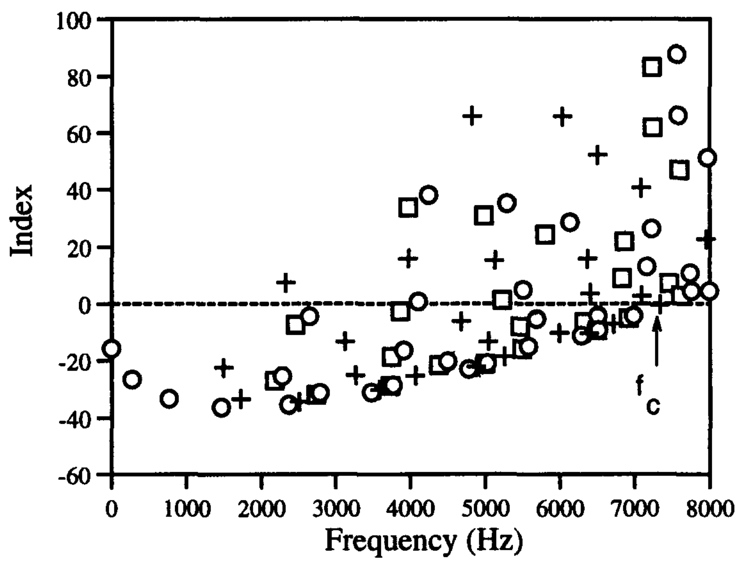


Fig.5. The index of each vibration mode of a cylindrical shell: +, simply-supported; o, free-free; □ , clamped-clamped.