FIFTH INTERNATIONAL CONGRESS ON SOUND AND VIBRATION

DECEMBER 15-18, 1997

ADELAIDE, SOUTH AUSTRALIA

COMBINED FEA/SEA VIBRATION ANALYSIS

Anders M Wilson

Division of Solid Mechanics
Chalmers University of Technology
SE-412 96 Göteborg, Sweden
wils@solid.chalmers.se

ABSTRACT

Local and global energy flow in structures built up by domains with different rigidities is studied. A combined FEA/SEA approach is advanced where stiffer domains (with low modal density) are analyzed using finite element analysis (FEA) and weaker domains (with high modal density) are analyzed using Statistical Energy Analysis (SEA). The approach proposed employs an iterative optimization procedure where the difference between externally supplied active power and the total power dissipated in the structure is minimized. Sectional forces in points connecting the stiff and weak domains are used as design variables. Exact dynamic substructuring is introduced, to reduce the number of degrees of freedom in the FE domain during the optimization procedure. The potential to cover a large frequency interval is demonstrated in a numerical example where the harmonic response of a truck is studied.

INTRODUCTION

Finite Element Analysis[1,2] (FEA) is frequently and mostly successfully used to model stress and low frequency vibration behavior in built up structures. Within the level of discretization, the method can provide the exact pressure level, or vibration amplitude at any point in space and time for any given dynamic input.

Although FEA has proven successful in these areas, it has been found to have severe limitations for noise prediction when the contribution from higher eigenmodes becomes important. For thin walled steel structures, FE calculated results will be poor at about frequencies corresponding to the 10'th to 20'th eigenmode. The important acoustic frequency range, however, often extends beyond the 100'th mode of vibration. To further extend the frequency range, in which FEA is acceptably accurate, some form of model reduction may be performed (Wilson and Josefson[3]). By using this approach, the number of nodes and elements can be increased, and higher modes may be represented more accurately.

The Statistical Energy Analysis[4,5] (SEA) technique has become increasingly interesting and important as an alternative and a complement to FEA, sofar specially to the aerospace and ship industry, for high frequency vibration and noise prediction. The successful application of SEA in its standard form relies on high modal density, high modal overlap, and short wave lengths. These are all factors that make FEA inaccurate at high frequencies. Up to now, SEA and FEA have mostly been used separately.

Since FEA and SEA have their computational strengths in different frequency ranges, a method of combining these two methods and taking advantage of each method's strengths would be useful. An advantage of a combination of FEA and SEA would be the possibility to analyze a generic structure, which may consist of components or regions having different rigidities. At one frequency, the modal densities of some components could be too high for FEA to be practically applicable, whereas another component is to rigid to permit use of SEA successfully. One may also note that coupling between a SEA component and an acoustic cavity is fairly straight forward, which makes a fluid structure interaction calculation easier. However, using the two methods together is not entirely straightforward due to the differences in the two methods natures.

PROPOSED METHOD

The present work is aimed at combining the two methods of analysis, SEA and FEA. To allow for large FEA models in the analysis, dynamic substructuring (Wilson and Josefson[3]) is used to reduce the number of degrees of freedom (DOFs) in the FE model. In the proposed approach separate SEA and FEA models of different components of the structure are first created. Sectional boundary forces in the model connecting the FEA and SEA components are then iteratively determined so that an objective function, involving the difference between the energy supplied to the structure and the energy dissipated in the structure, is minimized, see also Lu[6].

TWO DOMAIN APPROACH

Consider a structure built up by two domains V_{SEA} and V_{FEM}, see Fig. 1. The domain V_{SEA} has high modal density and is to be represented by SEA, and the domain V_{FEM}, the more stiff domain, is modeled by FEM and reduced using dynamic substructuring.

As indicated in Fig. 1 the two domains are connected along a common boundary. In the FEA domain the connection between the domains is described by connection forces and displacements. Displacements are due to both external and coupling loads. In the SEA domain the coupling is described by the sub-system energy flow, which in turn is described by power supplied by coupling forces between the domains. The governing state equations for the two domains are then solved separately at every frequency step iteratively, with the common connection forces as iterative variables, until the energy based objective function is fulfilled.

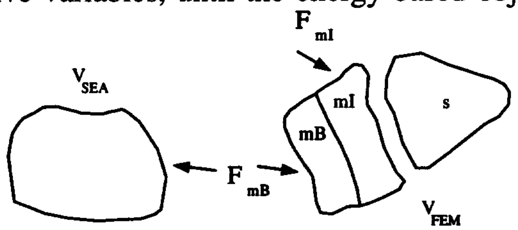

Figure 1. Domain V_{SEA} is analyzed by SEA, and domain V_{FEM} with sub-domains mB (master boundary), mI (master internal) and s (slave) is represented by FEA.

SEA DOMAIN V_{SEA}

A SEA energy balance for domain V_{SEA} built up by k subsystems at the analysis center band frequency ω may be formulated in matrix form according to Eq. (1):

$$\omega \boldsymbol{\Delta}\ \mathbf{E} = \boldsymbol{\Pi} \tag{1}$$

where $\boldsymbol{\Delta}$ is the coupling loss factor (kxk) matrix, $\mathbf{E}$ is the unknown subsystem modal energy (kx1) vector and $\boldsymbol{\Pi}$ is the (time averaged) supplied power (to the domain V_{SEA}) (kx1) vector.

The power input to V_{SEA} is given by $\Pi_{\mathrm{in}}^{\mathrm{SEA}} = \frac{1}{2}\mathrm{Re}\left\{\underline{\mathbf{F}}_{\mathrm{mB}}^{\mathrm{H}}\underline{\dot{\mathbf{X}}}_{\mathrm{mB}}\right\}$ where $\underline{\dot{\mathbf{X}}}_{\mathrm{mB}}$ contains velocity amplitudes of the points of the connection forces and $\underline{\mathbf{F}}_{\mathrm{mB}}$ contains the corresponding force amplitudes. Superscript $^{\mathrm{H}}$ denotes Hermitian transpose (i.e. transpose and complex conjugate). The time average response for the system is then obtained by solving the energy balance equation, Eq. (1) at every frequency step, giving the power dissipated in V_{SEA} as a function of $\underline{\mathbf{F}}_{\mathrm{mB}}$, to be used in the objective function. The size of the matrix and vectors in Eq. (1) is fairly small, due to the fundamental modeling approach in SEA.

FEA AND DYNAMIC SUBSTRUCTURING OF V_{FEM}

The domain V_{FEM}, as shown in Fig. 1, is modeled by FEA. This domain is divided into two sub-domains, one with interior slave (index s) DOFs, $\underline{\mathbf{X}}_s$ and one with master boundary DOFs (index m), $\underline{\mathbf{X}}_m$. The domain m is in turn divided in two sub-domains; one sub-domain mB in which boundary (or coupling) loads are applied, and sub-domain mI in which external loads are active. The equation of motion for domain V_{FEM} is expressed as

$$\underline{\mathbf{D}}(\omega)\begin{Bmatrix}\underline{\mathbf{X}}_s\\ \underline{\mathbf{X}}_{mI}\\ \underline{\mathbf{X}}_{mB}\end{Bmatrix} = \begin{Bmatrix}\underline{\mathbf{F}}_s\\ \underline{\mathbf{F}}_{mI}\\ \underline{\mathbf{F}}_{mB}\end{Bmatrix} \quad \Leftrightarrow \quad \underline{\mathbf{D}}(\omega)\begin{Bmatrix}\underline{\mathbf{X}}_s\\ \underline{\mathbf{X}}_m\end{Bmatrix} = \begin{Bmatrix}\underline{\mathbf{F}}_s\\ \underline{\mathbf{F}}_m\end{Bmatrix} \tag{2}$$

where underline denotes a complex entity, $\underline{\mathbf{D}}(\omega)$ is the frequency dependent dynamic stiffness matrix derived from the element stiffness (K), mass (M) and damping (C) matrixes, $\underline{\mathbf{F}}_s$ are forces applied at the slave DOFs, $\underline{\mathbf{F}}_{mI}$ are externally applied master forces, and $\underline{\mathbf{F}}_{mB}$ are optimization variable forces at the connection points between V_{SEA} and V_{FEM}.

The reason for dividing the domain V_{FEM} in to two areas is to eliminate the (slave) DOFs, that are of no primarily interest in the analysis, to reduce the number of active DOFs in the optimization procedure described below.

There are a great variety of dynamic substructuring reduction methods available. Generally, for a system under harmonic load with angular frequency ω, the system of equations for the domain containing $\underline{\mathbf{X}}_m$ can be partitioned into parts containing the slave DOFs and the master DOFs. When assuming that no loads are applied to the eliminated slave degrees of freedom, $\mathbf{F}_s = \mathbf{0}$

$$\begin{bmatrix}\underline{\mathbf{D}}_{ss} & \underline{\mathbf{D}}_{sm}\\ \underline{\mathbf{D}}_{ms} & \underline{\mathbf{D}}_{mm}\end{bmatrix}\begin{Bmatrix}\underline{\mathbf{X}}_s\\ \underline{\mathbf{X}}_m\end{Bmatrix} = \begin{Bmatrix}\mathbf{0}\\ \underline{\mathbf{F}}_m\end{Bmatrix} \tag{3}$$

$$\underline{\mathbf{D}}_{sm}(\omega) = \underline{\mathbf{S}}_{sm}(\omega) - \omega^2\mathbf{M}_{sm} \qquad \underline{\mathbf{D}}_{ms}(\omega) = \underline{\mathbf{D}}_{sm}^{T}(\omega) \tag{4}$$

$$\underline{\mathbf{D}}_{mm}(\omega) = \underline{\mathbf{S}}_{mm}(\omega) - \omega^2\mathbf{M}_{mm} \qquad \underline{\mathbf{D}}_{ss}(\omega) = \underline{\mathbf{S}}_{ss}(\omega) - \omega^2\mathbf{M}_{ss} \tag{5}$$

An exact dynamic representation of an undamped substructure is given by the reduced system of equations in Eq. (6):

$$(\mathbf{K}_{ex} - \omega^2\mathbf{M}_{ex})\mathbf{X}_m = \mathbf{F}_m \tag{6}$$

The exact reduced stiffness and mass matrices are given by a matrix transformation, according to Eqs. (7-8)

$$\mathbf{K}_{ex} = \mathbf{T}_{ex}^{T}\,\mathbf{K}\,\mathbf{T}_{ex} \qquad \mathbf{M}_{ex} = \mathbf{T}_{ex}^{T}\,\mathbf{M}\,\mathbf{T}_{ex} \tag{7}$$

$$\begin{Bmatrix}\mathbf{X}_s\\ \mathbf{X}_m\end{Bmatrix} = \mathbf{T}_{ex}\mathbf{X}_m \qquad \mathbf{T}_{ex} = \begin{bmatrix}-\mathbf{D}_{ss}^{-1}\mathbf{D}_{sm}\\ \mathbf{I}\end{bmatrix} \tag{8}$$

where $\mathbf{T}_{ex}$ (formally) is the exact transformation matrix.

However, the inverse of the matrix $\mathbf{D}_{ss}$ in Eq. (8) is computationally quite expensive to calculate. If the dynamic system is undamped ($\mathbf{C} = \mathbf{0} \rightarrow \mathbf{S}_{ij}(\omega) = \mathbf{K}_{ij}$), one has exact by expansion (Wilson and Josefson[3], Leung[7])

$$\mathbf{D}_{ss}^{-1}(\omega) = \mathbf{K}_{ss}^{-1} + \omega^2\mathbf{K}_{ss}^{-1}\mathbf{M}_{ss}\mathbf{K}_{ss}^{-1} + \omega^4\mathbf{\Phi}\mathrm{diag}\left[\Omega_i^{-4}(\Omega_i^2 - \omega^2)^{-1}\mathbf{I}\right]\mathbf{\Phi}^T \tag{9}$$

where $\mathbf{\Omega}^2 = \mathrm{diag}\left[\Omega_1^2,\ \Omega_2^2,\ \ldots,\ \Omega_n^2\right]$ contains the undamped fixed interface natural frequencies, and $\mathbf{\Phi}$ is the corresponding fixed interface modal matrix, that is $\mathbf{\Omega}$ and $\mathbf{\Phi}$ are calculated for the case when the master degrees of freedom are constrained, and normalized so that $\mathbf{\Phi}^T\mathbf{M}_{ss}\mathbf{\Phi} = \mathbf{I}$ and $\mathbf{\Phi}^T\mathbf{K}_{ss}\mathbf{\Phi} = \mathbf{\Omega}^2$. The fixed interface modes may be calculated by an Arnoldi algorithm with spectral transformation (Ericsson and Ruhe[8]).

If non-proportional damping is present, the damped fixed interface modal matrix $\mathbf{\Phi}$ is calculated in a different manner. The $2n$, where n is the number of slave degrees of freedom, damped fixed interface modes are calculated from the eigenvalue problem (see Leung[7])

$$[\mathbf{S}_{ss}(\omega) - \Omega^2\mathbf{M}_{ss}]\mathbf{\Phi} = \mathbf{0} \tag{10}$$

In this case the master degrees of freedom are constrained and normalized so that $\mathbf{\Phi}^T(2i\omega\mathbf{M}_{ss} + \mathbf{C}_{ss})\mathbf{\Phi} = \mathbf{I}$. Formally one then has the exact inverse of $\mathbf{D}_{ss}(\omega)$ by expansion in the damped fixed interface eigenpairs, according to Eq. (11)

$$\mathbf{D}_{ss}^{-1}(\omega) = \sum_{r=1}^{2n}(i\omega - \Omega_r)^{-1}\mathbf{\Phi}_r\mathbf{\Phi}_r^T \tag{11}$$

COUPLING OF THE FEA AND SEA DOMAINS

To obtain a solution, the complex amplitudes of the master boundary forces $\underline{\mathbf{F}}_{\mathrm{mB}}$ are chosen as design variables at the coupling points connecting the V_{SEA} and V_{FEM} domains. For the V_{FEM} domain these forces will act as external forces (together with the true external forces $\underline{\mathbf{F}}_{\mathrm{mI}}$ which supply the input power to the stiff component). At each frequency ω the complex harmonic response at the connecting points, $\underline{\mathbf{X}}_{\mathrm{mB}}$, is calculated using Eq. (2), assuming $\mathbf{F}_s = \mathbf{0}$. For the same frequency, a SEA calculation is performed for the weak component subject to an energy input at the connecting points, determined by the current values of $\underline{\mathbf{F}}_{\mathrm{mB}}$ and the impedance corresponding to $\underline{\mathbf{X}}_{\mathrm{mB}}$. In an iterative procedure the magnitudes and phases (e.g. $\underline{\mathbf{F}}_{\mathrm{mB}}$ is complex) of the forces $\underline{\mathbf{F}}_{\mathrm{mB}}$ are varied until the following objective function is minimized

$$\min_{\underline{\mathbf{F}}_{\mathrm{mB}}}\left|\frac{\Pi_{\mathrm{in}} - \Pi_{\mathrm{damp}}}{\Pi_{\mathrm{in}}}\right| \leq \varepsilon \tag{12}$$

where ε is taken as 10^{-4}.

The power expressions in Eq. (12) are

- $\Pi_{\mathrm{in}} = \frac{1}{2}\mathrm{Re}\left\{\underline{\mathbf{F}}_{\mathrm{mI}}^{\mathrm{H}}\dot{\underline{\mathbf{X}}}\right\}$ which is the energy input to the structure V_{FEA} by external forces $\underline{\mathbf{F}}_{\mathrm{mI}}$
- Π_{damp} is the sum of the energy dissipated in the stiff component $\Pi_{\mathrm{damp}}^{V_{\mathrm{FEM}}} = \frac{1}{2}\mathrm{Re}\left\{\dot{\underline{\mathbf{X}}}^{\mathrm{H}}\mathbf{C}_{\mathrm{ex}}\dot{\underline{\mathbf{X}}}\right\}$ and in the weak component $\Pi_{\mathrm{damp}}^{V_{\mathrm{SEA}}} = \sum_i E_i\,\omega\,\eta_i$.

Here $\mathbf{C}_{\mathrm{ex}}$ is the reduced damping matrix for the stiff domain V_{FEM}, and η_i is the damping loss factor for subsystem i. Note that the displacements at the boundary between V_{FEM} and V_{SEA}, $\underline{\mathbf{X}}_{\mathrm{mB}}$, are calculated from the FE analysis of V_{FEM} and used, together with the design forces $\underline{\mathbf{F}}_{\mathrm{mB}}$, to obtain the subsystem energies E_i in V_{SEA}.

The calculation of the starting vector of the unknown force amplitudes $\tilde{\underline{\mathbf{F}}}_{\mathrm{mB}}$ in the optimization process is based in the impedance for infinite structures (Cremer et. al.[9]) connected to the FE model. These approximate subsystems are chosen as the same type of subsystems as those of the actual connected V_{SEA} domain.

The complete reduced system of equations may be written as

$$\underline{\mathbf{D}}(\omega)_{\mathrm{ex}}\begin{Bmatrix}\underline{\mathbf{X}}_{\mathrm{mI}}\\ \tilde{\underline{\mathbf{X}}}_{\mathrm{mB}}\end{Bmatrix} = \begin{Bmatrix}\underline{\mathbf{F}}_{\mathrm{mI}}\\ \tilde{\underline{\mathbf{F}}}_{\mathrm{mB}}\end{Bmatrix} \tag{13}$$

Using the expression for the impedance $\mathbf{Z}$ for the connected approximate SEA subsystems (to approximate the connection between the FEA and the SEA domains) Eq. (13) may be rewritten as

$$\mathbf{D}_{\mathrm{ex}}\begin{Bmatrix}\underline{\mathbf{X}}_{\mathrm{mI}}\\ \tilde{\underline{\mathbf{X}}}_{\mathrm{mB}}\end{Bmatrix} = \mathbf{Z}i\omega\begin{Bmatrix}\mathbf{0}\\ \tilde{\underline{\mathbf{X}}}_{\mathrm{mB}}\end{Bmatrix} + \begin{Bmatrix}\underline{\mathbf{F}}_{\mathrm{mI}}\\ \mathbf{0}\end{Bmatrix} \tag{14}$$

which can be solved for the unknown coupling displacements. This in turn gives the starting forces $\tilde{\underline{\mathbf{F}}}_{\mathrm{mB}}$ as $\tilde{\underline{\mathbf{F}}}_{\mathrm{mB}} = i\omega\mathbf{Z}\tilde{\underline{\mathbf{X}}}_{\mathrm{mB}}$.

The minimization is performed in MATLAB[10] using the `fmins` function, thus employing an iterative BFGS Quasi-Newton minimization with a mixed quadratic and cubic line search procedure, which is adapted until the objective function in Eq. (12) is achieved. Since dynamic substructuring is employed, the size of the problem used in the iterations, to fulfill the energy balance for the structure, is fairly small.

NUMERICAL EXAMPLE: TRUCK STRUCTURE

To investigate the method proposed above, a harmonic vibration analysis of a structure built up by one stiff and one weak domain connected at a number of discrete points is performed.

The structure analyzed should resemble a conceptual model of a truck structure as shown in Fig. 2. In the model the frame and engine constitutes the stiff domain V_{FEM} having 21 eigenmodes up to 100 Hz. The weak domain V_{SEA} is here the cabin, which has more than 20 eigenmodes below 10 Hz. The truck is excited at the engine by a vertical point force.

FE MODEL OF FRAME

The FE model of the frame, as shown in Figs. 2 and 3, is modeled by 138 two-node Timoshenko beam elements. This part of the structure is taken as undamped. The wheel suspension is modeled as 8x3 two-node spring elements with translational stiffness in three directions and hysteretic damping in the vertical direction with loss factor $\eta = 0.05$. The frame-to-cabin connection is modeled by two-node connection springs at four locations. These springs have translational and rotational stiffness in three directions respectively, and translational hysteretic damping, with $\eta = 0.08$ in the vertical direction. Concentrated 100 kg masses are also added at the four connection points. The engine is modeled as two rigidly connected 500 kg concentrated masses.

As discussed above, in order to perform many solution steps efficiently, the frame structure shown in Fig. 3a is reduced using the exact dynamic substructuring approach, Eq. (6—9). The undamped frame (Fig. 3b) is reduced, retaining the node of load application (at the engine), the wheel suspension spring elements connection nodes and the cabin to frame connection points (where damping is introduced). Totally 13 nodes out of 146 are retained. The reduced FE model is shown in Fig. 3c. It is in the frame-to-cabin connection nodes that the iteration forces $\underline{\mathbf{F}}_{\mathrm{mB}}$ are introduced.

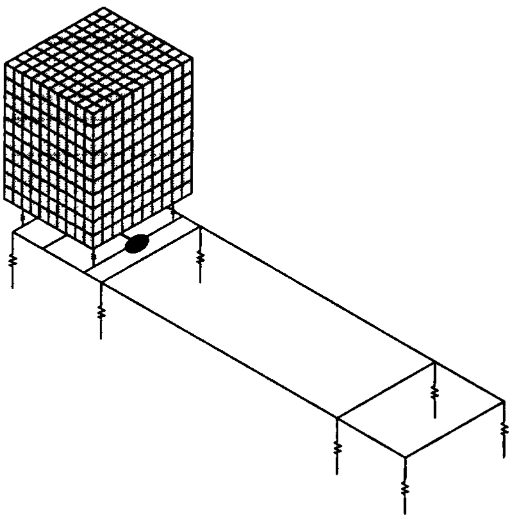

Figure 2. FEM model of truck structure consisting of engine, frame, suspension and cabin.

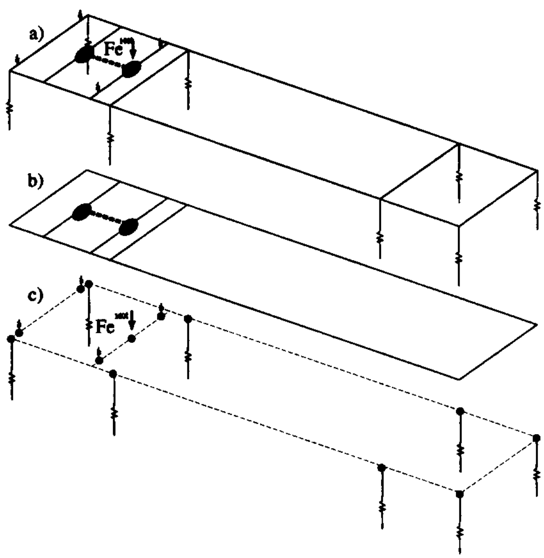

Figure 3. a) FE model of engine and frame structure.
b) Unreduced (undamped) frame structure.
c) Reduced FEM model (dashed) with retained nodes (dots).
Damped springs added.

SEA CABIN MODEL

The cabin is preferably modeled by SEA subsystems since it has a large number of resonant modes at low frequencies. The cabin is built up from the geometry in Fig. 4, and represented by a number of subsystems; six flexural plates according to Fig. 5. The cabin is modeled in the commercial SEA software `AutoSea`[4], by which the SEA parameters (coupling loss factors, modal densities etc.) are calculated. Material parameters used in the cabin and in the frame should resemble steel with Young's Modulus E = 207 MPa, density ρ = 7820 kg/m^3 and Poisson's ratio $\nu = 0.29$. The plate thickness is $t = 2$ mm. The cabin is taken as hysteretically damped with a constant damping loss factor $\eta = 0.01$. To fully model the plates used in the model all three wave types should be included; i.e. flexural, extensional and shear waves. However, when calculating the number of modes in the frequency bands, it is seen that the number of resonantly excited modes is substantially lower for the shear and extensional waves compared to the flexural waves. This means that the SEA assumptions are not valid at low frequencies for these two types of subsystems.

To investigate the validity of only including the flexural modes, one may calculate the power supplied to a three-wave type plate subsystem through a perpendicular flexural plate by a point force (which is similar to the subsystem network in the numerical example). Calculating the vibrational energy for the different wave types in the "three wave type plate", shows that the main part of vibrational energy is stored in flexural modes, which indicates that the assumption to only include these modes in the subsystem network is reasonable.

For the structure used in this analysis, one of the criteria for SEA to be valid, (i.e. the number of modes in band being >> 1) is well met for the flexural wave type. For the plates in the SEA model the number of modes in band are for the front/rear 784, left/right 700 and for floor/roof 580, for a frequency band $\Delta\omega$ = 100 Hz.

The subsystems are coupled along the connecting lines, where certain assumptions have to be made concerning plates joined perpendicular to each other regarding conditions at the connecting edges. The restriction used in `AutoSea`[4] is that no displacements are allowed at the joint, and that energy transmission is only occurring through bending moment (Cremer et. al.[9]).

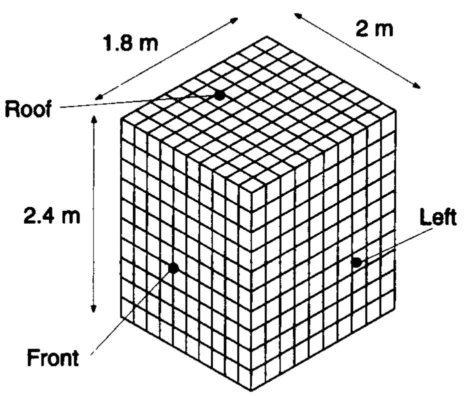

Figure 4. Geometry of cabin, and mesh of reference cabin FE model A containing 600 four-node shell elements.

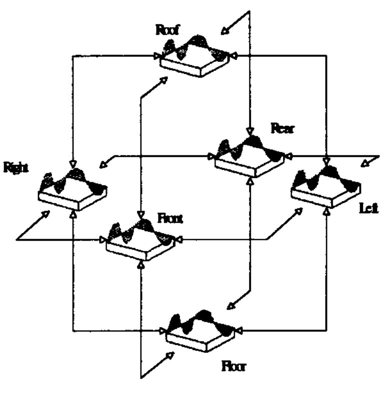

Figure 5. SEA network of cabin built up by six subsystems representing flexural modes in thin plates.

REFERENCE CABIN FE MODEL

Reference solution models, where both the frame and the cabin are modeled using FEA, are obtained using the commercial FE code `SOLVIA`[11]. The model of the frame, engine and suspension is the same as described above, and the cabin is modeled in two reference models. Reference model A with 600 iso-parametric four-node shell elements of Mindlin type, see mesh in Fig. 4, and reference model B with 3936 elements of the same kind (not shown). The element size used is 0.20 m in reference model A, and 0.08 m in reference model B, which gives a maximum reliable frequency (Blevins[12]) of 61 Hz and 380 Hz with four elements/wave length, respectively.

CALCULATED RESULTS

Figure 6 shows the calculated frequency variation of the real part of the amplitude of the vertical displacement at the point of load application. Results using the FEA/SEA approach, using dynamic substructuring with 30 undamped fixed interface modes (with the highest mode corresponding to 150 Hz) included in the reduction, are compared with results from the full reference A FE model. One finds that the FEA/SEA approach seems to miss two resonances; FEA mode 5 at 25 Hz and mode 7 at 30 Hz. These modes involve in-plane motion in the floor. Otherwise the agreement is good.

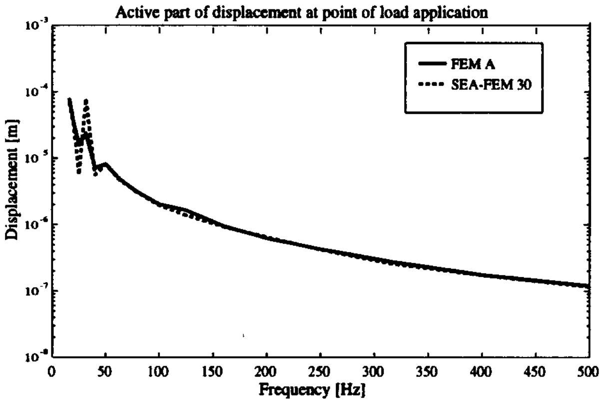

Figure 6. Calculated frequency variation of the real part of the vertical displacement at the load application point on the frame. 30 fixed interface modes are included in substructuring of frame used in combined FEA/SEA model. Compared to reference model A.

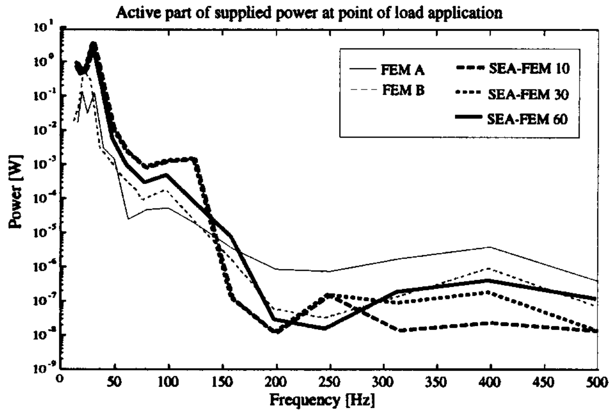

Figure 7. Calculated frequency variation of the real part of the supplied power to the truck structure. Different number of fixed interface modes used in substructuring of FE part.

Figure 7 shows the calculated frequency variation of the real part of the power supplied to the truck structure, that is the active power input using the present FEA/SEA approach compared to using the reference FEA models A and B with the cabin modeled using some 600 and 3936 shell finite elements, respectively. For the FEA/SEA approach results are presented for three different levels of dynamic substructuring, including 10, 30 and 60 fixed interface modes corresponding to 51, 150 and 300 Hz respectively, in the reduction of the frame structure.

One finds good agreement between the FEA/SEA approach and the reference A model up to frequencies of approximately 60 Hz where the cabin part of the reference FE model becomes inaccurate. When comparing to reference model B, which is valid up til approximately 380 Hz, one finds that including more interface modes in the reduction does seem to extend the validity of the calculated frequency response. Note, however, that there exist no exact reference solution to the present problem for the frequency range plotted. Such a solution would require a much denser FE mesh of the cabin part.

DISCUSSION AND SUMMARY

By combining a finite element analysis and a statistical energy analysis it seems possible to use the advantages of both approaches and obtain a harmonic response over a large frequency interval. The present approach is particularly useful when the structure analyzed consists of domains having considerable different rigidities. The state description of the structure is more detailed in FEA than in SEA. Because SEA is based on energy transfer, phase angles for displacements and forces are not included in the SEA model. And since this information need to be included in a FE analysis, the present approach to couple a FEA and a SEA model involves minimization of the difference between the total power input to the structure and the total power dissipated within the structure. The procedure is iterative at each frequency. It is therefore desirable to employ dynamic substructuring in the FE model in order to reduce the size of the model involved in the minimization procedure. The results show that dynamic substructuring may be used to reduce the size of the FEA domain system of equations, when care is taken to which modes that are included in the reduced model. It is seen that as the mesh of the FE modeled domain is made more dense the results, at high frequencies, from the combined model get more similar to those of a pure FE reference model. However, it is believed that the present approach in some aspects need to be further developed; the optimal level of substructuring in the FE model and the use of robust starting vectors for the design variables in the iterative minimization procedure need to be further investigated.

ACKNOWLEDGMENTS

The work presented here was initiated and supervised by Professor Lennart Josefson at the Division of Solid Mechanics Chalmers University of Technology, who's support is gratefully acknowledged. The work was funded by The Swedish Research Council for Engineering Sciences (TFR) through their contract 94-300 and by The Volvo Corporation, Göteborg, Sweden, through the Volvo-Chalmers Vehicle Research Program.
Special thanks to Dr. Mikael Enelund at the Division of Solid Mechanics, Chalmers University of Technology.

REFERENCES

[1] K.-J. Bathe. *Finite element procedures*. Prentice-Hall, Englewood Cliffs, NJ, USA, 1996.

[2] R. D. Cook, D. S. Malkus, and M. E. Plesha. *Concepts And Applications Of Finite Element Analysis*. John Wiley & Sons, Inc., New York, NY, USA, third edition, 1989.

[3] M. A. Wilson and B. L. Josefson. Mechanical intensity fields in reduced structures. In *Proceedings of the 37th AIAA/ASME/ASCE/AHS/ASC Structures, Structural Dynamics and Materials Conference in Salt Lake City, UT, USA.*, pages 1585–1593, 1996.

[4] Vibro-Acoustics Sciences Ltd. *AutoSea User Guide Rev 1.5*. San Diego, CA, USA, 1996.

[5] R. H. Lyon and R. G. DeJong. *Theory and Application of Statistical Energy Analysis*. Butterworth-Heinemann, Newton, MA, USA, second edition, 1995.

[6] L. Lu. Dynamic substucturing by FEA/SEA. In *Proceedings of Vehicle Noise*, volume NCA–9, pages 9–12, The American Society of Mechanical Engineers, New York, NY, USA, 1990.

[7] A. Y. T. Leung. *Dynamic Stiffness and Substructures*. Springer Verlag, London, UK, 1993.

[8] T. Ericsson and A. Ruhe. The spectral transformation for the numerical solution of large sparse generalized symmetric eigenvalue problems. *Mathematics of Computation*, 35:1251–1268, 1980.

[9] L. Cremer, M. Heckl, and E. E. Ungar. *Structure-Bourne Sound*. Springer Verlag, Berlin, Germany, second edition, 1988.

[10] The MathWorks, Inc., Natick, MA, USA. *MATLAB Reference Guide*, 1993.

[11] Solvia Engineering AB, Västerås, Sweden. *SOLVIA-PRE 95.0 Users Manual for Stress Analysis*, 1995.

[12] R. D. Blevins. *Formulas for Natural Frequency and Mode Shape*. Van Nostrand Reinhold Company, New York, NY, USA, 1979.