FIFTH INTERNATIONAL CONGRESS ON SOUND AND VIBRATION

DECEMBER 15-18, 1997

ADELAIDE, SOUTH AUSTRALIA

DYNAMIC BEHAVIOR OF THE AIRPLANE HORIZONTAL TAIL: SELECTION OF THE MODEL FOR CORRECT SIMULATION

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Abstract

Computer experiments were undertaken to study dynamic behavior of a horizontal tail during the flight in turbulent atmosphere. This work was carried out to select acceptable variant of fixation and clearance between the stabilizer and the elevator to avoid strikes.

The method of correction by elastic and inertial connection insertion in Galerkin formulation was used to obtain a set of ordinary differential equations representing the structure motion. Under consideration is a sequence of mathematical models making each other more accurate. The following factors were successively taken into account: bending of the stabilizer and the elevator, geometrical non-linearity of the structure response, effect of deformations of the whole airplane, and torsion of the elevator. This approach made it possible to assess how important and correct each of these model modifications for the estimations in question.

To check the simulation correctness, computed results were compared with available experimental data obtained by ground vibration and flight testing. The estimations in question allowed aerospace designers to select optimal modifications of the horizontal tail.

INTRODUCTION

Selection of correct mathematical models is one of the most important stages of computer simulation. Whatever the effectiveness of numerical methods in use, if an idealized model does not meet the corresponding real structure, all the results of computer experiments lose their value. To confirm the model correctness, one has to carry out the comparisons with experimental data.

Several models of some typical airplane horizontal tail making each other more accurate are considered here to determine the model capability that is necessary to ensure correct simulation. This work was necessary to select an acceptable variant of fixation and clearance between the stabilizer and the elevator to avoid strikes during the flight in turbulent atmosphere. Structure response to atmospheric turbulence and wind gusts for different aircraft speeds was estimated.

The following factors were successively taken into account:

- bending of the stabilizer and the elevator,

- geometrical non-linearity of the elevator response,
- effect of the motion of a whole airplane,
- rotation of the elevator.

This approach made it possible to assess how important and correct each of these model modifications from the viewpoint of correct estimation of horizontal tail dynamics.

To check the simulation correctness, computed results were compared with available experimental data obtained by ground vibration and flight testing. The conformity of dynamic characteristics and the presence of strike signs were applied as the correctness criteria.

The method of correction by elastic and inertial connection insertion in Galerkin formulation was used to obtain a set of ordinary differential equations representing the structure motion. To simulate and analyze behavior of this system, a special version of the DYSSAN software package [1,2] was used.

Since an object of the present study is selection of the mathematical model, any method of calculation might be employed. The aforementioned approach was chosen because of its flexibility, moderate performances of hardware to be used, and high rate of computations.

1. STRUCTURE MODELS

1.1. Nomenclature

a_s - coordinate of the s-th concentrated elastic connection,
$\mathbf{E}$ - identity matrix,
$E_i J_i = E_i J_i(x)$ - beam bending stiffness,
$E_{0i} J_{0i} = const,\ \mu_{0i} = const,$
I_2 - elevator moment of inertia,
J_{xe} - tail moment of inertia,
k_s - stiffness of the s-th concentrated elastic connection,
(s - number of a concentrated elastic connection),
l_i - beam length,
$M_{xe}(t)$ - moment of external forces which influence the tail,
$q_i = q_i(t)$ - external load per unit length of the beam,
t - time,
$w_i = w_i(x,t)$ - beam deflection,
x - spartial coordinate,
$\alpha_2(t)$ - angle of elevator rotation.
β - angle of rotation of the whole horizontal tail,
$\delta(x)$ - Kronecker delta,
ε - non-dimensional damping coefficient,
θ_2 - angle of elevator rotation,
$\mu_i = \mu_i(x)$ - beam mass per unit length of the beam,
$\xi_{ij} = \xi_{ij}(t)$ - normal coordinates,
($j=1,\ldots,N$ - numbers of natural modes).
σ_2 - distance between the elevator section gravity center and the axis of elasticity,
$\varphi_i = \varphi_i(x)$ *u* $\psi_i = \psi_i(x)$ - auxiliary functions,
($i=1$ for the stabilizer and $i=2$ for the elevator),
ω_{ij} - beam natural frequencies,
ω_{xe} - frequency of free angular oscillations of the tail,

In differential equations, a point denotes $\partial(\,)/\partial t$, a prime denotes $\partial(\,)/\partial x$.

A scalar product $(f_a(x), f_b(x))$ is $\int_L (f_a(x) \bullet f_b(x))\,dx$.

$u_{ij} = u_{ij}(x)$ are beam natural mode shapes for the following free vibration problems:

$$E_{01}J_{01}\, w_1'''' + \mu_{01}\, \ddot{w}_1 = 0 ,$$

$$E_{02}\, J_{02}\, w_2'''' + \mu_{02}\, \ddot{w}_2 = 0$$

(boundary conditions are described in Sect. 1.2).

Natural mode shapes are normalized to meet the equation $(u_{ij}, u_{ij}) = 1/\mu_{0i}$.

1.2. Boundary conditions

$w_1(0,t) = w_1'(0,t) = 0$ (clamped boundary conditions),

$w_1''(l_1,t) = w_1'''(l_1,t) = 0$ (free edge),

$w_2(0,t) = w_2''(0,t) = 0$ (simply supported boundary conditions),

$w_2''(l_2,t) = w_2'''(l_2,t) = 0$ (free edge).

1.3. Basic motion equations: bending

The elevator and the stabilizer are represented by beams suffered bending (Figure 1). Motion equations take into account bending of these beams and rotation of the elevator as an absolutely solid body (to take into account their mutual influence, concentrated elastic connections are defined at the points of elevator fixation):

$$\begin{aligned}
&(E_1J_1\, w_1'')'' + \mu_1\, \ddot{w}_1 = q_1 + \sum_s k_s\, (w_1(a_s,t) - w_2(a_s,t))\delta(a_s) ,\\
&(E_2J_2\, w_2'')'' + \mu_2\, \ddot{w}_2 = q_2 - \sum_s k_s\, (w_1(a_s,t) - w_2(a_s,t))\delta(a_s),\\
&I_2\ddot{\theta}_2 = - \sum_s k_s\, (w_1(a_s,t) - w_2(a_s,t))\, a_s .
\end{aligned} \tag{1}$$

Bending stiffness and mass of the beams are represented as:

$E_i J_i = E_{0i} J_{0i} + \varphi_i(x), \quad \varphi_i(l_i) = 0,$

$\mu_i = \mu_{0i} + \psi_i(x), \quad \psi_i(l_i) = 0 \quad (i = 1,2)$.

Obtained by using the Galerkin method with the modal expansion

$$w_i = \sum_{j=1}^{N} \xi_{ij}\, u_{ij} \tag{2}$$

is the following set of ordinary differential equations which describes the beam bending:

$$\begin{aligned}
&\ddot{\xi}_{1j} + \omega_{1j}^2 \xi_{1j} =\\
&(q_1 , u_{1j}) + \sum_s k_s\, ((w_1(a_s,t) - w_2(a_s,t))\, \delta(a_s), u_{1j}) - ((\varphi_1\, w_1'')'', u_{1j}) - (\psi_1\, \ddot{w}_1 , u_{1j}) ,\\
&\ddot{\xi}_{2j} + \omega_{2j}^2 \xi_{2j} =\\
&(q_2 , u_{2j}) - \sum_s k_s\, ((w_1(a_s,t) - w_2(a_s,t))\, \delta(a_s), u_{2j}) - ((\varphi_2\, w_2'')'', u_{2j}) - (\psi_2\, \ddot{w}_2 , u_{2j})
\end{aligned} \tag{3}$$

$(j = 1, \dots , N)$.

For the elevator, a rotation mode shape is added to expansion (2).

It is convenient to use the following notation:

$$\begin{aligned}
&(1 , u_{ij}) = s_{ij} ,\\
&(\varphi_i''\, u_{in}'', u_{ij}) + 2(\varphi_i'\, u_{in}''', u_{ij}) + (\varphi_i\, u_{in}'''', u_{ij}) = t_{ijn} ,\\
&(\psi_i\, u_{in}, u_{ij}) = v_{ijn} ,\\
&u_{1j}(a_s)\, u_{1n}(a_s) = u_{sjn,11} ,
\end{aligned} \tag{4}$$

$u_{2j}(a_s)\, u_{2n}(a_s) = u_{sjn,22}\,,$
$u_{1j}(a_s)\, u_{2n}(a_s) = u_{sjn,12}\,,$
$u_{2j}(a_s)\, u_{1n}(a_s) = u_{sjn,21}\,,$
where n is a current index, j is an equation number, i is a beam number, s is a number of a concentrated elastic connection. Scalar products which are referred to in notation (4) are to be calculated by any suitable numerical or analytical method.

Using notation (4), equations (3) may be rewritten in the matrix form as:

$$\begin{aligned}(\mathbf{E}+\mathbf{V}_1)\,\ddot{\xi}_1 + 2\varepsilon\mathbf{W}_1\dot{\xi}_1 + (\mathbf{W}_1^2+\mathbf{T}_1)\,\xi_1 &= q_1\mathbf{S}_1 + \sum_s k_s(\ \mathbf{U}_{s11}\xi_1 - \mathbf{U}_{s12}\xi_2)\,,\\ (\mathbf{E}+\mathbf{V}_2)\,\ddot{\xi}_2 + 2\varepsilon\mathbf{W}_2\dot{\xi}_2 + (\mathbf{W}_2^2+\mathbf{T}_2)\,\xi_2 &= q_2\mathbf{S}_2 + \sum_s k_s(-\mathbf{U}_{s21}\xi_1 + \mathbf{U}_{s22}\xi_2)\,.\end{aligned} \tag{5}$$

(Terms $2\varepsilon\mathbf{W}_i\dot{\xi}_i$ are added to take into account the effect of structural damping.)

To simplify (5), block matrices are used:

$$\mathbf{S}=\begin{bmatrix}\mathbf{S}_1\\ \mathbf{S}_2\end{bmatrix},\quad \mathbf{V}=\begin{bmatrix}\mathbf{V}_1 & 0\\ 0 & \mathbf{V}_2\end{bmatrix},\quad \mathbf{T}=\begin{bmatrix}\mathbf{T}_1 & 0\\ 0 & \mathbf{T}_2\end{bmatrix},\quad \mathbf{W}=\begin{bmatrix}\mathbf{W}_1 & 0\\ 0 & \mathbf{W}_2\end{bmatrix},$$

$$\mathbf{U}_s=\begin{bmatrix}\mathbf{U}_{s11} & -\mathbf{U}_{s12}\\ -\mathbf{U}_{s21} & \mathbf{U}_{s22}\end{bmatrix},\quad \xi=\begin{bmatrix}\xi_1\\ \xi_2\end{bmatrix}.$$

Finally, equations (5) may be rewritten as

$$(\mathbf{E}+\mathbf{V})\,\ddot{\xi} + 2\,\varepsilon\mathbf{W}\,\dot{\xi} + (\mathbf{W}^2+\mathbf{T})\,\xi = q\,\mathbf{S} + \sum_s k_s\mathbf{U}_s\xi\,.$$

1.4. Geometrical non-linearity

In case of deflections which are close to the elevator thickness, special corrections taking into account geometrical non-linearity of response are added to equations (6). It is equivalent to complementing system (1) with terms $F(x)w''$ where $F(x)$ is the tension created by stretching of the beam due to bending.

1.5. Effect of deformation of the whole airplane

Deformation of the whole airplane has some influence on dynamics of the horizontal tail. To estimate how important this factor, an elastic connection between the tail and the other part of an airplane, which allowed rotation, was added to the structure model.

Motion of the whole horizontal tail is described by the equation:

$$\ddot{\beta} + 2\,\varepsilon\,\omega_{x\theta}\,\dot{\beta} + \omega_{x\theta}^2\,\beta = M_{x\theta}(t)\,/\,J_{x\theta}$$

To simplify the equations, it is convenient to consider bending of the beams in a non-inertial coordinate system strictly fastened to the horizontal tail. Added to the motion equations are the distributed inertia forces $\mu_i(x)\ddot{\beta}(t)x$ $(i=1,2)$. (Since only transversal deflections are considered here, there is no need to take into account the Coriolis inertia force and longitudinal part of the transfer inertia force).

It is assumed that tail vibrations are excited by the homogeneous random loading fields which influence the right and left tail parts independently.

1.6. Rotation of the elevator

To estimate how important the factor of torsion oscillations, motion equations were modified to take into account rotation of the elevator. An equation describing rotation of this unit was added to the dynamic system under consideration. It included the following distributed load conditioned by the interaction of bending and torsion oscillations: $\mu_2\sigma_2\ddot{w}_2$. Added to the quation describing the elevator bending oscillations was the term $-\mu_2\ \sigma_2\ \ddot{\alpha}_2$.

2. EXTERNAL LOAD

Vertical velocity component v due to atmosphere turbulence or wind gust cause the additional external pressure p that can be calculated as follows:

$$p = c_\alpha'\rho Uv/2,$$

where $c_\alpha'=dc_y/d\alpha$ - the derivative of lift force coefficient with respect to attack angle, ρ - air density, U - aircraft velocity.

The external load at flight in a turbulent atmosphere was approximated by a uniform pressure field changing in accordance with a harmonic law on background noise. Frequencies and mean-root values of pressure fluctuations were 0.2, 0.2 and 0.3 Hz and 156, 200 and 238 Pa for aircraft velocity 586, 800 and 100 km/h, correspondingly.

The action of single vertical gust with the velocity 15 m/s and extent 30 m was approximated by uniform distributed symmetric pressure pulses of triangular form with maximum values and duration as follows: at 586 km/h - 3668 Pa and 0.184 s, at 800 km/h - 4995 Pa and 0.135 s, at 1000 km/h - 6255 Pa and 0.108 s.

The specified parameters of external load were selected thus way to have an acceptable approximation to available experimental data [3].

3. COMPUTER EXPERIMENTS

Computer experiments were carried out for 2- and 3-point fixation of the elevator. Five natural modes (***N=5***) were used to represent motion of both the elevator and the stabilizer.

Typical positions of the system under consideration are shown in Figures 2 and 3 (deflections were multiplied by a special factor to get acceptable views). Given in Figures 4 and 5 are distributions of root-mean-square deflections during the flight in turbulent atmosphere. Thick curves describe differences between the displacements of the stabilizer back edge and the elevator front edge. Since the greatest differences were at the centers of spans between the fixation points, these points were used as representative ones.

As illustrations, typical deflection power spectral densities (for 2- and 3-point fixation) and responses for wind gusts are presented in Figures 6-7 and 8-9 correspondingly.

Table 1 shows how the estimated greatest differences between the stabilizer and elevator displacements depend on the model employed in computer experiment.

Acceptable clearances were calculated using the data of Table 1 and geometrical considerations. Results of technical surveys of the stabilizer and elevator edges after test flights (strike marks were looked for) were compared with the aforementioned estimations. Such a comparison makes it possible to conclude:

- model 3 does not ensure correct simulation for the flight in turbulent atmosphere in case of the 2-point fixation;
- model 1 gives obviously unreal results in case of a wind gust.

Calculated ranges for natural frequencies of the tail and its natural frequencies obtained by ground vibration testing are presented in Table 2. Their comparison shows:

- on the whole, bearing in mind capability of the model in use, computer simulation that has been carried out satisfactorily predicts the experimental frequencies;
- capability of model 4 is sufficient for correct representation of dynamic characteristics of the structure under consideration.

Data presented in Tables 1 and 2 show that to obtain correct estimations of clearances between the elevator and the stabilizer,

- model 1 is sufficient for simulation of their dynamics in turbulent atmosphere (continuous turbulence);
- model taking into account their bending, geometrical non-linearity of elevator response, and elevator rotation is necessary for simulation of response to a wind gust.

Estimations of the clearances under consideration allowed aerospace designers to select optimal modifications of the horizontal tail.

Table 1. Differences between the stabilizer and elevator displacements
(as the ratio ***greatest difference / maximal elevator thickness***)

Notation: CT-continuous turbulence, WG-wind gust, 2p- 2-point fixation, 3p- 3-point fixation

Model No.	Model: factors taken into account	Type of loading	Airplane speed, km/hour					
			586		800		1000	
			2p	***3p***	***2p***	***3p***	***2p***	***3p***
1	Bending	***CT***	0.09	0.01	0.10	0.01	0.12	0.01
		WG	1.42	0.07	1.85	0.09	2.23	0.12
2	Bending + geometrical non-linearity	***CT***	0.09	0.01	0.10	0.01	0.12	0.01
		WG	0.20	0.07	0.23	0.09	0.27	0.12
3	Bending + geometrical non-linearity + airplane deformation	***CT***	0.10	0.01	0.12	0.01	0.13	0.01
		WG	-	-	-	-	-	-
4	Bending + geometrical non-linearity + airplane deformation + elevator rotation	***CT***	0.09	0.01	0.10	0.01	0.12	0.01
		WG	0.09	0.07	0.10	0.09	0.16	0.12

Table 2. Comparison of experimental and calculated vibration frequencies *(2-point fixation)*

Mode No.	Calculated natural frequencies (accuracy ±2.5Hz), Hz	Natural frequencies obtained by ground testing, Hz	Mode shape type
0	9.8	8 - 9	deformation of the airplane
1	17.1	20	stabilizer bending - mode 1
2	31.8	30 - 34	elevator bending
3	58.1	55	elevator rotation
4	75.7	78 - 79	stabilizer bending - mode 2

CONCLUSIONS

The following qualitative conclusions may be drawn:

1. Bending of elements of a horizontal tail is the most important model factor that is sometimes sufficient to ensure correct estimation of clearances between these elements during the flight in turbulent atmosphere.

2. To estimate correctly the response of a horizontal tail to a wind gust, it may be necessary to take into account bending of its elements, geometrical non-linearity of elevator deformation, and elevator rotation.
3. To ensure correct representation of dynamic characteristics of a horizontal tail, it may be necessary to take into account bending of its elements, geometrical non-linearity of elevator deformation, elevator rotation, and the effect of deformation of the whole airplane.
4. A beam structure model and the method of correction by elastic and inertial connection insertion are useful for simulation of dynamics of aircraft cantilever structures.

References

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2. L. S. Kuravsky, *«The DYSSAN package: application to the problem of analysis and optimization of acoustic vibrations»,* Journal of Technical Acoustics, 1995, vol. 2, No. 4, p. 3 - 12.
3. J. Taylor, Manual on aircraft loads, Pergamon Press, 1971.

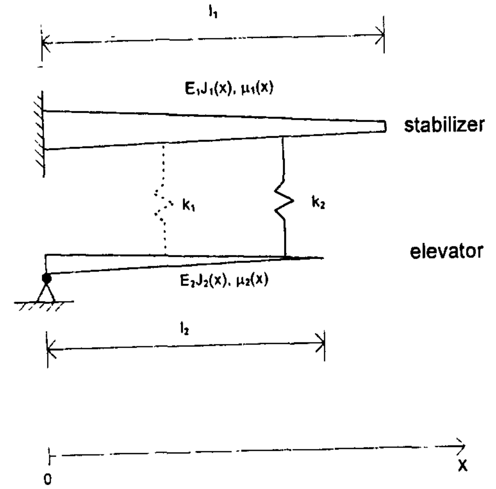

Fig. 1

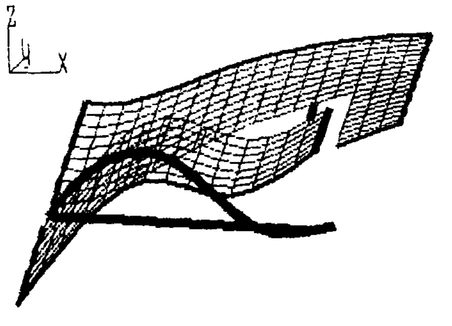

Fig. 2

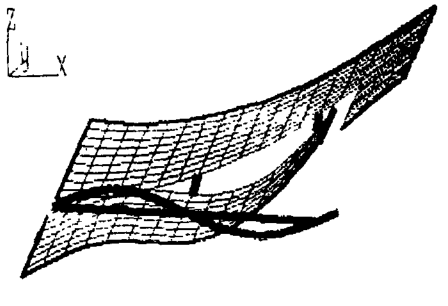

Fig. 3

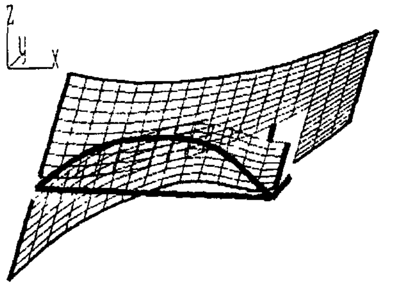

Fig. 4

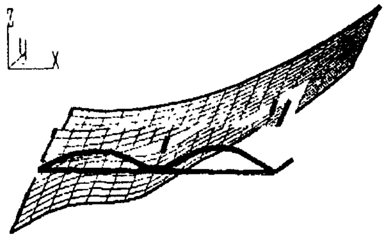

Fig. 5

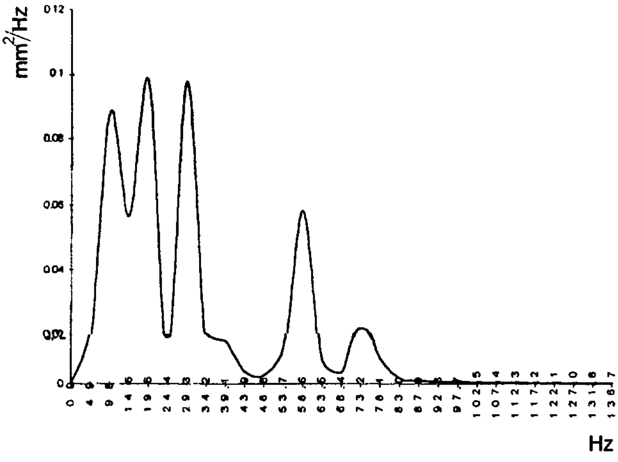

Fig. 6

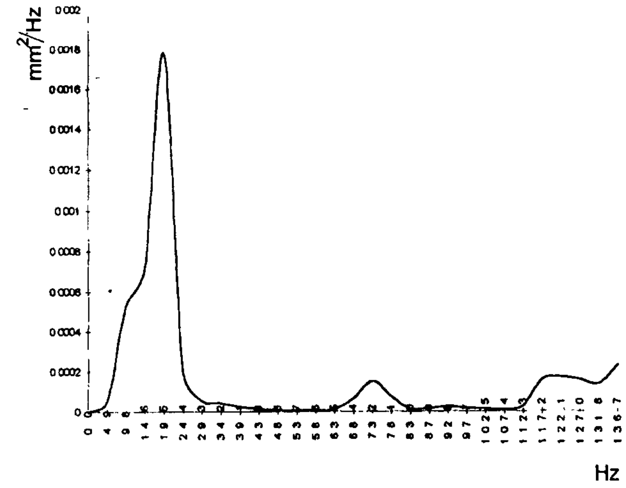

Fig. 7

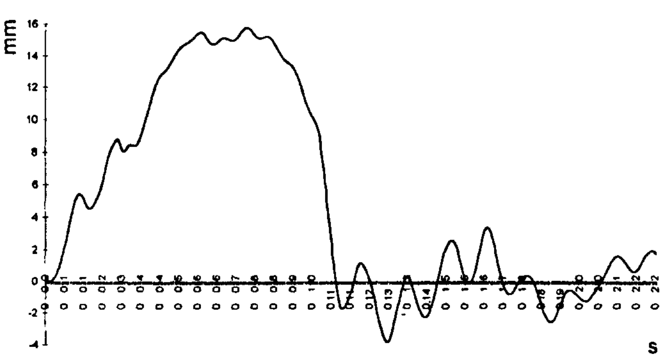

Fig. 8