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Updating of Non-conservative Structure Via Inverse Methods with Parameter Subset Selection

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Abstract

The paper deals with the improvement of the updating method proposed in [4], for the case of incomplete modal and spectral data with incomplete eigenvector. Missing modal data is given by some of expansion methods. The parameter subset selection is proposed as an improvement of updating method via inverse problem.

1. Introduction

The discipline and practice of finite element modeling (FEM) of structures has become a sophisticated technology. The techniques of experimental modal analysis (EMA) have developed into a formal and well developed technology. Both these disciplines imply specific rules and structure to be used to develop a model. Both claim great successes in their ability to provide consistent and useful models. In principal, of course, the analytical model should be consistent with, or predict, the results obtained from vibration tests. However, this rarely happens. As a result, the finite element model must often be adjusted or modified, until it agrees with the test data.

The approach proposed here is to use the results of inverse eigenvalue problems to develop methods for correcting models.

2. Mathematical Models

Consider a linear lumped parameter system which can be modelled by a differential equation of the form

$$\mathbf{M}\ddot{\mathbf{v}}(t)+\mathbf{D}\dot{\mathbf{v}}(t)+\mathbf{K}\mathbf{v}(t)=\mathbf{f}(t) \tag{1.1}$$

where $\mathbf{v}(t)$ is an n vector of time-varying elements representing the displacement of the masses.. The matrix coefficients **M**, **D** and **K** are *NxN* symmetric matrix of constant real elements representing the mass, damping and stiffness coefficients of the system, respectively. The mass matrix **M** is taken to be positive definite.

The first order $2N$ space form is

$$\mathbf{N}\,\dot{\mathbf{u}}(t) - \mathbf{P}\,\mathbf{u}(t) = \mathbf{g}(t) \tag{1.2}$$

where the vector $\mathbf{u}(t) = \begin{bmatrix} \mathbf{v}(t) \\ \dot{\mathbf{v}}(t) \end{bmatrix}$, $\mathbf{g}(t) = \begin{bmatrix} \mathbf{f}(t) \\ \mathbf{0} \end{bmatrix}$

and the matrices **P** and **N** are defined by the partitioned form

$$\boldsymbol{P} = \begin{bmatrix} -\mathbf{K} & \mathbf{0} \\ \mathbf{0} & \mathbf{M} \end{bmatrix}, \boldsymbol{N} = \begin{bmatrix} \mathbf{D} & \mathbf{M} \\ \mathbf{M} & \mathbf{0} \end{bmatrix}, \tag{1.3}$$

The standard state space formulation

$$\dot{\mathbf{x}}(t) - \mathbf{A}\,\mathbf{x}(t) = \mathbf{h}(t), \tag{1.4}$$

where the state vector $\mathbf{x}(t) = \mathbf{u}(t)$, $\mathbf{h}(t) = \mathbf{N}^{-1}\mathbf{g}(t)$, and the state matrix A is given by

$$\boldsymbol{A} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{D} \end{bmatrix}, \tag{1.5}$$

Next consider solution of Eqs. (1.1), (1.2), and (1.4) of the form $\mathbf{v}(t) = \mathbf{v}\,e^{st}$, $\mathbf{u}(t) = \mathbf{u}\,e^{st}$, and $\mathbf{x}(t) = \mathbf{x}\,e^{st}$, respectively, for the homogeneous case $\mathbf{f}(t) = \mathbf{g}(t) = \mathbf{h}(t) = \mathbf{0}$.

Each of these "eigenvalue"problems in $\mathbf{v}_i$, $\mathbf{u}_i$, and $\mathbf{x}_i$ can be restarted as a matrix equation by defining the matrix Λ to be the $2Nx2N$ diagonal matrix of eigenvalues λ. In addition if the $Nx2N$ matrix **V** is defined by taking the $2N$, $Nx1$ vectors $\mathbf{v}_i$ as its columns, if the $2Nx2N$ matrix **U** is defining by taking $2N$, $2Nx1$ vectors $\mathbf{u}_i$ as its columns.

From the Eq. (1.3), (1.5) and orthogonality conditions after some manipulation follows [4]:

$$\mathbf{M} = (\mathbf{V}\ \Lambda\ \mathbf{W}^{T})^{-1}, \tag{1.6}$$

$$\mathbf{D} = -\mathbf{M}\,(\mathbf{V}\ \Lambda^{2}\,\mathbf{W}^{T})\,\mathbf{M}, \tag{1.7}$$

$$\mathbf{K} = -\mathbf{M}\,(\mathbf{V}\ \Lambda^{2}\,\mathbf{W}^{T})\,\mathbf{D} + \mathbf{M}\,(\mathbf{V}\ \Lambda^{3}\,\mathbf{W}^{T})\,\mathbf{M}, \tag{1.8}$$

with modal condition

$$\mathbf{0} = \mathbf{V}\,\mathbf{W}^{T} \tag{1.9}$$

The relation among the matrix coefficients **M**, **D**, and **K** and design parameters and the element matrices can be expressed follows

$$\mathbf{K} = \sum_{j=1}^{Nk} \mathrm{k_j}\,\mathbf{K}_{\mathrm{ej}},\quad \mathbf{D} = \sum_{j=1}^{Nd} \mathrm{d_j}\,\mathbf{D}_{\mathrm{ej}},\quad \mathbf{M} = \sum_{j=1}^{Nm} \mathrm{m_j}\,\mathbf{M}_{\mathrm{ej}}, \tag{1.10}$$

where k_j, d_j, m_j are the design parameters amount of which is N_k, N_d, and Nm, and the element matrices $\mathbf{K}_{ej}$, $\mathbf{D}_{ej}$, $\mathbf{M}_{ej}$ are known.

2. Systems with Proportional Damping

For the case of symmetric coefficient matrices **K, D, M** and system of simple structure ($\Lambda = \boldsymbol{S} = \Lambda^{T}$) from eigenvalue problem follows that $\mathbf{W} = \mathbf{V}$.

In the case of proportional damping where the damping matrix is of the form

$$\mathbf{D} = \alpha\,\mathbf{M} + \beta\mathbf{K} \tag{2.1}$$

there exists such real matrix V_o, for which is valid

$$V_0^T K\, V_0 = \Omega_0^{\,2}, \qquad V_0^T D\, V_0 = 2\Delta\,, \qquad V_0^T M\, V_0 = I, \qquad (2.2)\text{-}(2.4)$$

$$V = V_0\,[C, \bar{C}\,], \tag{2.5}$$

where the matrices $\Omega_0^{\,2}$ and 2Δ are diagonal matrices of squared undamped natural frequencies $\omega_j^{\,2}$ and damping ratios $2\,\zeta_j$.

By comparison of (2.1)-(2.4) and an orthogonality condition it follows:

$$\Omega_0^{\,2} = S_s\,\bar{S}_s, \qquad 2\,\Delta = -\,(S_s + \bar{S}_s), \qquad V_0 = 2\,V_{sR}\,(S_{sI})^{-0,5}, \qquad V_{sR} = -\,V_{sI} \qquad (2.6)\text{-}(2.9)$$

where in the underdamped case

$$S = diag\,([S_s, \bar{S}_s]), \qquad S_s = S_{sR} + i\,S_{sI}, \qquad V = [V_s, \bar{V}_s], \qquad V_s = V_{sR} + i\,V_{sI}.$$

3. Complete Modal and Spectral Data Case

For known modal and spectral data **V, W**, and Λ the matrix coefficients **M, D**, and **K** are given by Eqs.(1.6), (1.7), and (1.8). Then by using relations in Eq.(1.10) for known element matrices $\mathbf{M}_{ej}$, $\mathbf{D}_{ej}$, and $\mathbf{K}_{ej}$ we yield the design parameters m_j, d_j, and k_j of the system.

By using (1.10), entries in the matrices **M, D**, and **K** are given by linear combination of the design parameters as follows

$$\mathbf{A}_{KI}\,\{k_l\} = \mathbf{b}_{KI}, \qquad \mathbf{A}_{DI}\,\{d_l\} = \mathbf{b}_{DI}, \qquad \mathbf{A}_{MI}\,\{m_l\} = \mathbf{b}_{MI}, \tag{3.1}$$

where $\mathbf{A}_{KI}$, $\mathbf{A}_{DI}$, $\mathbf{A}_{MI}$ to be (N^2,Nk), (N^2,Nd), and (N^2,Nm) matrices, and $\mathbf{b}_{KI}$, $\mathbf{b}_{DI}$, b_{MI} to be $(N^2,1)$ vectors. For j,k entry of the matrices **K**, **D**, **M** and j,k entry of the matrices $\mathbf{K}_{el}$, $\mathbf{D}_{el}$, $\mathbf{M}_{el}$ is valid

$$K_{j,k} = b_{KI_{j+N(k-1)}}, \quad D_{j,k} = b_{DI_{j+N(k-1)}}, \quad M_{j,k} = b_{MI_{j+N(k-1)}}, \tag{3.2}$$

$$K_{el\,j,k} = A_{KI_{j+N(k-1),l}}, \quad D_{el\,j,k} = A_{DI_{j+N(k-1),l}}, \quad M_{el\,j,k} = A_{MI_{j+N(k-1),l}}. \tag{3.3}$$

In the case, that is valid : $Nk,\ Nd,\ Nm \le n_{bK},\ n_{bD},\ n_{bM}$,

where n_{bK}, n_{bD}, n_{bM}, are nonzero rows of the matrices $\mathbf{A}_{KI}$, $\mathbf{A}_{DI}$, $\mathbf{A}_{MI}$ it is possible to determine the design parameters k_j, b_j, m_j by using Eq. (3.1)

Let us have the analytical model given by the coefficient matrices $\mathbf{M}_a$, $\mathbf{D}_a$, and $\mathbf{K}_a$ with in advance given structure (some entries are zeros). For this model the modal and spectral data are representing by matrices $\mathbf{V}_a$, $\mathbf{W}_a$, and Λ_a. Let the experimental spectral and modal data are given by the matrices $\mathbf{V}_e$, $\mathbf{W}_e$, and Λ_e. By using Eqs.(1.6)-(1.9) we yield the experimental coefficient matrices $\mathbf{M}_e$, $\mathbf{D}_e$, and $\mathbf{K}_e$ which will have different structure as analytical. Then the design parameters determined from $\mathbf{M}_e$, $\mathbf{D}_e$, and $\mathbf{K}_e$ by using Eq.(3.1) will create coefficient matrices $\mathbf{M}_u$, $\mathbf{D}_u$, and $\mathbf{K}_u$ which will be different from both analytical $\mathbf{M}_a$, $\mathbf{D}_a$, and $\mathbf{K}_a$ and experimental. $\mathbf{M}_e$, $\mathbf{D}_e$, and $\mathbf{K}_e$. Structures of matrices $\mathbf{M}_u$, $\mathbf{D}_u$, and $\mathbf{K}_u$ and matrices $\mathbf{M}_a$, $\mathbf{D}_a$, and $\mathbf{K}_a$ will be the same. Nonzero entries of matrices $\mathbf{M}_u$, $\mathbf{D}_u$, and $\mathbf{K}_u$ and nonzero entries of matrices $\mathbf{M}_e$, $\mathbf{D}_e$, and $\mathbf{K}_e$ will be the same $(Nk,\ Nd,\ Nm = n_{bK},\ n_{bD},$

n_{bM}), or will be different in the sense of minimum of Euclid norm (Nk, Nd, Nm < n_{bK}, n_{bD}, n_{bM}).

Example 1

Let the coefficient matrices of the analytical system with proportional damping are

$$\mathbf{M}_a = \mathrm{diag}([m_1, m_2, m_3, m_4, m_5]),$$

$$\mathbf{K}_a = \begin{bmatrix} k_1+k_2 & -k_2 & 0 & 0 & 0 \\ -k_2 & k_2+k_3+k_6 & -k_3 & 0 & -k_6 \\ 0 & -k_3 & k_3+k_4 & -k_4 & 0 \\ 0 & 0 & -k_4 & k_4+k_5 & 0 \\ 0 & -k_6 & 0 & 0 & k_6 \end{bmatrix},$$

where k_{aj} = 1.0e+003 [1.6, 30, 1.2, 25, 1.5, 1.0],
m_{aj} = [0.5, 5.5, 5.5, 0.8, 3.0].

and

$$\mathbf{D}_a = \alpha\,\mathbf{M} + \beta\mathbf{K}$$

where α_a= 0.01, β_a = 0.0001

Let us simulate errors in some design parameters and than build coefficient matrices $\mathbf{M}_r$, $\mathbf{D}_r$, and $\mathbf{K}_r$, where

k_{rj} =1.0e+003 [1.84, 25.5, 1.056, 29, 1.68, 1.14]
m_{rj} = [0.57, 4.95, 5.94, 0.76, 3.3]

and

α_r= 0.011, β_r = 0.0001

This system will be represented by modal and spectral matrices $\mathbf{V}_r$, $\mathbf{W}_r$, and $\mathbf{\Lambda}_r$ or V_{0r} and S_r. By using Eqs.(1.6), (1.7), and (1.8), or (2.2)-(2.4) we compute the effective design parameters with errors.

Let us define some error matrices $\mathbf{eV_0} = \mathbf{eW_0}$, $\mathbf{eS}$ representing a noise in measurement and multiply every entry of the error matrices with corresponding entry of effective modal and spectral ones. These will represent experimental modal and spectral data $\mathbf{V}_{0e} = \mathbf{V}_{0r}\, eV_0$ and $\mathbf{S}_e = \mathbf{S}_r eS$

By using Eqs. (2.2) - (2.4) we yield the experimental coefficient matrices $\mathbf{K}_e$, $\mathbf{M}_e$ and α_e= 0.009, β_e = 0.0001

From (3.1) - (3.2) we yield the new design parameters

k_{uj} = 1.0e+004[0.2686, 2.3665, 0.1397, 3.0793, 0.0951, 0.0959]
m_{uj} = [0.5308, 4.8068, 6.2764, 0.7737, 3.5202]

Comparison of analytical, effective and updated design parameters is given in the next table

k_{aj}	1.6	30	1.2	25	1.5	1.0
k_{rj}	1.84	25.5	1.056	29	1.68	1.14
k_{uj}	2.686	23.665	1.397	30.793	0.951	0.959

m_{aj}	0.5	5.5	5.5	0.8	3.0
m_{rj}	0.57	4.95	5.94	0.76	3.3
m_{uj}	0.5308	4.8068	6.2764	0.7737	3.5202

$\alpha_a = 0.01$, $\beta_a = 0.0001$, $\alpha_r = 0.011$, $\beta_r = 0.0001$, $\alpha_e = 0.009$, $\beta_e = 0.0001$

4. Incomplete Modal and Spectral Data with Complete Eigenvector Case

In the case in which for modal and spectral matrices $\boldsymbol{V}$, $\boldsymbol{S}$ there are known only m eigenvectors and m eigenvalues (modal matrix V_N of dimension $(N,2m)$ and spectral matrix $\boldsymbol{S}_m$ of dimension $(2m,2m)$) for finding of design parameters k_j, m_j, α and β can be used equations (2.1)-(2.4) in the form

$$V_{0N}^T \boldsymbol{K} V_{0N} = \Omega_{0m}^2,\quad V^T_{0N} \boldsymbol{D} V_{0N} = 2\,\Delta_m,\quad V_{0N}^T \boldsymbol{M} V_{0N} = \boldsymbol{I}_m, \qquad (4.1)-(4.3)$$

where the matrices Ω_{0m}^2, $2\,\Delta_m$, dimension of (m,m), are given by (2.6)-(2.7) and the matrix V_{0N} is given by (2.5) and (2.8).

By using (1.10), entire elements of multiplication in equations (4.1) and (4.3) can be given as linear combination of design parameters as follows

$$\boldsymbol{A}_{Ko}\,\{k_l\} = \boldsymbol{b}_{Ko}, \qquad \boldsymbol{A}_{Mo}\,\{m_l\} = \boldsymbol{b}_{Mo}, \qquad (4.4)-(4.5)$$

where, $\boldsymbol{A}_{Ko}$, $\boldsymbol{A}_{Mo}$, to be (m^2,Nk), (m^2,Nm) matrices and $\boldsymbol{b}_{Ko}$, $\boldsymbol{b}_{Mo}$ are $(m^2,1)$vectors. For j,k entry of matrices Ω_{0m}^2, $\boldsymbol{I}_m$, and j,k entry of matrices

$$\boldsymbol{Ko}_l = V_{0N}^T \boldsymbol{Ke}_l\, V_{0N}, \qquad \boldsymbol{Mo}_l = V_{0N}^T \boldsymbol{Me}_l\, V_{0N}, \qquad (4.6)-(4.7)$$

is valid:

$$\Omega_{0m}^2{}_{j,k} = b_{Ko_{j+m(k-1)}}, \qquad I_{m\,j,k} = b_{Mo_{j+m(k-1)}}. \qquad (4.8)-(4.9)$$

$$Ko_{l\,j,k} = A_{Ko_{j+m(k-1),l}}, \qquad Mo_{l\,j,k} = A_{Mo_{j+m(k-1),l}}. \qquad (4.10)-(4.11)$$

In the case that is valid : $Nk \le n_o$, $Nm \le n_o$,
where $n_o = m(m+1)/2$, it is possible to determine the design parameters k_j, m_j (number of which is Nk, Nm) by using equations (4.4)-(4.5) .

From comparison of equations (2.1) and (4.2) it follows:

$$\boldsymbol{A}_{ab}\begin{bmatrix}\alpha\\ \beta\end{bmatrix} = \boldsymbol{b}_{ab}, \qquad (4.13)$$

where, $\boldsymbol{A}_{ab}$ is dimension of $(m,2)$, $\boldsymbol{b}_{ab}$ is dimension of $(m,1)$ and it is valid :

$$b_{\alpha\beta_j} = 2\,d_j, \qquad A_{\alpha\beta\,j,1} = 1, \qquad A_{\alpha\beta\,j,2} = \omega_{0j}^2. \qquad (4.14)-(4.16)$$

By solution of the equation (4.13) for the case $2 \le m$, are given the design parameters α and β.

5. Case of Incomplete Modal and Spectral Data with Incomplete Eigenvector

Matrices V_N and V_{0N} can be given as follows:

$$V_N = \begin{bmatrix} \mathbf{V}_r \\ \mathbf{V}_o \end{bmatrix}, \; V_{0N} = \begin{bmatrix} \mathbf{V}_{0r} \\ \mathbf{V}_{0o} \end{bmatrix} = \boldsymbol{T}\, V_{0r}, \tag{5.1)-(5.2}$$

where V_r, V_o, V_{0r}, V_{0o}, are *(n,2m)*, *(N-n,2m)*, *(n,m)*, *(N-n,m)* matrices. From equations (2.5) a (2.8) follows:

$$V_r = V_{0r}\, [C_m, \overline{C}_m], \tag{5.3}$$

where $C_m = (2\, i\, S_{sIm})^{-1/2}$.

For known matrices V_r, S_m, T by using (5.1)-(5.2), (4.4)-(4.5) and (4.13) design parameters k_j, m_j (number of which is *Nk*, *Nm*), α and β can be computed.

Matrix T can be given as follows:

$$\boldsymbol{T} = \begin{bmatrix} \mathbf{I} \\ \mathbf{V}_{0o}\, \mathbf{V}_{0r}^{\;+} \end{bmatrix} = \begin{bmatrix} \mathbf{I} \\ \mathbf{T}_s \end{bmatrix}. \tag{5.4}$$

For determining design parameters k_j, m_j, α and β for the case of unknown matrix V_{0o} it is necessary to estimate the matrix T. Two methods (Expansion using analytical modes and Physical expansion) of determining of the matrix T or matrix V_{0o}, are described in [2].

6. Parameter Subset Selection

Solving equations (4.4)-(4.5) with the full design parameter set sometimes leads to meaningless results due to the bad condition of the coefficient matrix A. Thus the problem of the parameter subset selection is essential for the whole procedure.

Numeric significant rank of matrices A_K, A_M is less than the number of determining design parameters N_k, N_m.

Singular value decomposition of matrices A_K, A_M is given by:

$$(A = U\, \Sigma\, V^T)_{K,M}, \tag{6.1)-(6.2}$$

where $U_{K,M}$, $V_{K,M}$ are the (m^2,m^2), $(N_{k,m},N_{k,m})$, orthonormal matrices and $\Sigma_{K,M}$ is diagonal and real matrix containing the singular values σ_j, $j=1,2,\ldots,N_{k,m}$, of the matrix A. From sudden decrease singular values ($\sigma_j >> \sigma_{j+1}$), can be indicated significant numeric rank of matrices A_K, A_M and simultanuelosly the number of dominant design parameters N_k', N_m'.

The selection of dominant design parameters k'_j, m'_j can be found by means of the Q-R decomposition [3] of matrices A_K, A_M,

$$(A\, \Pi = Q\, R)_{K,M}, \tag{6.3)-(6.4}$$

where $Q_{K,M}$ are (m^2,m^2) orthonormal matrices and $R_{K,M}$, are $(m^2,N_{k,m})$ upper triangular matrices. The $(N_{k,m},N_{k,m})$ permutation matrices $\Pi_{K,M}$ contain only unit vectors as columns and describes the interchange of columns of A_K, A_M in such a way, that diagonal entries of matrices $R_{K,M}$ are in ascending order (in order of optimal linear independent columns). After having performed the Q-R decomposition the solution of Least Squares problem (4.4)-(4.5) can be easily obtained by backward substitution:

$$(R\, \Pi^T x = Q^T b)_{K,M}, \tag{6.5)-(6.6}$$

where $(\boldsymbol{R} = \begin{bmatrix} \boldsymbol{A}_{11} & \boldsymbol{A}_{12} \\ \boldsymbol{0} & \boldsymbol{A}_{22} \end{bmatrix})_{K,M}$, $(\boldsymbol{Q}^T \boldsymbol{b} = \begin{bmatrix} \boldsymbol{b}_1 \\ \boldsymbol{b}_2 \end{bmatrix})_{K,M}$, $(\boldsymbol{\Pi}^T \boldsymbol{x} = \begin{bmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \end{bmatrix})_{K,M}$,

where $(\boldsymbol{A}_{11}, \boldsymbol{A}_{12}, \boldsymbol{A}_{22})_{K,M}$ are of dimension $(N_{k',m'}, N_{k',m'})$, $(N_{k',m'}, N_{k,m}-N_{k',m'})$, $(m^2-N_{k',m'}, N_{k,m}-N_{k',m'})$ and $(\boldsymbol{b}_1, \boldsymbol{b}_2)_{K,M}$ are of dimension $(N_{k',m'}, 1)$, $(m^2-N_{k',m'}, 1)$.

Assuming only dominant design parameters $(\boldsymbol{x}_1)_{K,M}$ number of which is N_k', N_m', $(\boldsymbol{x}_2 \cong \boldsymbol{0})_{K,M}$, we yield:

$$(\boldsymbol{x}_1 \cong \boldsymbol{A}_{11}^{-1} \boldsymbol{b}_1)_{K,M}. \qquad (6.7)\text{-}(6.8)$$

Example 2

Let us have the same system as in example 1 and we have only 3 eigenvalues and 3 eigenvectors measured. The experimental modal and spectral data $\mathbf{V_{0e}}$ and $\mathbf{S_e}$ are taken from example 1.

$$\mathbf{V_{0em}} = \begin{bmatrix} 0.2032 & 0.0284 & 0.3540 \\ 0.2202 & 0.0310 & 0.3562 \\ 0.1433 & 0.3437 & -0.1182 \\ 0.1414 & 0.3391 & -0.1116 \\ 0.4085 & -0.2764 & -0.2333 \end{bmatrix}$$

$$\mathbf{S_{em}} = \mathrm{diag}([\ 172.9,\ 392.7,\ 936.46\ ])$$

Now we will make parameter subset selection. SVD and Q-R decomposition of matrices A_K and A_M give the following results:

-dominant stiffness design parameters are k_6, k_3, k_5 and k_1

-dominant mass design parameters are m_5, m_2, and m_3

Comparison of analytical, effective and updated design parameters is given in the next tables

1.0e+003*

k_{aj}	1.6	1.2	1.5	1.0
k_{rj}	1.84	1.056	1.68	1.14
k_{uj}	1.7888	1.1751	1.5042	1.2189

m_{aj}	5.5	5.5	3.0
m_{rj}	4.95	5.94	3.3
m_{uj}	5.2377	5.6646	3.384

$\alpha_a = 0.01$, $\beta_a = 0.0001$, $\alpha_r = 0.011$, $\beta_r = 0.0001$, $\alpha_e = 0.0111$, $\beta_e = 0.0001$

Example 3

Let us have the same system as in example 1. Number of measured modes m=3, number of measured degree of freedom N=4. We used SVD and Q-R decomposition for removing one line from modal matrix (it is the second line). The experimental modal and spectral data $\mathbf{V_{0e}}$ and $\mathbf{S_e}$ are taken from example 1.

Missing modal data V_{0o} in equation (5.2) is given by equation (5.4)

$V_{0o} = [\ 0.2135\ 0.0300\ 0.3682\ 0.0030\ 1.2931]$

Spectral and modal matrices are the same as in example 2 except 2nd line where are 1st three entries of V_{0o}.
Now we will make parameter subset selection. SVD and Q-R decomposition of matrices A_K and A_M give the following results:
-dominant stiffness design parameters are k_6, k_3, k_5 and k_1
-dominant mass design parameters are m_5, m_2 and m_3
Comparison of analytical, effective and updated design parameters is given in the next tables

1.0e+003*

k_{aj}	1.6	1.2	1.5	1.0
k_{rj}	1.84	1.056	1.68	1.14
k_{uj}	1.8127	1.0955	1.5829	1.1657

m_{aj}	5.5	5.5	3.0
m_{rj}	4.95	5.94	3.3
m_{uj}	5.0211	5.6643	3.4097

$\alpha_a = 0.01$, $\beta_a = 0.0001$, $\alpha_r = 0.011$, $\beta_r = 0.0001$, $\alpha_u = 0.0111$, $\beta_u = 0.0001$

7. Conclusion

Updating methods for the cases of complete spectral and modal data, incomplete spectral and modal data with complete eigenvectors and incomplete modal and spectral data with incomplete eigenvectors are presented. From comparison of analytical, effective and updated design parameters which have been determined in examples for all cases follows that the parameter subset selection cuts down the sensitivity of design parameters on noise errors. The full design parameter set sometimes leads to meaningless results due to the bad condition of the coefficient matrix A

8.References

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