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A QUALITATIVE DYNAMICS-BASED DISCRETE HOLOGRAPHICS METHOD FOR VIBRATION SIGNAL ANALYSIS

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ABSTRACT

Presented in this paper is a graphics method called discrete holographic method which can rapidly interpret the results from the plethora of vibration data. The method is developed based on the theory of qualitative dynamics, the subject of which is to extract in a qualitative sense the characteristics of the underlying dynamics that governs the behavior of a dynamical system. The proposed discrete holographics is constructed in a three-dimensional space by three sets of vibration data sampled by time delays. The three sets of data are called visible vectors in qualitative dynamics. It is shown in the paper that the number of visible vectors sufficient for vibration signal analysis is three, which is proposed here to construct a discrete holographics. The proposed method is used to analyze various types of vibration signals including transient signals, modulated signals, repetitive signals. The results show that the method can efficiently reveal the distinctions for different types of vibration signals.

1. INTRODUCTION

There are many methods that have been proposed in the literature for vibration signal analysis, the subject of which is to extract the characteristics of a vibration system from vibration measurement data. In general, vibration signals can be classified as deterministic and non-deterministic. The former can be further classified as linear and non-linear, and latter as stationary and non-stationary. Most of vibration signal analysis methods are meant to provide quantitative analysis, such as for structure identification. However, certain applications may only need qualitative analysis, such as condition monitoring and diagnosis of machines and processes based on vibration signals.

Until recently, the development in qualitative dynamics has given rise to the introduction of qualitative analysis of complex dynamical signals. The subject of qualitative dynamics is to extract in a qualitative sense from measured signals the characteristics of the underlying dynamics that governs the behavior of a dynamical system. Qualitative dynamics has been extensively studied and used [1][2] in physics for extracting the complex nonlinear dynamics of physics experiment. In computer science, research has been carried out on solving qualitative differential equations (QDE) to develop qualitative reasoning methods for extracting information with incomplete knowledge [3].

In this paper, we extend qualitative dynamics to vibration signal analysis. A method called the discrete holographics method is proposed which is constructed by three sets of vibration measurement data collected by time delays. Various types of vibration signals including transient signals, modulated signals, repetitive signals, are analyzed by using the proposed method. The results show that the discrete holographic method can graphically reveal differences for different types of vibration signals. This gives rise to the possibility of utilizing computer graphics to graphically extract the distinct patterns from the plethora of complex vibration data.

2. QUALITATIVE DYNAMICS

2.1 Dynamical System

Consider a dynamical system described by the following differential equations:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}) \tag{1}$$

where $\mathbf{x}$ is a vector representing a state of the dynamical system. If $\mathbf{f}(\mathbf{x})$ is locally Lipschitz [1], we may write the solution of eqn. (1) as

$$\mathbf{x} = \phi_t \mathbf{x}_o \tag{2}$$

where $\mathbf{x}_o$ is an initial value of $\mathbf{x}$ and ϕ_t is a function of time. For all possible initial values, we may obtain a set of solutions. If the dynamical system is dissipative, the solution set will contract onto a set of lower dimension, called *attractor.* On the attractor, the dynamical system has fewer degrees-of-freedom and hence requires less information to specify its state. This gives rise to the possibility of describing a complex dynamical system qualitatively by measurement data.

The theorem for embeddings serves as a basis for extracting qualitative dynamics from measurement data. An embedding is referred to as a map Φ from manifold M to a space U such that image $\Phi(M)$ is a smooth submanifold of U, and Φ is a diffeomorphism between M and $\Phi(M)$. It was proven [1] that in Euclidean space, a smooth $m-$dimensional manifold, which is Hausdorff, may be embedded in a $n(> 2m+1)-$dimensional space.

2.2 Method of Delays

Based on Whitney's theorem, a method called *method of delays* was proposed by Broomhead and King[1] to reconstruct an embedding space from a time series of measurement data. To describe this method, let us consider a time series of measurement data at a single measurement point

$$x(1),\ x(2),\ ...,\ x(i),\ x(i+1),\ ... \tag{3}$$

and define a visible vector $\mathbf{u}_1$ by means of the so-called $(n,\ J)$ window as

$$\mathbf{u}_1 = [x_1\ x_2\ ...\ x_k\ ...\ x_n] \tag{4}$$

where

$$x_k = x(1 + (k-1)J) \tag{5}$$

Note that $\mathbf{u}_1$ is defined as a row vector, n is the dimension of the visible vector and J is the time interval. A visible vector means a vector containing n visible data points separated by time interval J. For a total of N_T data points, the number of the visible vectors that can be constructed is equal to

$$N = \frac{N_T - (n-1)J}{J} \tag{6}$$

These vectors are used to form the $N \times n$ trajectory matrix $\mathbf{U}$ given below

$$\mathbf{U} = [\mathbf{u}_1^T,\ \mathbf{u}_2^T, ...\mathbf{u}_n^T]^T \tag{7}$$

where the kth element of $\mathbf{u}_l$ equals to $x(1 + (l + k - 2)J)$.

A visible vector defined in the $(n,\ J)$ window constitutes a vector in the embedding space. As the time series advances in a step of J through the window, a sequence of vectors in the embedding space is generated, forming a discrete trajectory. A discrete phase plane can be constructed by plotting two adjacent visible vectors against each other in a two-dimensional plane. As shown later, the discrete phase plane is one of three projections of the discrete holographics proposed in this paper. For given sampling frequency f_s, the sampling length τ_w of the $(n,\ J)$ window should be $\tau_w \geq (2m+1)/f_s$.

3. RELATION TO VIBRATION SIGNAL ANALYSIS

We use the method presented by Ibrahim [4] for modal analysis in time domain to show how the method of delays can be related to vibration signal analysis. Let us start with considering the following vibration equations

$$\dot{\mathbf{v}} = \mathbf{A}\mathbf{v} \tag{8}$$

where $\mathbf{v} = [\mathbf{x}^T,\ \dot{\mathbf{x}}^T]^T$ is the n-dimensional vector; $\mathbf{x}$ is $n/2$-dimensional displacement vector; $\dot{\mathbf{x}}$ is $n/2$-dimensional velocity vector; and $\mathbf{A}$ is the $n \times n$ matrix consisting of mass, damping and stiffness. The response $\mathbf{x}$ can be expressed as follows

$$\mathbf{x} = \mathbf{P}\mathbf{e} \tag{9}$$

where $\mathbf{P}$ is the $n/2\times n$ mode matrix; $\mathbf{e} = [e^{\lambda_1 t}\ e^{\lambda_2 t}\ \dots\ e^{\lambda_r t}\ \dots\ e^{\lambda_n t}]^T$; and λ_r is the rth eigenvalue.

For the sake of modal analysis, Ibrahim suggested to construct the following three matrices by carrying out measurement at m measurement points and collecting the data in three difference periods of time separated by a time delay Δt, that is

$$\begin{aligned} \mathbf{X} &= [\mathbf{x}_1\ \mathbf{x}_2\ \dots\ \mathbf{x}_i\ \dots\ \mathbf{x}_n] \\ \mathbf{Y} &= [\mathbf{y}_1\ \mathbf{y}_2\ \dots\ \mathbf{y}_i\ \dots\ \mathbf{y}_n] \\ \mathbf{Z} &= [\mathbf{z}_1\ \mathbf{z}_2\ \dots\ \mathbf{z}_i\ \dots\ \mathbf{z}_n] \end{aligned} \tag{10}$$

where $\mathbf{x}_i = \mathbf{Pe}(t_i)$; $\mathbf{y}_i = \mathbf{Pe}(t_i + \Delta t)$ and $\mathbf{z}_i = \mathbf{Pe}(t_i + 2\Delta t)$.

Considering eqn. (10) in row vectors, in analog to eqn. (7), eqn. (10) can be considered based on the method of delays formed by $(n, \Delta t)$ windows for all measurement points, and the number of visible vectors for each window corresponding to each measurement point is three.

To show how eqn. (10) can be used for modal analysis, we may re-write it in view of eqn. (9) as follows

$$\begin{aligned} \mathbf{X} &= \mathbf{PE} \\ \mathbf{Y} &= \mathbf{QE} \\ \mathbf{Z} &= \mathbf{RE} \end{aligned} \tag{11}$$

where $\mathbf{Q} = \mathbf{P}\boldsymbol{\Delta}$; $\mathbf{R} = \mathbf{Q}\boldsymbol{\Delta}$; $\mathbf{E} = [\mathbf{e}_1\ \mathbf{e}_2\ \dots\ \mathbf{e}_{2m}]$ and $\boldsymbol{\Delta} = diag[e^{\lambda_r \Delta t}]$. Then we form the following two matrices

$$\boldsymbol{\Phi} = [\mathbf{X}^T,\ \mathbf{Y}^T]^T;\ \hat{\boldsymbol{\Phi}} = [\mathbf{Y}^T,\ \mathbf{Z}^T]^T \tag{12}$$

which can be re-written in view of eqn. (11) as

$$\boldsymbol{\Phi} = \boldsymbol{\Psi}\mathbf{E};\quad \hat{\boldsymbol{\Phi}} = \hat{\boldsymbol{\Psi}}\mathbf{E} \tag{13}$$

where

$$\boldsymbol{\Psi} = [\mathbf{P}^T,\ \mathbf{Q}^T]^T;\ \hat{\boldsymbol{\Psi}} = [\mathbf{Q}^T,\ \mathbf{R}^T]^T = \boldsymbol{\Psi}\boldsymbol{\Delta} \tag{14}$$

In view of eqns. (13) and (14), we can have the following characteristic equation

$$\mathbf{B}\boldsymbol{\Psi} = \boldsymbol{\Psi}\boldsymbol{\Delta} \tag{15}$$

where $\mathbf{B}$ is the characteristic matrix given as

$$\mathbf{B} = \hat{\boldsymbol{\Phi}}\boldsymbol{\Phi}^{-1} \tag{16}$$

The eigenvectors of eqn. (15) represent vibration modes. Natural frequencies and damping ratios can be determined from the eigenvalues of eqn. (15). In other words, data construction of eqn. (10) is sufficient for modal analysis. Vibration signal analysis may be considered as a subset of modal analysis as it is generally concerned about analysis of vibration signals for individual measurement points, without referring to vibration modes.

4. DISCRETE HOLOGRAPHICS

4.1 Description of the Method

As discussed in the previous two sections, based on qualitative dynamics, the data matrices constructed by time delays can be used to extract the characteristics of the underlying dynamics of a dynamical system. In light of modal analysis in time domain, three sets of vibration data collected by time delays are sufficient for extracting the characteristics of vibration signals. On the basis of these two concepts, we propose to use three sets of vibration data to form a discrete trajectory in a three-dimensional space. We call this method discrete holographic method, as it uses the discrete (sampled) data to provide an overall information for vibration signal analysis by using a three-dimensional graphics.

4.2 Selection of Window Parameters

As described in the previous subsection, the discrete holographics is constructed by three visible vectors formed based on the method of delays by means of $(n,\ J)$ window. For given total of N_T data points, since $N = 3$, from eqn. (6) we know that n is bound by

$$n \leq \frac{(N_T - 2J)}{J} \tag{17}$$

The parameter J is two fold. First, J is the time interval between two adjacent data points for the same visible vector. By virtue of signal processing theory, for $J > 1$, it represents sub-sampling. To obtain smooth trajectories J should be in general close to the following ratio

$$\alpha = f_s/f_n \tag{18}$$

where f_n stands for the highest frequency of interest. It is said undersampled when $J > \alpha$ and oversampled when $J < \alpha$. As an example, Fig. 1 shows the discrete holographics for the vibration response of an un-damped free vibration system, with $\alpha = 4.5495$. Fig. 1(b) is for $J = 4,\ \approx \alpha$, showing a smooth limit cycle; Fig. 1(d) is for $J = 2,\ < \alpha$, oversampled, showing a dense and wide limit cycle; Fig. 1(f) is for $J = 20,\ > \alpha$, undersampled, showing a coarse and wide limit cycle.

Second, J is related to the lag time between visible vectors, as it can be seen by referring to eqn. (7). The lag time causes the phase delay between two visible vectors. If there is no phase delay, the two visible vectors are said positive correlated; if the phase delay is 180^o, they are said negative correlated. For the un-damped free vibration system, the major axis of the limit cycle is close to the diagonal line when the phase delay is within the range from -90^0 to 90^o, whereas it is close to the antidiagonal line when the phase delay is from 90^o to 270^o. This is demonstrated in Fig. 2(c)(e) and (f) with the phase delay equal to 316.52^o, 158.26^o, 142.60^o, respectively.

5. VIBRATION SIGNAL ANALYSIS

5.1 Modulated Signal

There are two types of modulated vibration signals, amplitude and phase modulated. The modulated vibration signals are produced when the response of a vibration system is affected by that of another vibration system. For example, in gear transmission boxes, the vibrations due to gear faults could be modulated by the vibrations of the rotating shaft due to imbalance.

Shown in Figs. 2(a) and (c) are the amplitude and phase modulated signals of Fig. 1(a), respectively. As compared to the discrete holographics of the original signal shown in Fig. 1(b) which is disk-shaped, that of the amplitude modulated signal shown in Fig. 2(b) is stretched like a spiral, whereas that of the phase modulated signal shown in Fig. 2(d) becomes like a pile of straws. These distinct patterns can be used to identify graphically the modulation in the vibration signals.

5.2 Transient Signal

Transient signals could change in amplitude or in frequency or both, which could be caused by a changing external disturbance to the vibration system. For example, blasting generates an amplitude transient (damped) vibration in the mining machine, whereas starting-up of the engine changes the frequency of the forced vibration of the airplane due to rotor imbalance.

Figs. 3(a) and (c) shows the vibration signals with changing amplitude and frequency changing. As shown in Fig. 3(b), in case of amplitude changing vibration, the trajectory in the discrete holographics will converge. As shown in Fig. 3(d), in case of frequency changing vibration, the multi rings (for un-damped free vibration) appear in the discrete holographics, indicating the different frequencies.

5.3 Repetitive Signal

Repetitive signals are those caused repetitive impulses. Examples include bearing and gear defect-induced vibrations, plasma torch vibration due to arc striking, veneer peeling knife vibration due to wood grain variation, etc.

Fig. 4(a) shows a series of repetitive damped vibration signals and Fig. 4(b) shows its discrete holographics. In this case, one should choose smallest $J(=1)$ in order to show the nature of the damped vibration signals.

Fig. 5 shows the analysis of defect gear vibration signals. Fig. 5(a), (b), (c) and (d) represent baseline (normal condition), pitting, spalling, and scuffing, and clearly their patterns are different.

6. CONCLUSIONS

In this paper, we have proposed a graphical method called the discrete holographics for vibration signal analysis. We have laid down the theoretical foundation for the method based on the qualitative dynamics and the modal analysis in the time domain. The three sets of data, namely three visible vectors, are sufficient to extract the vibration parameters. The results of analyses of the simulation and measurement data show that the proposed method can graphically extract the patterns for different types of vibration signals.

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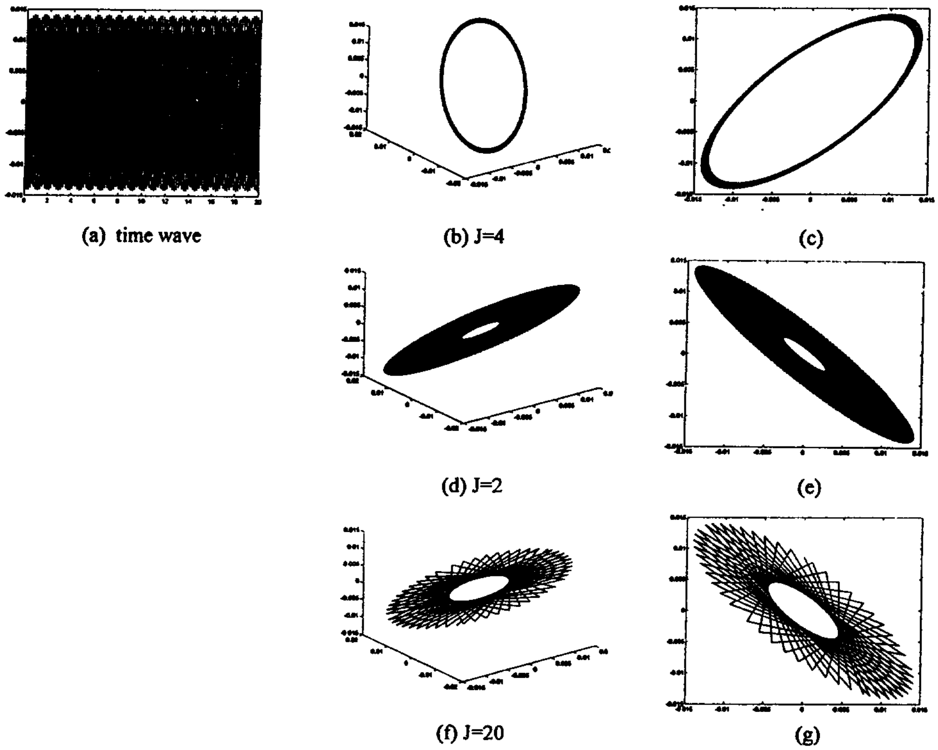
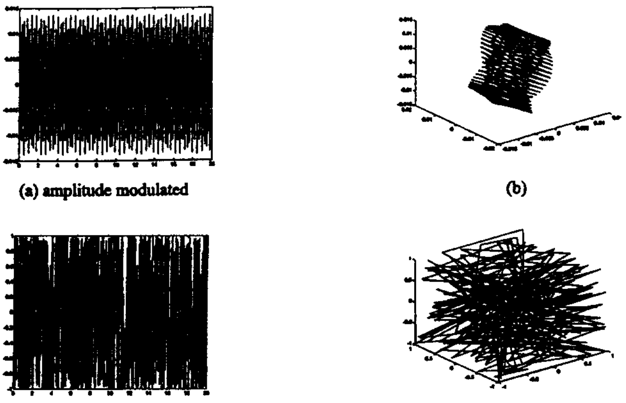


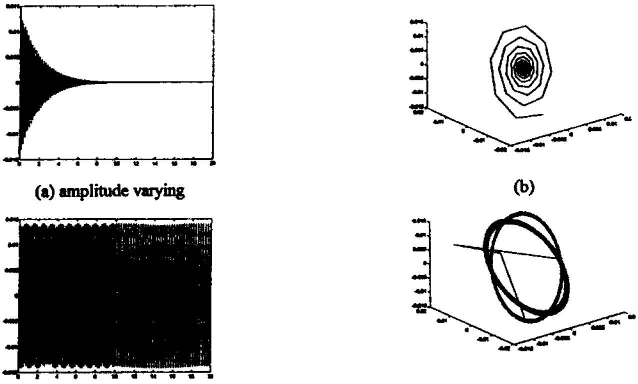
Figure 1 Un-damped vibration signals and their discrete holographics



(a) amplitude modulated (b)

(c) phase modulated (d)

Figure 2 Modulated signals and their discrete holographics



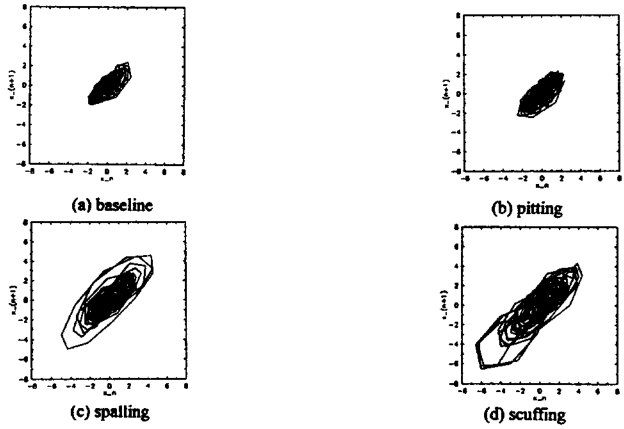
(a) amplitude varying (b)

(b) frequency varying (d)

Figure 3 Transient signals and their discrete holographics

(a) (b)

Figure 4 Repetitive signals and their discrete holographics



(a) baseline (b) pitting

(c) spalling (d) scuffing

Figure 5 Discrete holographics of defect gear vibration signals