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NONSTATIONARY TRANSIENT VIBROACOUSTIC RESPONSE OF A BEAM STRUCTURE

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ABSTRACT

This study consists of an investigation into the nonstationary transient response of the Verification Test Article (VETA) when subjected to random acoustic excitation. The goal is to assess excitation models that can be used in the design of structures and equipment when knowledge of the structure and the excitation is limited. The VETA is an instrumented cantilever beam that was exposed to acoustic loading during five Space Shuttle launches. The VETA analytical structural model response is estimated using the direct averaged power spectral density and the normalized pressure spectra methods. The estimated responses are compared to the measured response of the VETA. These comparisons are discussed with a focus on prediction conservatism and current design practice.

INTRODUCTION

The problem of structural vibroacoustic response of ground structures and equipment to intense rocket acoustics is investigated. The goals of this investigation are to utilize and assess two structural vibroacoustic response estimation methods. Those methods are to be used in the design of structures and equipment when knowledge of both the structure and the excitation are limited. The methods must be sufficiently conservative to provide a reasonable factor of safety. At the same time, the methods must avoid gross over-design and false failure predictions. Intense arguments between designers on how to approach this problem provided the impetus for this effort. The results of this work show two load modeling methods that are used for design applications, and the maximum and minimum prediction bounds that can result when loading assumptions are introduced when applying the methods.

The two methods for modeling the loading are designated the direct averaged power spectral density (PSD) and the normalized pressure spectra. Each method is used to estimate the predicted maximum and minimum vibroacoustic response of the Verification Test Article (VETA). The VETA is a structure that was subjected to Space Shuttle launch acoustics on launch pad 39A at Kennedy Space Center. Details of the VETA structure and its modal test results may be found in reference 1. The analytical model of the VETA and the joint acceptances are derived prior to applying the response methods.

ANALYTICAL MODEL

The structure is modeled as a slender, prismatic, cantilevered beam with several assumptions. Fixed end conditions are assumed at the base. The entire length of the beam is used in the calculations. Light damping and lack of modal coupling are assumed. This is justified from the results of the modal test. Only the first three bending modes are calculated for use in the response analysis. This is based on the expectation that almost the entire response will be due to the first few bending modes.

The Euler-Bernoulli beam is used to model the VETA. The effects of shear and rotational inertia are neglected. The beam model is depicted in figure 1. Its equation of motion is,

$$\frac{\partial^2 w}{\partial t^2} + \beta \frac{\partial^4 w}{\partial x^4} = q \quad \text{where} \quad \beta = \frac{EI}{\rho A}$$

The variables in the above equation are:

> w = transverse displacement   t = time   x = axial coordinate
> q = distributed loading   E = Young's modulus   I = moment of inertia
> ρ = mass density   A = Area

The eigenvalues for the first three bending modes of the VETA along with the modal test derived natural frequencies and dampings are presented in table 1. The mode shapes are depicted in figure 2.

JOINT ACCEPTANCE

The excitation of a structure by distributed random pressure loads can act in some instances with the mode, and in some cases against the mode resulting in a lower response. The instantaneous correlation of pressures over the structure is required to describe the total effect of the pressure loading. In addition, the mode shape of interest must be accounted for (reference 2). The joint acceptance "is a factor which describes the proportion of [the total] force which a particular mode of distortion can 'accept' and convert into the corresponding generalized force" (reference 3). The word joint is used to state that the acceptance is a function of both the trace wavelengths of the pressure loading and the wavelength of the particular mode in question.

Note that the VETA has light damping and little modal coupling for the modes in question. Thus, the contribution of the cross modal terms to the joint acceptance is negligible (reference 2). In addition, the pressure loading is uniform over the VETA, so the cross power spectrum of the uniform load is equal to its auto power spectrum. Taking these facts into account yields the following result for the joint acceptance.

$$j_{rr}^2 = \frac{1}{A^2} \iint_A \Phi_r(\bar{r})\Phi_r(\bar{r}')d\bar{r}d\bar{r}' \Rightarrow Aj_{rr} = \iint_A \Phi_r(\bar{r})\Phi_r(\bar{r}')d\bar{r}d\bar{r}'$$

where j_{rr} = joint acceptance for mode r, r = mode number, Φ_r = mode shape, and $\bar{r}, \bar{r}'$ = position vectors.

The joint acceptances for the first three modes of the VETA are presented in table 2. The area in table 2 was set equal to 1 for convenience.

DIRECT AVERAGE POWER SPECTRAL DENSITY METHOD

This method uses an assumed data model and the fast Fourier transform (FFT) to estimate the maximax power spectrum of the pressure loading (reference 4). The estimated PSD is subsequently used in conjunction with the structural modal response function to arrive at a response PSD (reference 5). The response PSD is integrated to yield the root-mean-square (rms) estimate of the response of a particular mode. The overall response is the summation of the individual responses.

The model assumed for the nonstationary PSD is known as the evolutionary spectral density (reference 6). This model requires that the PSD be a slowly varying function of time. The model is expressed mathematically as follows.

$$p(t) = r(t)u(t)$$

where p(t)= the nonstationary time history, r(t)=a slowly varying time function, and u(t)= a time history with a stationary PSD.

This model applies to both the input and the response, since the response of a system to a nonstationary input will in turn be nonstationary. A certain degree of experience is required in order to apply this model to the type of nonstationary data generated in a space shuttle launch. Typical pressure and strain time histories are shown in reference 1. These demonstrate that r(t) is generally not a slowly varying function of time. A subset of the time history is chosen in order to comply with the slowly varying requirement. The subset time window choice is made to encompass a time slice in which the rms pressures peak. There are two such intervals in the data. Only the first of these intervals is chosen for this analysis. The reasoning is that the first interval is due to acoustic inputs only, and is typical of most structures on the launch pad. The second time interval is due to impingement by deflected plume gases and particulate. The nature of this loading is not of interest here.

The power spectrum of the generalized load, w_{Lrr}, is related to the power spectrum of the incident pressure, w_{p_0}, by the following relation. Here ω is the circular frequency.

$$w_{Lrr}(\omega) = w_{p_0}(\omega)A^2 j_{rr}^2(\omega)$$

The displacement response to random excitation is given by the following relation using the above form for the generalized load.

$$w_{wr}(\omega) = \frac{\Phi_r^2 |H_r(\omega)|^2 w_{p_0}(\omega)A^2 j_{rr}^2(\omega)}{m_r^2}$$

The term $H_r(\omega)$ is the impulse complex frequency response of mode r, and m_r is the generalized mass for mode r. The mean square displacement is obtained from the relation,

$$w^2 = \sum_{r=1}^{N} \int w_{wr}(\omega)d\omega$$

The predicted analytical strains have to be computed for comparison with the measured strains. An assumption is made as to the deflected shape of the structure. Observing the measured strain data spectrum (figure 3) indicates that the response is dominated by the first beam bending mode. Also, the deflected shape in the first mode is very similar to the shape taken by a uniformly loaded cantilever beam. Therefore, the equations for the bending stress and the elastic curve of a uniformly loaded cantilever can be recast to give the relation between deflection and strain at any point along the beam. The final form of the equation is,

$$\varepsilon = \frac{12w(L-x)^2 c}{x^4 - 4Lx^3 + 6L^2x^2}$$

where c is the maximum fiber distance, L is the length of the beam, ε is the strain, and x is as previously defined. This equation was used to compute the strains presented in table 3 for both the direct average PSD and the normalized pressure spectra methods.

NORMALIZED PRESSURE SPECTRA METHOD

The normalized pressure spectra method takes a different approach to estimating the load due to an random nonstationary pressure. The method stems from the solution to the equation of motion for a mode shape to an arbitrary pressure loading (reference 7). The equation of motion is

$$\ddot{q}_r + 2\zeta_r\omega_r\dot{q}_r + \omega_r^2 q_r = \frac{Aj_{rr}}{m_r}p(t)$$

where ζ_r = damping ratio q_r = modal coordinate $p(t)$ = pressure load time history

This equation may be solved by setting the the Aj_{rr}/m_r ratio equal to one. The resulting quantity is recast into the following form and plotted versus frequency. A sample plot of this quantity is shown on figure 4.

$$Y(\omega_r) = q_{r\,max}\left(\frac{m_r \omega_r^2}{Aj_{rr}}\right)$$

where Y is called the load modal coordinate.

RESULTS

Both methods were used to estimate the minimum and maximum strain responses of the VETA. The VETA was instrumented with both front and rear pressure transducers. Sample plots of the pressure time histories may be found in reference 1. Thus, the response contributions from each side were assumed to add to estimate the maximum, and the response contributions were subtracted to estimate the minimum responses. This approach guarantees that the measured response of the VETA will be bounded by the estimates. In addition, this provides insight into how much conservatism may be introduced by assuming a worst case contribution to the response from the incident pressure waves. The measured and estimated strains from both methods are presented in table 3.

CONCLUSIONS

The above results encompass the measured strain gage data. The direct averaged PSD approach was more conservative than the normalized pressures spectra approach. This will always be the case due to the fact that the normalized pressure spectra method accounts for the transient nature of the loading, and the direct average PSD method does not. Thus the maximum response estimate from the normalized pressure spectra method will always be more accurate. The normalized pressure spectra approach overpredicted the response by ~220 percent. The direct average PSD overpredicted by ~270 percent. It should be noted that this only constituted the response measurements for five launches. Therefore, it is very likely that future strains will exceed the measured values, so these methods provide safety margins that are within the range typically used at Kennedy Space Center for design.

In conclusion, reference 1 shows how the normalized pressure spectra approach can be used to yield a very accurate estimate of the response when the loading is well understood. This paper has shown the use of the normalized pressure spectra and the direct averaged PSD methods. The methods provide reliable response estimates of a structure exposed to nonstationary random acoustic loading when the loading is assumed to be poorly understood. The resulting conservative response bounds were calculated, and shown to be acceptable in the current design practice.
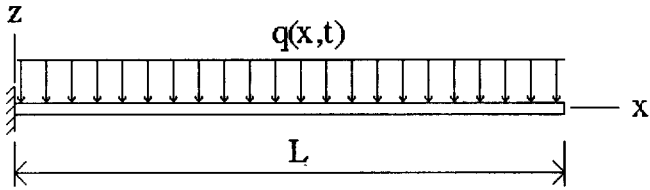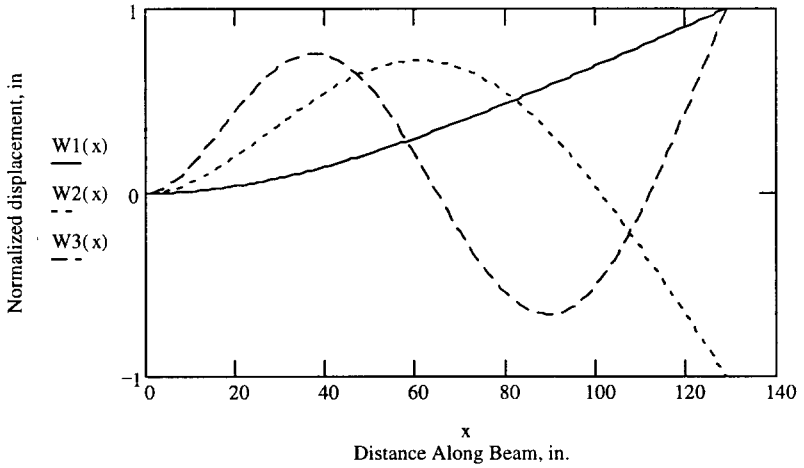
Figure 1. Beam Model

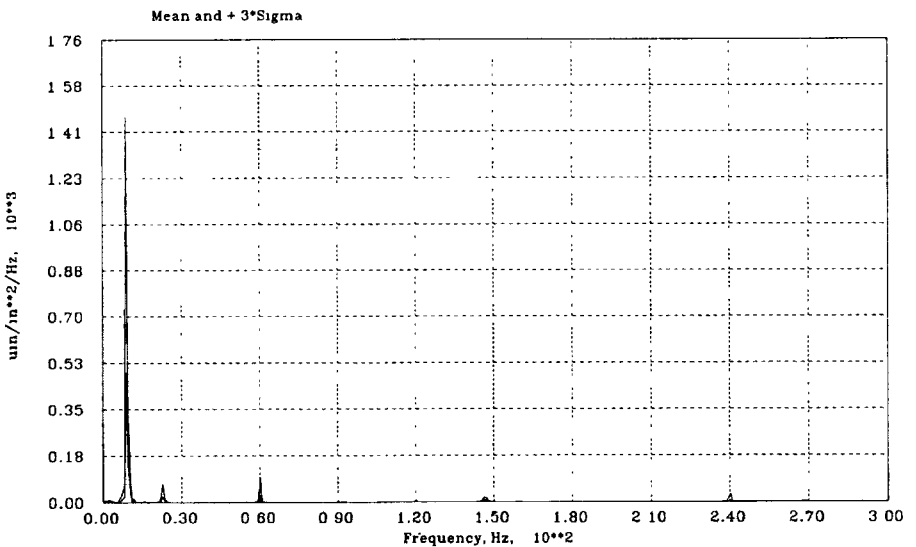Figure 2. First Three Analytical Bending Mode Shapes

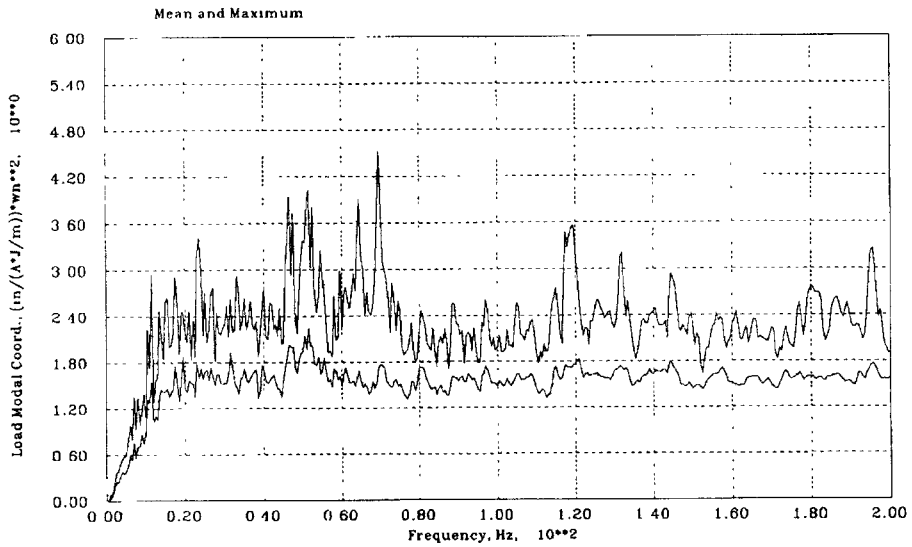Figure 3. Measured Strain Data Spectrum

Figure 4. Sample Load Modal Coordinate Plot

Table 1. Analytical Versus Test Natural Frequencies

| Mode Number | Natural Frequencies in Hz | | Modal Test Damping, % |
	Analytical	Modal Test	
1	8.94	8.84	0.45
2	56.0	54.3	0.17
3	157.	144.	0.17

Table 2. Joint Acceptances for First Three Modes (A=1)

Mode	Aj_{rr}
1	0.391
2	0.217
3	0.127

Table 3. Measured and Estimated Strains

| Location | Measured | Direct Averaged PSD | | Normalized Pressure | |
		Minimum	Maximum	Minimum	Maximum
14 inches	56	13	206	6	187

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