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IMPACT DAMPER WITH GRANULAR MATERIALS FOR MULTIBODY SYSTEM

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An efficient impact damper consists of a bed of granular materials(shot) moving in a container mounted on a multibody vibrating system. This paper deals with the damping characteristics of a multidegree of freedom(MDOF)system that is provided with the shot impact damper. In theoretical analysis, the particle bed is assumed to be a mass which moves unidirectionally in a container, and collides plastically with its ends. Equations of motion are developed for an equivalent single-degree-of-freedom(SDOF)system and attached damper mass with use made of the normal mode approach. The modal mass is estimated such that it represents the equivalent mass on the point of maximum displacement in each of the vibrating modes. The mass ratio is modified with the modal vector to include the effect of impact interactions. Results of the analysis are applied to the special case of a three-degree-of freedom(3DOF) system, and the effects of the damper parameters including mode shapes and the number of dampers are determined. A digital model is also formulated to simulate the damped motion of the physical system. Numerical and experimental studies are made of the damping performance of plural dampers located at selected positions throughout a multibody system. In this report, the impact vibration model, including the motion forms comprising contact or separation of damper masses on vibrating bodies is developed, and the resulting formulation is analyzed by using the modal synthesis method. For the N units of damper case, the existence of as many as 2^N combinations of motion forms are identified through digital simulation. Results of analytical and simulation studies, applied to a three degree-of-freedom system, were compared to, and were found to be in good agreement with, experimental results. It is found that shot impact dampers with properly selected mass ratios and with container clearances effectively suppress the resonance peaks over a wide frequency range.

1. INTRODUCTION

The impact damper is an effective vibration absorber using a solid particle free to move in a container attached to a primary vibrating body(Grubin, 1956; Masri, 1969). Some authors have reported the application to the actual machine parts to reduce the resonance vibrations(Panossian, 1992; Skipor, 1980). The objective of the present study is to investigate the damping performance of the impact damper with granular material(shot) when applied to multibody system. The authors have presented analytical approach for predicting the damping performance of the shot impact damper applied to a single-degree-of-freedom(SDOF) system(Araki et al., 1985). Masri(1973) has reported the response of a multidegree system(4 to 10 story building) equipped with a solid impact damper, and presented the exact solution for the steady-state motion of the system. Roy et al.(1975) also studied the motion of a beam system under the action of an impact damper by seminumerical technique.But for the multibody system subjected to impact and/or sustained contact of shot, it is obvious that an exact analysis of such a system is very complicated. In this report, therefore, we propose an approximate analysis for the motion of the multibody system provided with shot dampers by developing equations of motion for its equivalent SDOF system by means of modal analysis. Numerical simulations studies were carried out for the motion of the system by developing solutions valid between impact and contact of the damper mass. Experiments with a structure of three-degree-of-freedom(3DOF) system using lead shot as a damper mass were conducted to corroborate the theoretical results. The effects of the number of damper units, container clearance, and mode shapes on the damping performance were considered.

2. EQUATIONS OF MOTION AND ANALYSIS

2.1 FORMULATION The schematic model of the multidegree-of freedom(MDOF) oscillating system equipped with the shot impact damper is shown in Fig.1. The damper consists of a particle bed of mass m_{dj} constrained to move vertically, with clearance d_j, in a container attached to jth vibrating body. In the analysis, the particle bed is assumed to be a mass which moves unidiredtionally without friction and collides plastically with bottom or top end of damper container. Thus the possible motions of the damper system can be divided into the two kinds of motion segments. In the sustained contact segment, the damper mass moves together with the primary mass after impact at container ends, while in the separate segment, the damper mass moves freely in the clearance.

The equation of motion for each segment can be written in the matrix form: these are, respectively, for separate segment(free flight phase)

$$[m]\{\ddot{x}\}+[c]\{\dot{x}\}+[k]\{x\}=\{f\}\sin\omega t \qquad \text{(primary body)}$$

$$m_{dj}\,\ddot{y}=-m_{dj}g\,,\ (j=1,2,\ldots,N), \qquad \text{(damper mass)} \tag{1}$$

and for sustained contact segment (riding phase)

$$[m_c]\{\ddot{x}_c\}+[c]\{\dot{x}_c\}+[k]\{x_c\}=\{f\}\sin\omega t-\{m_d\}g$$

$$(m_{cjj}=m_{jj}+m_{dj}), \qquad \text{(primary body and damper mass)} \tag{2}$$

where $[m]$, [c], [k] are the mass, damping, stiffness matrices, $\{x\}$ denotes displacement vector of the separate segment, $\{f\}$ the exciting force vector, and $\{x_c\}$ that of sustained contact segment, $\{f\}^T=(0,\ldots,0,f_k,0,\ldots,0)$, $\{m_d\}^T=(m_{d1},\ldots,m_{dj},\ldots,m_{dN})$, t = time, ω = exciting frequency, and g = acceleration of gravity. Applying the coordinate transformations

$$\{x\}=[\phi]\{q\},\ \{x_c\}=[\phi_c]\{x_c\}, \tag{3}$$

where $[\phi]$ is the modal matrix, $\{q\}$ the normal coordinate for the separate segment, and $[\phi_c]$ and $\{q_c\}$ are those for the sustained contact segment, equations (1) and (2) are transformed, under proportional damping, into the form

$$[M]\{\ddot{q}\}+[C]\{\dot{q}\}+[K]\{q\}=[\phi]^T\{f\}\sin\omega t \tag{4}$$

$$[M_c]\{\ddot{q}_c\}+[C_c]\{\dot{q}_c\}+[K_c]\{q_c\}=[\phi_c]^T\{f\}\sin\omega t-[\phi_c]^T\{m_d\}g\,, \tag{5}$$

where $[M]([M_c]),[K]([K_c])$, and $[C]([C_c])$ are diagonal matrices corresponding to the modal mass, stiffness, and damping matrices, respectively. The ith mode equation of the system (4) and (5) are represented as follows:

$$M_i\ddot{q}_i+C_i\dot{q}+K_iq_i=\phi_{ki}f_k\sin\omega t \tag{6}$$

$$M_{ci}\ddot{q}_{ci}+C_{ci}\dot{q}_{ci}+K_{ci}q_{ci}=\phi_{cki}f_k\sin\omega t-g\sum_{j=1}^{N}\phi_{cji}m_{dj} \qquad (i=1,2,\ldots,n) \tag{7}$$

and their solutions, together with the motion of the damper mass y_j, are given by

$$q_i=\exp(-\theta_i\tau)[A_i\sin\alpha_i\tau+B_i\cos\alpha_i\tau]+A_{pi}\sin(\tau-\beta_i-\varphi) \tag{8}$$

$$q_{ci}=\exp(-\theta_{ci}\tau)[A_{ci}\sin\alpha_{ci}\tau+B_{ci}\cos\alpha_{ci}\tau]+A_{pci}\sin(\tau-\beta_{ci}-\varphi)-\gamma_i \tag{9}$$

$$y_{1j}=-K_R(\tau-\tau_{0j})^2/2+D_{1j}(\tau-\tau_{0j})+E_{1j} \qquad \text{(upwards)} \tag{10}$$

$$y_{2j}=-K_R(\tau-\tau_{2j})^2/2+D_{2j}(\tau-\tau_{2j})+E_{2j} \qquad \text{(downwards)} \tag{11}$$

where φ is phase lag of exciting force with respect to impact at bottom, τ_{0j} time of separation from bottom, and τ_{2j} time of separation from top, and $\tau = \omega t$. The parameters of the ith mode are given by $\omega_i = \sqrt{K_i / M_i}$, $\varsigma_i = C_i / 2M_i\omega_i$, $\theta_i = \varsigma_i / r_i$, $\alpha_i = \sqrt{(1-\varsigma_i^2)} / r_i$, $r_i = \omega / \omega_i$, $K_R = g / \omega^2$, $\gamma_i = K_R r_i \mu_i$, $\delta_i = \phi_{ki} f_k / K_i$, $A_{pi} = \delta_i / \sqrt{(1-r_i^2)^2 + (2\varsigma_i r_i)^2}$, $\tan\beta_i = 2\varsigma_i r_i / (1-r_i^2)$. The constants A_i, B_i, A_{ci}, B_{ci}, $D_{1,2j}$, and $E_{1,2j}$ are determined by the boundary conditions for each segment of motion. The complete behavior of the system, as given by Eqs. (8) to (11) can be very complex, even for single-unit damper. The exact solutions for the periodic motion are difficult to obtain due to need of solving $2N$ to $4N$ nonlinear equations (N is the number of damper), corresponding to the type of periodic motion. Hence, a method based on approximating the system by an equivalent SDOF system has been developed to study the damping effect of the impact damper on multibody system.

2.2 APPROXIMATE ANSALYSIS. Let us assume that the vibrating multibody system has well-separated resonances whose resonant peaks are primary concern of vibration reduction. Over relatively wide frequency ranges near natural frequencies, it can be represented as a SDOF model with parameters M_i, C_i, and K_i (as shown in Eq.(6)), forced harmonically at resonant frequency ω_i, as shown in Fig.2. In order to suppress ith mode vibration by applying the damper, it is necessary to estimate the modal mass M_i so that it can represent the physically equivalent mass on the damper location. To this end, we normalize the modal vector $\{\phi_i\}$ associated with every one of the natural modes, with the value of the largest element of the vector equal to unity. The resulting modal parameters are given by

$$[M_i] = [\bar{\phi}]^T[m][\bar{\phi}], [K_i] = [\bar{\phi}]^T[k][\bar{\phi}], [C_i] = \alpha[M_i] + \beta[K_i], \qquad (12)$$

where $[\bar{\phi}]$ is a modal matrix with normalized modal vectors $\{\bar{\phi}_i\}$'s, and α and β are constants used for specifying proportional-type damping.

Furthermore, in view of the fact that for a SDOF system provided with the shot damper, the damping effect varies with the acceleration level of the vibrating body(Araki et al., 1985), factors are involved in modifying the impacting mass ratio pertaining to a specified mode. Since the acceleration within the vibrating multibody system is proportional to the modal amplitude $\{\phi_i\}$, the effect of the damper location on the response of ith mode can be approximately estimated by the total mass ratio

$$\mu_i = (\sum_{j=1}^{N} \phi_{ji} m_{dj}) / M_i . \qquad (13)$$

Thus, using the assumed mode approach retaining only one mode of the vibrating multibody system, the same analysis as the one developed for the SDOF system(Araki et al.,1985) can be applied to Eqs.(6) and (7) to study the damper problem of the multibody system.

Among the major impact vibrations that exist in the vertical SDOF damper system are following:

(1) Type I motion. The damper mass leaves bottom end at time τ_0 when the acceleration of the primary body reaches -g. The mass then hits top end at time τ_1, and keep contact with it until the acceleration reaches g at time τ_2 (Fig.3(a)).

(2) Type II motion. Motions without sustained contact on top end; the damper mass rebounds just after impact at top end (Fig.3(b)).

(3) Type III motion. Motions in which the damper mass hits only bottom end when the clearance is too large (Fig.3(c)).

It remains now to find the steady-state solutions to the motions of the MDOF system subjected to the impact damper, using the approximate SDOF design technique.

The method for calculating the response of the equivalent SDOF system to the shot impact damper is based on adjoining the successive motion segments over one cycle of the excitation by the condition of periodicity. To illustrate the procedure, consider the periodic motion of Type II shown in Fig.3(b), where the displacement and velocity of Modal mass M_i and damper mass m_{dj} are shown for one period of the excitation. Due to the nature of the periodic motion, the solutions in Eqs.(8)-(11) must satisfy the following conditions:

$$At\ \tau = 0: q_{ci}(0) = \rho_{c1}, q'_{ci}(0) = u_{c1}, u_{c1} = (\mu_i v_{b1} + u_{b1}) / (1+\mu_i), \qquad (14)$$

$$At\ \tau = \tau_0: q_{ci}(\tau_0) = q_{i1}(\tau_0) = y_{1j}(\tau_0) = \rho_s, q'_{ci}(\tau_0) = q'_{i1}(\tau_0) = y'_{1j}(\tau_0) = u_s, \qquad (15)$$

$$At\ \tau = \tau_1 : q_{i1}(\tau_1) = q_{i2}(\tau_1) = \rho_{c2}, y_{1j}(\tau_1) = y_{2j}(\tau_1) = \rho_{c2} + d_j, q'_{i1}(\tau_1) = u_{b2},$$

$$y'_{1j}(\tau) = v_{b2}, q'_{i2}(\tau_1) = y_{2j}(\tau_1) = u_{c2}, u_{c2} = (\mu_i v_{b2} + u_{b2})/(1+\mu_i), \tag{16}$$

$$At\ \tau = 2\pi : q_{i2}(2\pi) = y_{2j}(2\pi) = \rho_{c1}, q'_{i2}(2\pi) = u_{b1}, y'_{2j}(2\pi) = v_{b1}. \tag{17}$$

Substituting Eqs.(8)-(11) into the relations(14)-(17) leads to a set of three coupled algebraic equations in the form:

$$\alpha_{ci} A_{ci} + \theta_{ci} B_{ci} + R_1 A_{2i} + R_2 B_{2i} + K_R(2\pi - \tau_1)\mu_i/(1+\mu_i) = 0 \tag{18}$$

$$R_3 A_{1i} + R_4 B_{1i} + a_{45} A_{2i} + a_{46} B_{2i} - K_R(\tau_1 - 2\pi)^2/2 - d_j = 0 \tag{19}$$

$$R_5 A_{2i} + R_6 B_{2i} - B_{ci} - K_R(2\pi - \tau_1)^2/2 + d_j + K_R \mu_i = 0, \tag{20}$$

together with the linear algebraic equation:

$$\begin{bmatrix} 0 & a_{12} & 0 & 0 & a_{15} & a_{16} \\ a_{21} & a_{22} & a_{23} & a_{24} & 0 & 0 \\ a_{31} & a_{32} & a_{33} & a_{34} & 0 & 0 \\ 0 & 0 & a_{43} & a_{44} & a_{45} & a_{46} \\ a_{51} & a_{52} & 0 & 0 & 0 & 0 \\ 0 & 0 & a_{63} & a_{64} & a_{65} & a_{66} \end{bmatrix} \begin{pmatrix} A_{ci} \\ B_{ci} \\ A_{1i} \\ B_{1i} \\ A_{2i} \\ B_{2i} \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \\ b_5 \\ b_6 \end{pmatrix} \tag{21}$$

that determines the integral constants in Eqs.(8) and (9). The remaining terms are available in Araki et al. (1985).

Using the values of the specified modal parameters and employing numerical techniques for the solution of nonlinear algebraic Eqs.(18)-(20), the values of τ_0, τ_1 and φ can be determined. Then, the displacement of the modal mass M_i can be found by evaluating $q_{i1}(\tau), q_{i2}(\tau)$, and $q_{ci}(\tau)$, respectively, for each motion segment.

2.3 SIMULATION STUDIES In view of the preceding approximate SDOF technique which requires the closed-form solution, an alternative approach to the exclusive reliance on analytical methods to estimate the response is to utilize simulation techniques. In motion of the vibrating multibody system under action of the shot damper, there may occur impact vibrations other than those with the three types discussed previously. Simulations of the damped motion are performed to investigate the transient and steady-state response of the damper system by formulating the digital model.

Integrating the equations of motion (1) and (2) yields the following general solutions:

For separate segment $(\tau \geq \tau_{sj})$:

$$\{x_s\} = [G_1(\tau - \tau_{sj})]\{x_{s0}\} + [G_2(\tau - \tau_{sj})]\{\dot{x}_{s0}\} - [G_3(\tau - \tau_{sj})]\{h_1(\tau_{sj})\} - [G_4(\tau - \tau_{sj})]\{h_2(\tau_{sj})\} + \{h_1(\tau)\}$$

$$y_j = -K_R(\tau - \tau_{sj})^2/2 + \dot{x}_{sj}(\tau - \tau_{sj}) + x_{sj}, \tag{22}$$

For sustained contact segment $(\tau \geq \tau_{cj})$:

$$\{x_c\} = [G_{1c}(\tau - \tau_{cj})]\{x_{c0}\} + [G_{2c}(\tau - \tau_{cj})]\{\dot{x}_{c0}\} - [G_{3c}(\tau - \tau_{cj})]\{h_{1c}(\tau_{cj})\} - [G_{4c}(\tau - \tau_{cj})]\{h_{2c}(\tau_{cj})\} + \{h_{1c}(\tau)\}, \tag{23}$$

where $(\{x_{s0}\}, \{\dot{x}_{s0}\})$ and $(\{x_{c0}\}, \{\dot{x}_{c0}\})$ are the initial conditions for the separate and contact segment, respectively, and τ_{sj} is time when jth damper leaves the container end, τ_{cj} time at impact, and where

$$[G_1(\tau)] = [\phi][U(\tau)][Q], [G_2(\tau)] = [\phi][V(\tau)][Q], [G_3(\tau)] = [\phi][U(\tau)],$$

$$[G_4] = [\phi][V(\tau)], [Q] = [M]^{-1}[\phi]^T[m]. \tag{24}$$

$[U], [V], \{h_1\}$, and $\{h_2\}$ are diagonal matrices and vectors with elements

$$u_{ii}(\tau) = \exp(-\theta_i \tau)[(\theta_i/\alpha_i)\sin\alpha_i\tau + \cos\alpha_i\tau)], v_{ii}(\tau) = \exp(-\theta_i\tau)(1/\alpha_i)\sin\alpha_i\tau$$

$$h_{1i} = A_{pi}\sin(\tau - \beta_i), h_{2i} = A_{pi}\cos(\tau - \beta_i), (i = 1,2,\ldots\ldots n) \tag{25}$$

and the undefined matrices and vectors with subscript c are those for the contact segment.

To proceed the computation of motion undergoing impacts and separations of the damper masses, it is necessary to evaluate the modal parameters, every time the damper masses change their respective phases of motion, that depend on the mass of the vibrating main body, varying spatially and temporally within the system. These modal parameters can be evaluated beforehand for 2^N possible combinations of motion phases just once and then stored for reuse. The calculation routine tests the phase of motion when any damper mass makes a transition from one phase to the next, and changes the current modal parameter to the one associated with a new combination of motion phase.

3. APPLICATION

The utility of the proposed equivalent SDOF design technique is demonstrated by considering an example structure with a 3DOF system shown in Fig.4(a) whose natural frequencies, mode shapes, and damping properties are obtained from experiments. Since in this case n=3, the natural frequencies are $\omega_1 = 81.9\ rad/s, \omega_2 = 243.9\ rad/s, and\ \omega_3 = 312.3\ rad/s$, and the corresponding three modal vectors are $\{\phi_1\}^T = (0.37, 0.77, 1), \{\phi_2\}^T = (1, 0.57, -0.91)$, and $\{\phi_3\}^T = (-0.96, 1, -0.47)$, respectively. Figure 4(b) shows the frequency response curves without damper for three masses m_1, m_2, and m_3.

If this system is provided with a single or three-unit shot impact dampers for properly selected clearance, the simulated frequency responses in its third mode are given by Fig.5. The displacement wave forms at the frequencies marked by a, b, and c in Fig.5 are shown in Fig.6. In Fig.6(a), a single damper is attached to mass m_2, i.e., a loop point, and in Fig.6(b) three dampers are applied simultaneously to mass m_1, m_2, and m_3, respectively, with the identical clearance. The results in Fig.5 show that what effects can be obtained by using plural dampers instead of the single unit damper, with all tuning and excitation parameters remaining the same. There are amplitude reductions around resonance by a factor 1/2.5 to 1/4 of the corresponding response of the single damper. The simulation wave forms indicate that three types of damper motions exist, and Type I motion shown in Fig.6(a) enhances the motion of main mass by sustained contact of damper mass at both ends; while, for plural dampers, Type II and III motions effectively attenuate the resonant amplitude, as shown in Fig.6(b) and 6(c).

4. EXPERIMENTS AND DISCUSSION OF RESULTS

Experiments with shot impact dampers were conducted on a 3DOF model of the resonant structure to correlate the theoretical and experimental results. Figure 7 shows a construction of the apparatus and measurement system. Each main body incorporates a container tube of 40 mm diameter by 120 mm long with two flange ends, and it is supported by leaf springs. Using a screw mounted on the top flange, the clearance can be adjusted. The dimensions of the structures are : $m_1=0.74\ kg$, $m_2=0.71\ kg$, and $m_3=0.64 kg$; $k_1=24.3\ kN/m$, $k_2=21.8\ kN/m$, and $k_3=19.8\ kN/m$. A bed of lead shot of 2 mm diameter is put in the container as damper mass. The sinusoidal force is put to m_1 by electrodynamic sweep shaker through a coil spring ($k_0=6.6\ kN/m$). Four piezo-electronic accelerometer with amplifier incorporating integrating circuits are used to measure displacements of main bodies and the shaker table. Transfer functions in terms of compliance are calculated between response and excitation points by a personal computer.

The effect of the number of damper on the damping performance up to the third resonance is shown in Fig.8 . The dotted curve **A** represents the measured response without damper, and solid curves **B**, **C**, and **D** are the ones for a single, two, and three-unit dampers, respectively, with d $=\infty$, and identical mass ratio $\mu_j (=m_{dj}/m_j)$=10 %. It can be observed from the figure that a single damper m_{d1} exerts damping action to the second and third resonant vibrations by virtue of amplification effects associated with corresponding modal amplitudes, while two and three-unit dampers can exert cumulative damping effects to each of the vibration modes. The effect of increasing the number of dampers is prominent for higher resonances except for fundamental one. This is because the damping performance of shot impact damper deteriorates with lower acceleration level of main vibrating body.

The equivalent SDOF design technique was applied to the 3DOF system with the impact damper for each of its vibration modes. The results of these analyses are compared with those obtained by experiments and simulations. Figure 9 shows the frequency response of the system with a single damper located at j=2 for the third mode, when the clearance is optimized such that only Type II motion appears within resonance. The left-hand side ordinate in Fig.9 is the compliance of the primary mass with and without damper, while the right-hand side ordinate represents the impact phase φ, time of separation from bottom τ_0, and time of impact at top τ_1. In the graph, the equivalent SDOF solution is shown as solid curves. For the sake of comparison, the graph also contains the two kinds of modal curves of the 3DOF system, one for the absence of the impact damper, the other for the main mass equal to $m_j + m_{dj}$ (i.e., with damper mass stuck on the primary mass). These are shown, respectively, as dotted and interrupted lines. It is seen that there are generally close agreements between experimental results and those of predictions by the equivalent SDOF technique as well as simulations. Figure 10(a) demonstrates the comparison between simulation and experimental results of the response of the system with mass ratios $\mu_j = 10\%$, and $d_j = \infty (j = 1,2,3)$, up to the third resonance, and Fig.10(b) shows its impulse response of acceleration by experiment. There is generally close agreement between them in the region of resonance except for antiresonant regions. The differences in these regions is attributed to the deteriorated noise level of the accelerometer used in experiments. In Fig.10(b), the three units shot damper attenuates the free oscillation faster and more effectively.

5. SUMMARY AND CONCLUSIONS

An approximate analysis has been presented for determining the damping characteristics of a multibody system that is provided with the shot impact damper attached to some arbitrary point of the system. The equations of motion are developed for the equivalent SDOF system and attached damper masses by means of the normal mode approach, and steady-state solutions to the motion of the MDOF system subjected to the impact damper was derived using SDOF design technique. Results of the analysis were applied to a 3DOF resonant structure, and the effect of the number of dampers on the damping performance were investigated. It is shown that multi-unit dampers with properly selected mass ratios and clearances effectively suppress the resonant peaks over a wide frequency range. Experimental studies with a resonant model and digital simulations were performed to verify the validity of the analysis. Good correlation was obtained between the theory, experiments and simulations.

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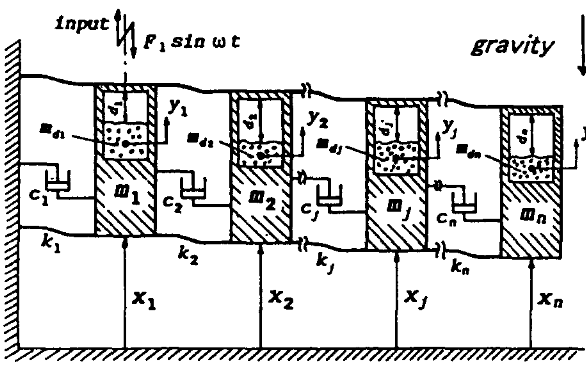


Fig.1 MDOF system with shot damper

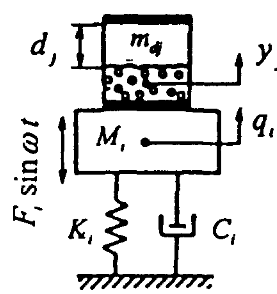
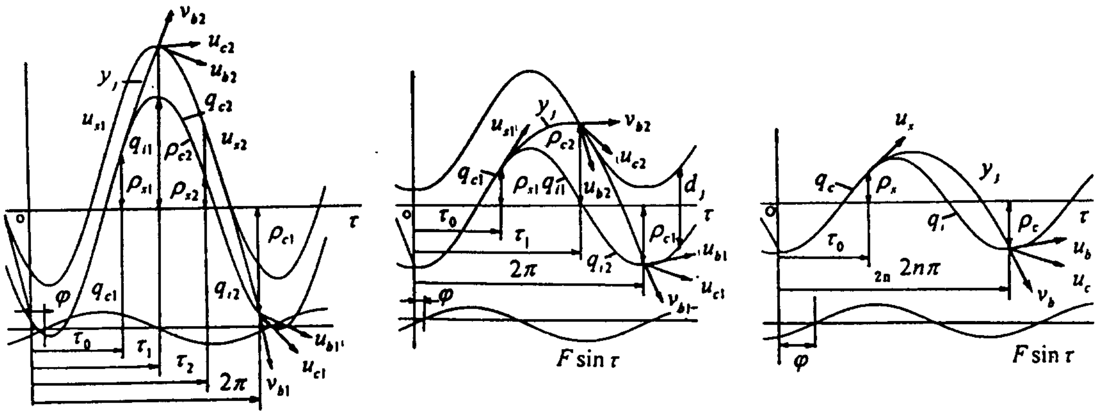
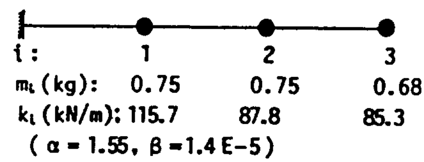


Fig. 2 Equivalent SDOF model with damper

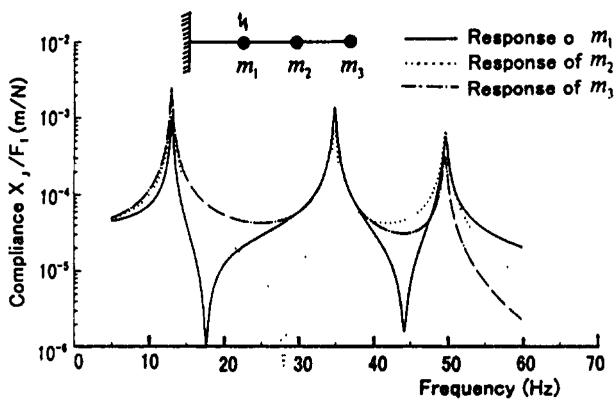


(a) Type Ⅰ periodic motion (b) Type Ⅱ periodic motion (c) Type Ⅲ periodic motion

Fig.3 Types of impact vibration of SDOF system

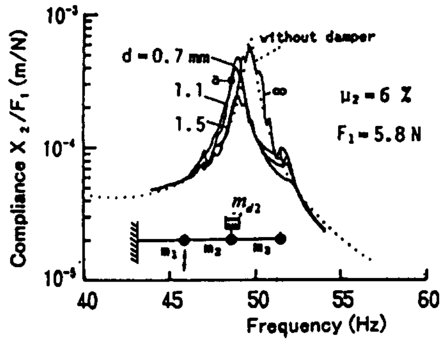


(a) 3DOF system

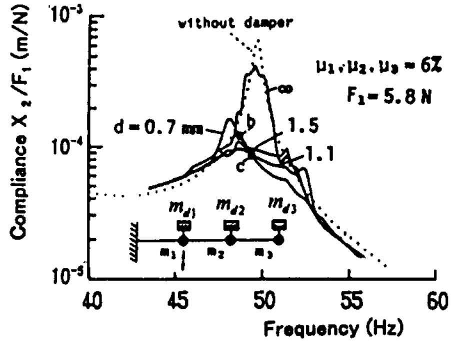


(b) Frequency response of Fig.3(a)

Fig.4 Example structure

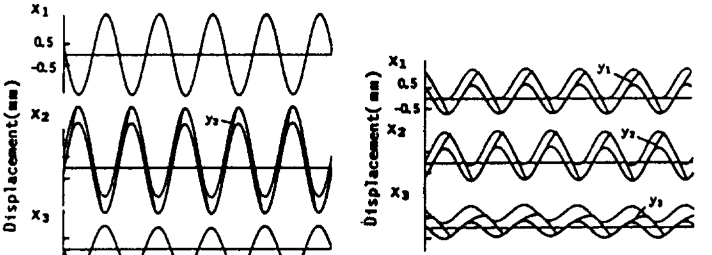


(a) Single-unit damper at m_2



(b) Three-unit dampers at m_1, m_2, m_3

Fig.5 Influence of the number of damper and clearance (3 rd mode)



(a) Wave form at point "a"
(48.6 Hz, d_2=0.7mm)

(b) Wave form at point "b"
(48.6 Hz, d_j=0.7mm,j=1,2,3)

(c) Wave form at point "c"
(49.6 Hz, d_j=1.1mm, j=1,2,3)

Fig.6 Simulation wave forms

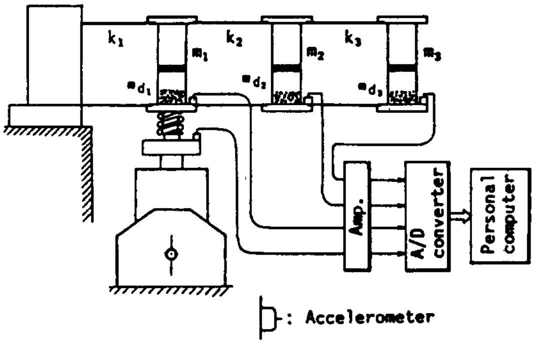


Fig.7 Construction of experimental apparatus

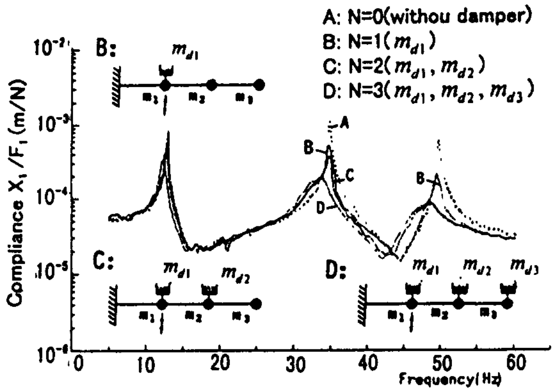


Fig.8 Variation of frequency response with number of damper unit N
(μ_j=0.1, d_j=∞; j=1,2,3: F_1=2.6 N)

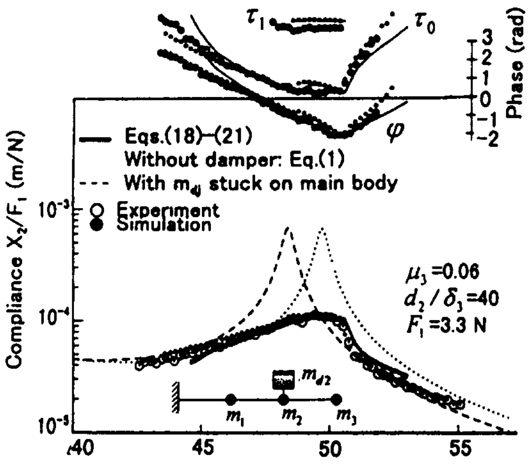
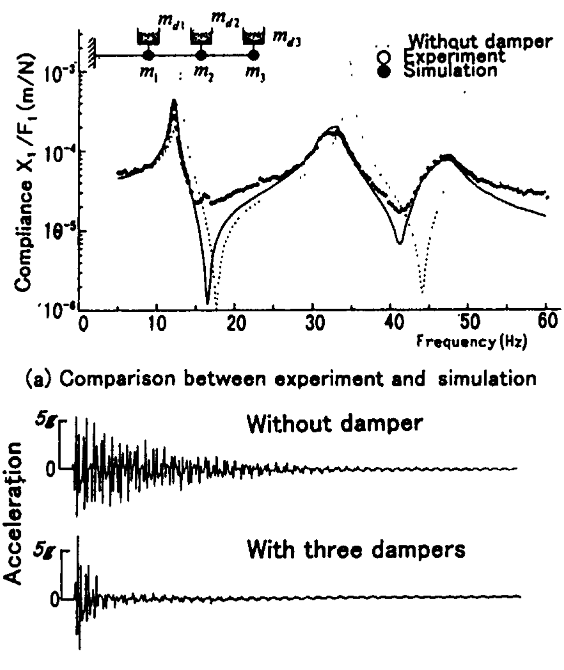


Fig.9 Comparison between theory, experiment and simulation of 3 rd mode response



(a) Comparison between experiment and simulation

(b) Impulse response of acceleration (experiment)

Fig.10 Effectiveness of plural dampers (N=3)
(μ_j=0.1, d_j=∞; j=1,2,3: F_1=2.6 N)