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Propagation of low-frequency sound in a hydroacoustic waveguide with the thin ice-floe of finite width laying on a surface

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Abstract

The paper is devoted to the problem of sound propagation in stratified waveguide which is a non–homogeneous liquid layer overlaying multi–layered liquid–elastic bottom with the thin ise–floe of finite width laying on a surface. Influence of the ice–floe to characteristics of an acoustic field is investigated. For the problem solution the ice layer is replaced by equivalent in some sense condition on a surface of ”defreesed ice”. Problem with this condition can be easily reduced to the equation of convolution on a finite interval or, in Fourier images, to so-called modified Wiener–Hopf equation. To solve this equation we use a method of matrix Riemann boundary problem. It allows to construct asymptotic formulas for elements of reflection and transmission matrices by a small parameter ε, characterizing thickness of ice. These formulas are uniform on ice–floe width a. Finally some numerical calculations are given to obtain quantitative estimates for so–called ”critical” ice–floe lengths and to illustrate influence of absorption in ice on attenuation of acoustic field.

Consider at first a problem of waveguide covered by a solid homogeneous ice layer with thickness d. Ice layer is simulated by homogeneous elastic layer ([1]). Suppose that the water layer has constant density ϱ_0 and ice layer has constant density ϱ_{ice}. Choose an origin of z coordinate at a level of "defreezed" ice, so that low edge of ice will be at level $z = h$, where $h = \frac{\varrho_{\text{ice}}}{\varrho_0} d$, and upper one will be at level $z = h - d$.

Let a complex amplitude of sound pressure $p(x, z)$ satisfies in a water layer $z \in (h, H)$ the Helmholtz equation with wave number k_0 and index of refraction $n(z)$. Using the Fourier transformation (in dimensionless form)

$$\Psi(\mu, z) = \int_{-\infty}^{\infty} p(x, z) \exp(ik_0 \mu x) dx, \quad \mu \in \mathbf{R}, \tag{1}$$

and taking into account the interface conditions we can replace the elastic layer by an equivalent ([8]) impedance condition on the lower surface of ice $z = h$:

$$\Psi(\mu, h) + \frac{f(\mu)}{k_0} \cdot \frac{\partial \Psi}{\partial z}(\mu, h) = 0, \tag{2}$$

where $f(\mu)$ is the impedance of ice layer.

"Defreezing" ice and replacing it at $z \in (0, h)$ by a homogeneous water layer with density ϱ_0 and index of refraction $n \equiv 1$, we shall transform the condition (2) to the equivalent condition at a level of thawing ice $z = 0$:

$$\Psi(\mu, 0) + \frac{f_0(\mu)}{k_0} \cdot \frac{\partial \Psi}{\partial z}(\mu, 0) = 0, \tag{3}$$

where $f_0(\mu) = -\frac{1}{\gamma(\mu)} \cdot \frac{\operatorname{tg}(k_0 \gamma(\mu) h) + f(\mu)\gamma(\mu)}{f(\mu)\gamma(\mu)\operatorname{tg}(k_0\gamma(\mu)h) - 1}, \gamma(\mu) = \sqrt{1 - \mu^2}$.

Acting on (3) by the inverse Fourier transformation and taking into account that product transforms into a convolution, we obtain:

$$p(x, 0) + \frac{1}{2\pi k_0} \int_{\mathbf{R}} F_0(x - s) \frac{\partial p}{\partial z}(s, 0) ds = 0, \quad x \in \mathbf{R}, \tag{4}$$

where $F_0(x)$ is the inverce Fourier transformation of function $f_0(\mu)$.

Let $\varepsilon = k_0 h$ be a parameter characterizing thickness of ice. Simple asymptotic analysis gives the formulas

$$f_0(\mu) = \varepsilon^3 \left(\frac{1 - \mu^2}{3} + M(\mu) \right) + O(\varepsilon^5), \text{where} \tag{5}$$

$$M(\mu) = -\frac{4\mu^4 \left(1 - \frac{n_1^2}{n_2^2}\right)(\mu^2 - n_2^2) + n_1^2 \mu^2 (n_1^2 - 2n_2^2) + n_2^4 n_1^2}{3 \left(\frac{\varrho_{\text{ice}}}{\varrho_0}\right)^2 [n_2^4 + 4\mu^2 (n_1^2 - n_2^2)]}. \tag{6}$$

Consider then a problem with ice–floe. Change solid homogeneous ice layer considered above by ice–floe of finite width a. The same computations in this case give the following conditions on the surface $z = 0$ that are valid only in asymptotic sense (at $\varepsilon \to 0$):

$$p(x, 0) = 0, \quad x \in \mathbf{R} \setminus (0, a), \tag{7}$$

$$p(x,0)+\frac{1}{2\pi k_0}\int_0^a F_0(x-s)\frac{\partial p}{\partial z}(s,0)ds=0,\quad x\in\mathbf{R},\tag{8}$$

So, the complex amplitude of sound pressure $p(x,z)$ in the problem with ice-floe satisfies the conditions (7)–(8) at a level $z=0$, the Helmholtz equation in $z\in(0,H)$ with wave number k_0 and index of refraction $n(z)$ ($n(z)\equiv 1$, in $z\in(0,h)$) and the standard impedance condition at a level $z=H$. In Fourier images this condition has a form

$$\frac{\partial\Psi}{\partial z}(\mu,H)+k_0q(\mu)\Psi(\mu,H)=0.\tag{9}$$

Our goal is to obtail asymptotic formulas on parameter ε for coefficients of reflection and transmission matrices.

After the Fourier transformation (1) the Helmholtz equation assumes a form

$$\frac{\partial^2\Psi}{\partial z^2}(\mu,z)+k_0^2(n^2(z)-\mu^2)\Psi(\mu,z)=0.\tag{10}$$

Consider now the spectral problem **(A)** for equation (10) with boundary conditions (9) and $\Psi(\mu,0)=0$ and spectral problem **(B)** for equation (10) with boundary conditions (9) and $\Psi(\mu,0)+\frac{f_0(\mu)}{k_0}\cdot\frac{\partial\Psi}{\partial z}(\mu,0)=0$. Denote as $(k_0^2\mu_s^2,\varphi_s(z))$ and $(k_0^2\mu_{s,\varepsilon}^2,\varphi_{s,\varepsilon}(z))$ eigenvalues and eigenfunctions of the problems **(A)** and **(B)** respectively.

Let the j–th mode $\varphi_j(x,z)=\varphi_j(z)\exp(ik_0\mu_jx)$ over-runs on an edge $x=0$. Denote the perturbated field by $\varphi(x,z)$, so that $p(x,z)=\varphi(x,z)+\varphi_j(x,z)$. For selection the physically correct solution we shall require that $\varphi(x,z)$ satisfied to a principle of limiting absorption.

Acting on (7)–(8) the dimensionless Fourier transform and making some computations, similar to [2]-[4] we have the following modified Wiener-Hopf equation:

$$\exp(iL\mu)X^+(\mu)+X^-(\mu)+G(\mu)X^+_{L,j}(\mu)=\varphi_j'(0)g(\mu),\tag{11}$$

where $G(\mu)=\dfrac{\Psi(\mu,0)+\frac{f_0(\mu)}{k_0}\cdot\frac{\partial\Psi}{\partial z}(\mu,0)}{\Psi(\mu,0)}$, $g(\mu)=\dfrac{e^{iL(\mu+\mu_j)}-1}{ik_0(\mu+\mu_j)}$, $L=k_0a$, $\Psi(\mu,z)$ satisfies the conditions (10), (9) and $\int_0^H|\Psi(\mu,z)|^2\,dz=1$,

$$X^-(\mu)=\int_\infty^0\frac{\partial\varphi}{\partial z}(x,0)e^{ik_0\mu x}dx;\quad e^{iL\mu}X^+(\mu)=\int_a^{+\infty}\frac{\partial\varphi}{\partial z}(x,0)e^{ik_0\mu x}dx;$$

$$X^+_{L,j}(\mu)=\int_0^a\frac{\partial\varphi}{\partial z}(x,0)e^{ik_0\mu x}dx+\varphi_j'(0)g(\mu).$$

Thus, for the function $\varphi(x,z)$ we have:

$$\varphi(x,z)=-\frac{1}{2\pi}\int_{-\infty}^{\infty}\frac{f_0(\mu)X^+_{L,j}(\mu)}{\Psi(\mu,0)}\Psi(\mu,z)e^{-ik_0\mu x}d\mu.\tag{12}$$

By extracting normal modes from this integral representation, we have the following formulas for coefficients of transmission and reflection matrices:

$$A_{tr}=\left\{-\frac{if_0(\mu_l)X^+_{L,j}(-\mu_l)}{\Psi_\mu(\mu_l,0)}+\delta_{l,j}\right\}_{l,j=1}^n;\ A_{ref}=\left\{-\frac{if_0(\mu_l)X^+_{L,j}(\mu_l)}{\Psi_\mu(\mu_l,0)}\right\}_{l,j=1}^n,$$

where $\delta_{l,j}$ is the Kronecker symbol, n is a number of guided modes.

Taking into account zeroes and poles of symbol $G(\mu)$ in equation (11) and using the matrix Riemann problem method ([5]-[7]), we shall obtain asymptotic formulas for $X^+_{L,j}(\mu_l)$ and, as a consequence, asymptotic formulas for coefficients of matrices A_{tr} and A_{ref} at $\varepsilon \to 0$:

$$[A_{tr}]_{j,j} = e^{iL(\mu_{j,\varepsilon}-\mu_j)} + o(\varepsilon^3); \tag{13}$$

$$[A_{tr}]_{j,l} \approx -\varepsilon^3 \cdot \frac{\Psi_z(\mu_j,0)}{k_0\Psi_\mu(\mu_l,0)} \cdot \frac{M(\mu_l)e^{-iL\mu_l}}{\mu_l-\mu_j}(e^{iL\mu_{l,\varepsilon}} - e^{iL\mu_{j,\varepsilon}}); \tag{14}$$

$$[A_{ref}]_{j,l} \approx -\varepsilon^3 \cdot \frac{\Psi_z(\mu_j,0)}{k_0\Psi_\mu(\mu_l,0)} \cdot \frac{M(\mu_l)}{\mu_l+\mu_j}(e^{iL(\mu_{j,\varepsilon}+\mu_{l,\varepsilon})} - 1). \tag{15}$$

Note that the formulas (13) – (15) are uniform on parameter $L \gg 1$. Pay attention also to the fact that the term of ε^3 order is absent in (13). The exact analysis shows that in (13) $o(\varepsilon^3) = O(\varepsilon^5)$.

Remark. *If the system of several ice-floes with widths $a_1,\dots,a_m$ $(m > 1)$ is considered, the transmission and reflection matrices for the whole system are obviously the same as the product of transmission and reflection matrices of each ice-floe separately. Hence, the diagonal elements of a transmission matrix for system with m ice–floes looks like:*

$$[A^{(m)}_{tr}]_{j,j} \approx e^{i(k_0\sum_{s=1}^{m} a_s)(\mu_{j,\varepsilon}-\mu_j)}. \tag{16}$$

The given formula is uniform on m and $L = k_0 \sum\limits_{s=1}^{m} a_s$.

As follows from (14),(15), the elements of reflection matrix and also non–diagonal elements of transmission matrix have the order $O(\varepsilon^3)$. The diagonal elements of $[A_{tr}]_{j,j}$ have the order $O(1)$ and are of greatest interest:

$$\begin{cases} [A_{tr}]_{j,j} \approx 1 \quad \text{if } \left|L - \frac{2\pi s}{|\mu_j-\mu_{j,\varepsilon}|}\right| \varepsilon^3 \ll 1, & s \in \mathbf{N}; \\ [A_{tr}]_{j,j} \approx -1 \quad \text{if } \left|L - \frac{\pi(2s-1)}{|\mu_j-\mu_{j,\varepsilon}|}\right| \varepsilon^3 \ll 1, & s \in \mathbf{N}. \end{cases} \tag{17}$$

Thus, in the first case (particularly at $L\varepsilon^3 \ll 1$) the influence of the ice-floe to a transmitted field is negligible, in second one (particularly at $L \approx \pi/|\mu_j - \mu_{j,\varepsilon}|$) it is maximal.

To obtain quantitative estimates we spent the numerical research of dependence of the first "critical" size of the ice-floe $a^j_1 = \frac{\lambda}{2|\mu_j-\mu_{j,\varepsilon}|}$, when $[A_{tr}]_{j,j} \approx -1$, from frequency and thickness of ice (here λ is a wave length). Model of "liquid homogeneous layer on liquid homogeneous half-space" with parameters $H = 200\,m$, $c_{liq} = 1500\,mps$, $\varrho_0 = 1\ g/sm^3$, $\varrho_{bot} = 1.4\ g/cm^3$, $c_{bot} = 2000\ mps$, $c_1 = 3500\,mps$, $c_2 = 1800\,mps$, $\varrho_{ice} = 0.9\,g/cm^3$ was considered.

The results of calculations at thickness of ice $d = 10\ m$ are illustrated on fig. 1.

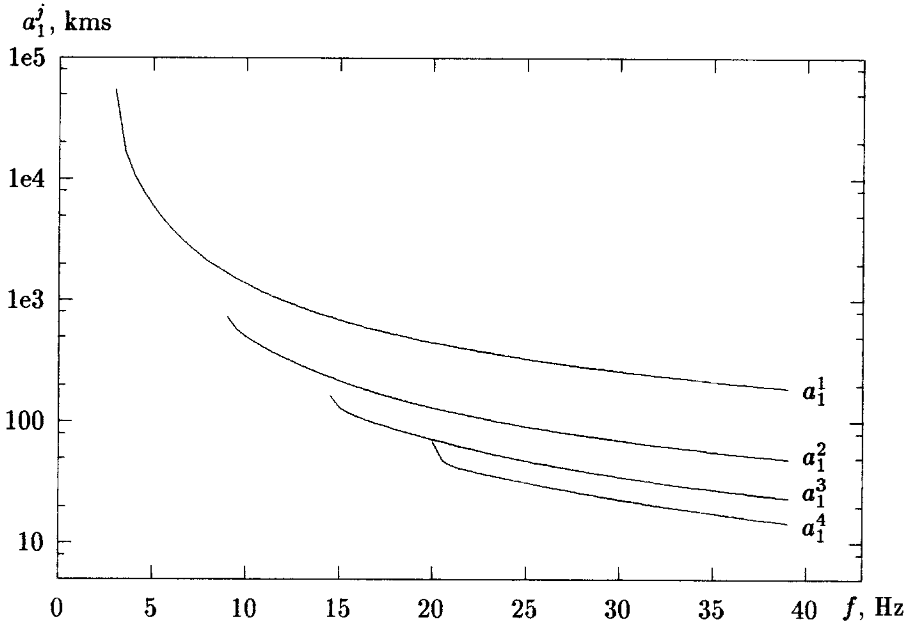

Fig. 1. Dependence of the first "critical" size of ice–floe from frequency for various modes.

The calculations show that in the shallow sea ($H = 200\ m$) the first "critical" length of the ice-floe a_1^j is great (at frequency 40 Hz it equals $\sim 15 - 200\ kms$ depending on mode number) and with increase of number j of surging mode a_1^j is decreased.

The following group of calculations is devoted to investigation the influence of absorption in ice on attenuation of an acoustic field. The absorption in ice is taken into account as the small image additives $i\delta_1$ and $i\delta_2$ to n_1 and n_2 accordingly ($\delta_1 > 0$, $\delta_2 > 0$). For quantitative estimates the same waveguide was chosen, with point monochromatic source located on depth 100 m. As a measure of attenuation of sound pressure on distance R from a source we consider the difference of average values of fields (taken in Db) without the ice–floe (p_{av}) and with it (p_{av}^{ice}), averaged by the rectangle $0 \leq z \leq H$, $R - r \leq x \leq R + r$ ($r \sim 2 - 5$ km, $R > a$).

In calculation shown on fig. 2 the length of ice-floe varies from 1000 up to 5000 km; the frequency f of a point source equals 11 Hz (two–moded waveguide), thickness of ice $d = 10\ m$ and distance $R = 5100\ km$. Here one can see a few calculations with various δ_1 and δ_2 satisfying the equality $\delta_1^2 + \delta_2^2 = 0.05^2$.

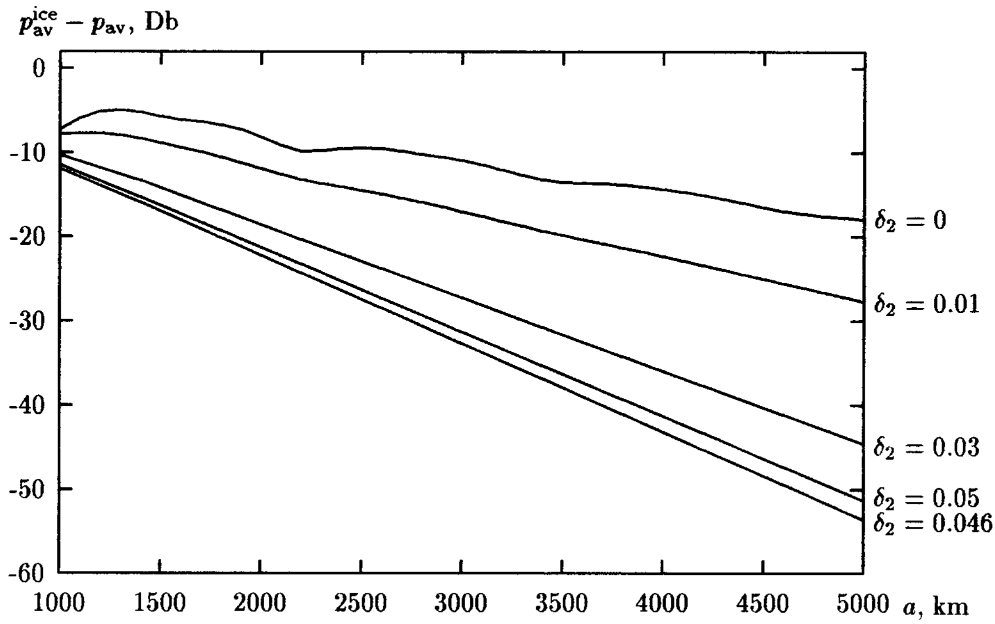

Fig. 2. Dependence of field attenuation from ice-floe length.

In calculation shown on fig. 3 frequency varies; thickness of ice $d = 10\ m$, distance $R = 1100\ km$; coeffitients of absorption $\delta_1 = 0.02$, $\delta_2 = 0.046$ satisfy the equality $\delta_1^2 + \delta_2^2 = 0.05^2$ and the attenuation is maximum (bottom diagram). For comparison similar calculation (top diagram) at absence of absorption in ice ($\delta_1 = \delta_2 = 0$) is given.

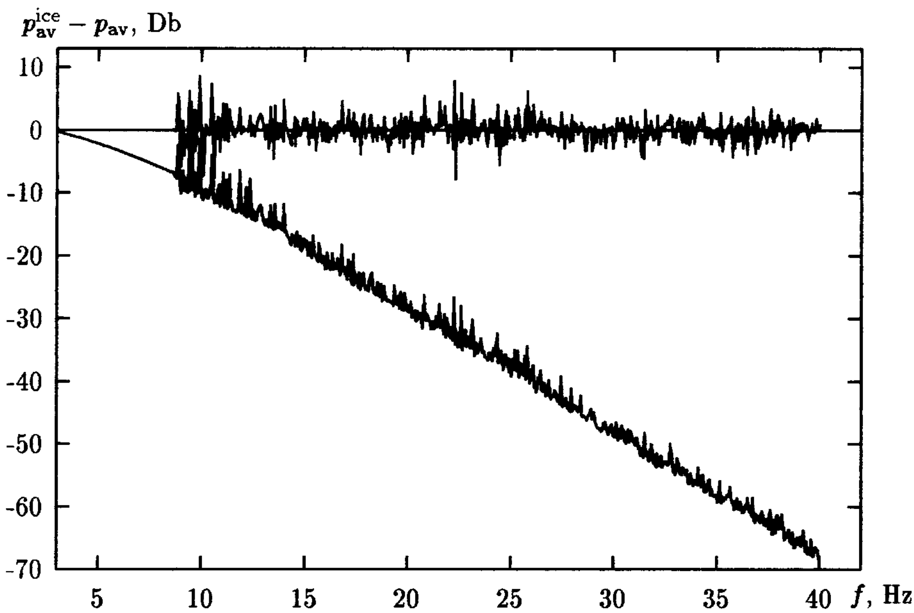

Fig. 3. Dependence of attenuation of a field from frequency.

Here the conversion from one–mode to two–mode waveguide ($f \sim 8.5\ Hz$) is clearly seen. After this convertion both curves begin to oscillate with amplitude $\sim 3 - 8\ Db$, and the bottom curve decreases with an average gradient $\sim 2\ Db$ on $1\ Hz$.

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