

## FIFTH INTERNATIONAL CONGRESS ON SOUND AND VIBRATION

DECEMBER 15-18, 1997  
ADELAIDE, SOUTH AUSTRALIA

### Dependence of Active Damping to Feedback Gain and Stiffness in Vibration Control of Smart Structures

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**ABSTRACT** - Interaction between the structure and the control is investigated numerically and verified experimentally for the active vibration control of carbon/epoxy laminated composite beams with collocated piezoceramic sensor and actuator. Finite element method is used for the analysis of dynamic characteristics of the laminated composite beams with and without the piezoceramic sensor/actuator. Damping and stiffness of the adhesive layer and the piezoceramics are taken into account in the finite element modeling. The impact of varying stacking sequence of  $[\theta_4/0_2/90_2]_s$ , where  $\theta = 0, 15, 30, 45, 60, 75$  and  $90$  degree of the laminated composite beam on the stiffness and the damping properties is studied. Experiments on the active vibration control of the laminated composite beams have been carried out by making use of velocity feedback with constant gain. Active and passive damping ratios( $\zeta$ ) and modal dampings( $2\zeta\omega$ ) of the first bending mode of the beams are measured experimentally. They are in good agreement with those predicted by the finite element analysis. When the feedback gain is small, the active control follows trends of the passive control but adds additional effects due to the active control. But for large feedback gains, the active control is dominant over the passive control. Active control is more effective to the structure with higher stiffness than to the structure with lower stiffness, when the feedback gain is large.

## INTRODUCTION

Methodology for vibration suppression of the structure can be categorized into two groups, namely the passive and the active control. In the passive control, material properties of the structure itself such as damping and stiffness are modified to change the structural response. In the active control, the structural response is controlled by adding external effort to the structure. Active control is complex and expensive, but it can achieve good performance in comparison with passive control.

Several studies regarding the active control of structural vibration are available<sup>1-6</sup>. A common feature of all of these studies is ignorance of the inherent damping of the structure when using piezoelectric sensors/actuators in the formulation. When a strain-rate sensor and a piezoelectric actuator is used, it increases damping of the entire system. Therefore, the damping must be taken into account in the finite element formulation and the validity of the formulation should be verified in the viewpoint of damping. Most research using the piezoceramic sensor/actuators has taken into account stiffness of the piezoceramics, but did not consider damping and stiffness of the adhesive layer and the piezoceramics at the same time.

In this paper, passive control by the tailoring and active control by the piezoceramic sensor/actuator are investigated numerically and verified experimentally. Characteristics of the optimal control in structural vibration of the smart structures are analyzed to study the dependence of the active damping on the feedback gain and the stiffness of the structure. It is aimed to address an analytical approach to evaluate the passive and the active vibration control of the laminated composite beams with piezoceramics. Damping and stiffness of the adhesive layer and the piezoceramics are taken into account simultaneously in the process of finite element formulation. Experiments of the active vibration control of the beams have been carried out by making use of the velocity feedback. Interaction between the passive and the active control has been studied analytically and experimentally.

## METHOD

Laminated composite beams with the piezoceramic sensor/actuator are modeled as 2-D plates. Hamilton's principle is used to derive the equation of motion for the plates. Piezoceramic sensor/actuator layers and adhesive layers are treated as other layers with different material properties in deriving the kinetic and the potential energies.

When the beam is not very thin and not very long, the vibrational response is transmitted dominantly in flexural type motion and the vibrational flow in wave type motion can be ignored. Thus the in-plane displacements can be ignored and transverse vibration is considered only.

The equation of motion for the system in terms of the nodal displacements is as follows.

$$(\mathbf{M}_S + \mathbf{M}_B + \mathbf{M}_P)\ddot{\mathbf{q}} + (\mathbf{K}_S + \mathbf{K}_B + \mathbf{K}_P)\mathbf{q} = \mathbf{F}_{Ext} \quad (1)$$

where the subscripts S, B, and P represent the main structure, the adhesive layer, and the

piezoceramic materials, respectively.

Damping properties of the composite materials exhibit anisotropic characteristics, low in the fiber direction and high in the transverse. It can be controlled by changing the fiber orientations and the stacking sequences. Damping of the laminated composite beams is analyzed using concept of the specific damping capacity (SDC) suggested by Lin, Ni and Adams<sup>7</sup>. Damping of the adhesive layer and the piezoceramic sensor/actuator are considered in the system modeling<sup>8</sup>. The SDC,  $\varphi$ , can be rewritten as follows :

$$\varphi = \frac{\mathbf{q}^T \mathbf{K}_D \mathbf{q}}{\mathbf{q}^T (\mathbf{K}_S + \mathbf{K}_B + \mathbf{K}_P) \mathbf{q}} \quad (2)$$

where

$$\mathbf{K}_D = \sum_{elem} \int_A \Psi_\kappa^T \mathbf{D}^\Delta \Psi_\kappa dA \quad (3)$$

and  $\mathbf{K}_D$  is the damped stiffness matrix and  $\mathbf{D}^\Delta$  the damped flexural stiffness matrix. The SDC can be derived for each vibration mode, which is called the modal SDC.

The equation of motion of a structure with multiple degrees of freedom in active control is expressed in discretized form as follows :

$$\mathbf{M}\ddot{\mathbf{q}} + \mathbf{C}\dot{\mathbf{q}} + \mathbf{K}\mathbf{q} = \mathbf{F}_{Ext} + \mathbf{F}_c \quad (4)$$

where  $\mathbf{q}(t)$  is the  $n \times 1$  displacement vector,  $\mathbf{M}$  ( $=\mathbf{M}_S + \mathbf{M}_B + \mathbf{M}_P$ ) the mass matrix,  $\mathbf{C}$  the structural damping,  $\mathbf{K}$  ( $=\mathbf{K}_S + \mathbf{K}_B + \mathbf{K}_P$ ) the stiffness matrix,  $\mathbf{F}_{Ext}$  the external force vector, and  $\mathbf{F}_c$  the control force.

By introducing the modal coordinates, the transformed equation of motion yields

$$\bar{\mathbf{M}}\ddot{\eta} + \bar{\mathbf{C}}\dot{\eta} + \bar{\mathbf{K}}\eta = \bar{\mathbf{F}}_{Ext} + \bar{\mathbf{D}}u_c \quad (5)$$

where  $\Phi$  is the open-loop modal matrix, and the transformed modal space mass, damping, stiffness, external forces and control influence matrices are, respectively, given by ;

$$\bar{\mathbf{M}} = \Phi^T \mathbf{M} \Phi = \mathbf{I} \quad (6a)$$

$$\bar{\mathbf{C}} = \Phi^T \mathbf{C} \Phi = \text{diag}(2\zeta_1 \omega_1, 2\zeta_2 \omega_2, \dots, 2\zeta_n \omega_n) \quad (6b)$$

$$\bar{\mathbf{K}} = \Phi^T \mathbf{K} \Phi = \text{diag}(\omega_1^2, \omega_2^2, \dots, \omega_n^2) \quad (6c)$$

$$\bar{\mathbf{F}}_{Ext} = \Phi^T \mathbf{F}_{Ext} \quad (6d)$$

$$\bar{\mathbf{D}} = \Phi^T \mathbf{D}_a \quad (6e)$$

The first-order state space form of the system equations equivalent to Eq. (5) is

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}u_c + \mathbf{B}_0 \quad (7)$$

where

$$\mathbf{A} = \begin{bmatrix} 0 & \mathbf{I} \\ -\bar{\mathbf{K}} & -\bar{\mathbf{C}} \end{bmatrix}, \mathbf{B} = \begin{bmatrix} 0 \\ \bar{\mathbf{D}} \end{bmatrix}, \mathbf{B}_0 = \begin{bmatrix} 0 \\ \bar{\mathbf{F}}_{Ext} \end{bmatrix}, \mathbf{x} = \begin{bmatrix} \eta \\ \dot{\eta} \end{bmatrix}. \quad (8)$$

In Eq.(7), the state vector  $\mathbf{x}(t) \in \mathbb{R}^{2n}$ , control vector  $u_c(t) \in \mathbb{R}^m$ , and the system matrix  $\mathbf{A}$  and the control influence matrix  $\mathbf{B}$  may contain time-varying elements.

The state space equation of Eq.(7) can be expressed as a complex eigenvalue problem

for the case of structural system without external forces

$$[\lambda I - (A - BG)]\{\phi\} = 0 \quad (9)$$

where G is the feedback gain in the active control.

The complex eigenvalue of Eq.(9) is expressed as follows:

$$\lambda = \mu + i\omega_d \quad (10)$$

Damping ratio and modal damping are defined as follows:

$$\zeta = \frac{-\mu}{\sqrt{\mu^2 + \omega_d^2}} \quad (11)$$

$$\text{Modal Damping}(2\zeta\omega) = -2\mu \quad (12)$$

## RESULTS AND DISCUSSIONS

Vibration control of the carbon/epoxy laminated composite beams with the piezoceramic sensor and actuator is numerically simulated using the finite element method and verified experimentally. The beam has stacking sequence of  $[\theta_1/0_2/90_2]_s$  where  $\theta = 0, 15, 30, 45, 60, 75$  and  $90$  degree. The beam is made of the carbon prepreg(CU125NS). Direction of the beam length is chosen as  $0$  degree of the fiber orientation. Thickness of the carbon prepreg is  $0.125\text{mm}$  and size of the specimen is  $230 \times 20 \times 2\text{mm}$ . Table 1 represents the mechanical properties of the carbon/epoxy laminates. Damping properties  $\phi_{S1}$ ,  $\phi_{S2}$  and  $\phi_{S12}$  are measured by the impulse technique. Finite element model of the beam consists of 46 elements with 72 nodes.

Fig. 1 shows experimental set-up for vibration control of the laminated composite beam with the piezoceramic sensor/actuator. Dimension of the piezoceramics in Fig. 1 is  $50 \times 20 \times 0.5\text{mm}$ . Adhesive material used to attach the piezoceramic sensor/actuator to the beam is cyanoacrylate adhesive<sup>9</sup>. Measured thickness of the adhesive layer is  $0.05 \pm 0.01\text{mm}$ . Material properties for the piezoceramics and the adhesive layer are shown in reference 8.

Stiffness and damping of the structure change as fiber orientation changes as shown in Fig. 2 and Fig. 3, respectively. Passive control by the tailoring shows that as the fiber orientation changes, natural frequency and damping ratio change. Stiffness of the beams decreases as the angle of the fiber orientation increases for both beams without and with the piezoceramics. From Fig. 2, one can realize that the stiffness of the adhesive layer does not affect the total structure very much. Measured frequencies agree very well with the predicted values for the beams without and with the piezoceramics. In Fig. 3, predicted damping ratios, in which we have taken into account both damping and stiffness of the adhesive layer and the piezoceramics, are in good agreement with the measured values. However, the prediction\*, in which damping and stiffness of the adhesive layer and damping of the piezoceramics are ignored, shows large discrepancy between the prediction and the measurement.

Piezoceramics are used to provide active damping. In order to achieve the desired

performance of the controlled structure, it is necessary to analyze the structural system and to consider the effects of changes in mass, damping and stiffness. Fig. 4 shows increase of modal damping with negative velocity feedback gain for the beams with the piezoceramics. Modal damping changes as the outer layer fiber orientation changes for a constant gain. When the feedback gain is small, they follow trends of the passive control but extra effect is added to the structure. As the magnitude of feedback gain increases, modal damping of the beam with higher stiffness increases more than the flexible beam does. Other words, the beam with higher stiffness is more effective than the beam with lower stiffness in the active control. Fig. 5 shows the time history of the  $[30_4/0_2/90_2]_S$  specimen with the piezoceramics. The FRF(frequency response function) before and after the active control is represented in Fig. 6.

## CONCLUSIONS

Active and passive control method is applied to investigate the control characteristics of the laminated composite beams in the vibration control of the first bending mode. The finite element method is used to analyze passive and active vibration control of the composite structures. The following conclusions can be drawn.

A systematic and numerical method that takes into account both damping and stiffness of the piezoceramics and the adhesive layer is proposed. Interaction between the structure and the control is analyzed for the laminated composite beams with the piezoceramic sensor and actuator. It shows good agreement between the analyses and the experimental results in the natural frequencies, the damping ratios and the modal dampings.

When the feedback gain is large, the active modal damping increases more in the beams with higher stiffness. While, for small feedback gain, the active modal damping is maximum when the passive modal damping is maximum, i.e. it follows trends of the passive control but adds additional effect due to the active control. Therefore, it can be said that for controlling the structure actively the beam with higher stiffness is more effective than the beam with lower stiffness.

## References

<sup>1</sup>Hanagud, S., Obal, M. W. and Calise, A. J., "Optimal Vibration Control by the Use of Piezoceramic Sensor and Actuators," *Journal of Guidance, Control, and Dynamics* Vol. 15, No. 5, 1992, pp. 1199 - 1206.

<sup>2</sup>Ha, S. K., Keilers, C., and Chang, F. K., "Finite Element Analysis of Composite Structures Containing Distributed Piezoceramic Sensor and Actuators," *AIAA Journal*, Vol. 30, No. 3, 1992, pp. 772-780.

<sup>3</sup>Chandrashekhara, K., and Agarwal, A. N., "Active Vibration Control of Laminated Composite Plates Using Piezoelectric Devices : A Finite Element Approach," *Journal of Intelligent Material Systems and Structures*, Vol. 4, 1993, pp. 496-508.

<sup>4</sup>Chen, C. I., Napolitano, M. R., and Smith, J. E., "Active Vibration Control Using the

Modified Independent Modal Space Control(M.I.M.S.C.) Algorithm and Neural Networks as State Estimators,” Journal of Intelligent Maetrial Systems and Structures, Vol. 5, 1994, pp. 550-558.

<sup>5</sup>Samanta, B., Ray, M. C., and Bhattacharyya, R., “Finite Element Model for Active Control of Intelligent Structures,” AIAA Journal, Vol. 34, No. 9, Sep., 1996, pp. 1885-1893.

<sup>6</sup>Callahan, J., and Baruh, H., “Active Control of Flexible Structures by Use of Segmented Piezoelectric Elements,” Journal of Guidance, Control, and Dynamics, Vol. 19, No. 4, 1996, pp. 808-815.

<sup>7</sup>Lin, D. X., Ni, R. G. and Adams, R. D., "Prediction and Measurement of the Vibrational Damping Parameters of Carbon and Glass Fibre-Reinforced Plastic Plates," Journal of Composite Materials Vol. 18, 1984, pp. 132 - 152.

<sup>8</sup>Kang, Y. K., Park, H. C., Hwang, W., and Han, K. S., “Prediction and Measurement of Modal Damping of Laminated Composite Beams with Piezoceramic Sensor/Actuator,” Journal of Intelligent Material Systems and Structures, Vol. 7, No. 1, 1996, pp. 25 - 32.

<sup>9</sup>Lee, J. K., and Marcus, M. A., “The Deflection-Bandwidth of Poly(Vinylidense Fluoride) Benders and Related Structures, Ferroelectrics, Vol. 32, 1981, pp. 93-101.

Table 1 Mechanical Properties of Carbon/Epoxy laminates(CU125NS)

Property	Symbol	Value
Young's modulus in fiber direction	$E_1$	$114.7 \times 10^9$ Pa
Young's modulus in transverse direction	$E_2$	$7.589 \times 10^9$ Pa
Shear Modulus	$G_{12}$	$4.77 \times 10^9$ Pa
Poisson Ratio	$\nu_{12}$	0.28
Volume Density	$\rho$	$1510 \text{ kg/m}^3$
Damping capacity in fiber direction	$\varphi_{s1}$	0.013966
Damping capacity in transverse direction	$\varphi_{s2}$	0.049120
Damping capacity in shear direction	$\varphi_{s12}$	0.074344

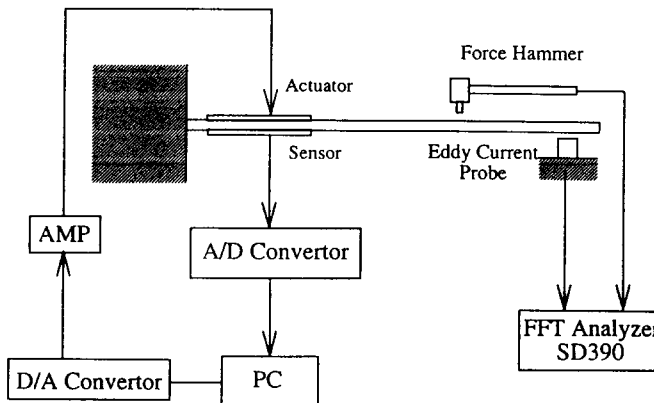


Fig. 1 Experimental Setup for Vibration Control of Laminated Composite Beam

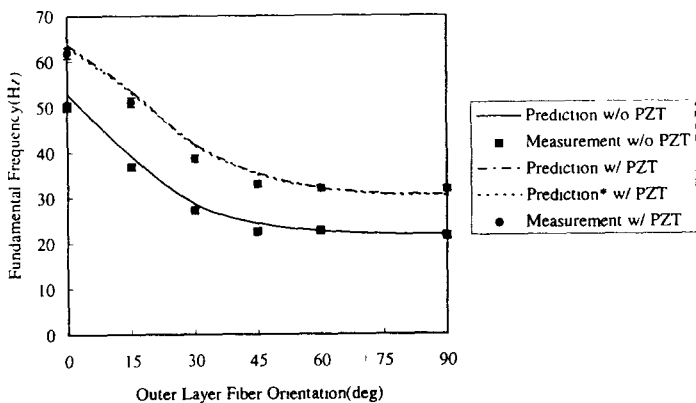


Fig. 2 Fundamental Frequency vs. Outer Layer Fiber Orientation

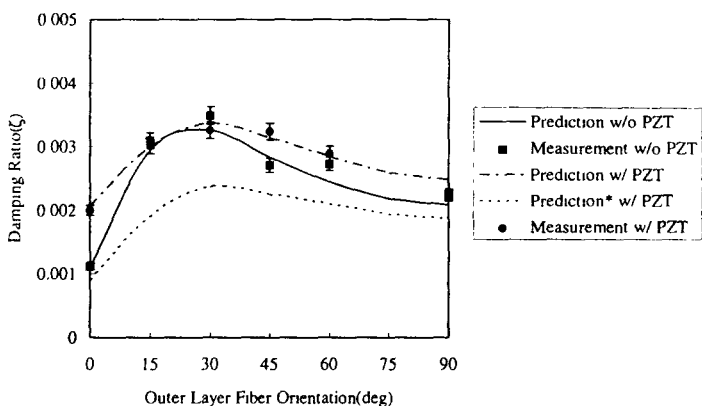


Fig 3 Passive Damping Ratio vs. Outer Layer Fiber Orientation

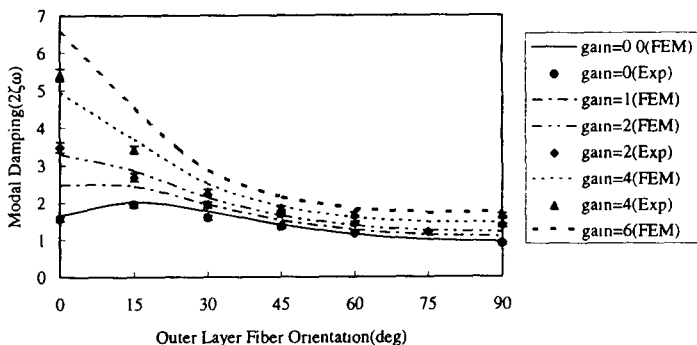
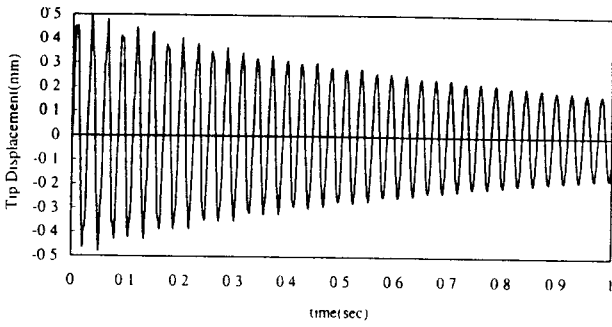
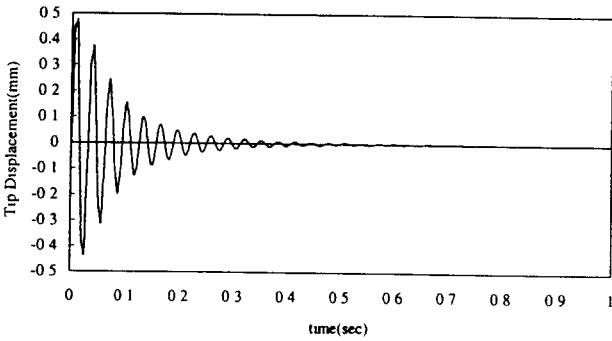


Fig 4 Active Modal Damping vs. Outer Layer Fiber Orientation



(a)



(b)

Fig. 5 (a) Free Vibration and (b) Controlled Vibration of the  $[30_4/0_2/90_2]_s$  Beam

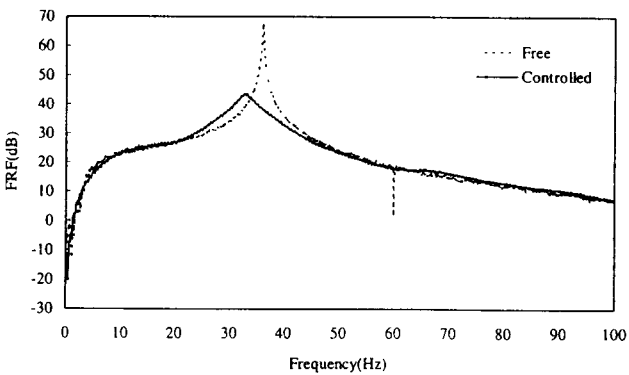


Fig 6 Frequency Response Function of the  $[30_4/0_2/90_2]_s$  Beam