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OPTIMAL CONTROL OF A SPACE ROBOT BY THE NONHOLONOMIC STRUCTURE

Koichi Matsuda, Yoichi Kanemitsu, and Shinya Kijimoto

Department of Intelligent Machinery & Systems, Kyushu Univeristy
6–10–1 Hakozaki, Fukuoka 812, JAPAN

ABSTRACT

Owing to the nonholonomic structure, a free–flying space robot can reorient its attitude only by actuating the manipulators. We formulate the approach to plan the optimal path of the manipulator movement to achieve such reorientation. This formulation has been done as an optimal control problem in a more general way than that of the problem to have been solved so far. That is, the approach can afford to solve the case where the initial and final posture of the manipulators are different and where the optimal solution is not a smooth and continuous function of time. The optimal control problem is converted via discretization with high–order integration methods into a finite dimensional problem and solved by the periodically preconditioned conjugate gradient–restoration algorithm.

INTRODUCTION

Free–flying space robots would be conceived to play many important roles in future developments in space activity, and much attention has been devoted to their dynamics and control in recent years. Being different from the robots on the earth, the space robots cannot be fixed anywhere. As a special feature, the space robots have nonholonomic mechanical structure; owing to this structure, a free–flying space robot can reorient its attitude only by actuating the manipulators.

From this point of view, there have been many researches[4–12] on the optimal path planning of the manipulator movement to achieve such reorientation. Most of these researches, however, have concentrated on the situation where the initial posture of the manipulator are the same as the final one. This limited situation is taken because this optimal control problem is significantly simplified to have a periodical solution and corresponds to the falling–cat phenomenon, that is, a cat changes her shape in such a way as to land on her feet when she is dropped from upside–down with no angular momentum. Such optimal path–planning problems have been solved by many approaches based on a Fourier series. The original infinite–dimensional problem is reduced to be a finite one using the Fourier series. The solution to the original problem would not always be a smooth and continuous function, that is, the solution is not necessary represented by a Fourier series owing to selecting the performance index. Moreover, many powerful computational methods have recently developed for solving optimal control problems. In particular, we would like to describe briefly the periodically preconditioned conjugate gradient–restoration algorithm[1–3] to be used in this paper. The infinite dimensional problem is converted via discretizing the problem with high–order integration methods into a finite dimensional problem, that is, a mathematical programming problem. The conjugate gradient–restoration algorithm is applied to the resulting mathematical programming problem while an optimal preconditioner is periodically used to accelerate the convergence. The preconditoner is the positive–definite reflection of the Hessian matrix and efficiently constructed using a singular–value decomposition.

In this paper, we numerically solve this optimal control problem when the initial and final postures of the manipulator are different, not using the periodicity at all. It would be very difficult to optimize the manipulator movement without the periodicity owing to the nonlinearities of the robot. The problem is solved by the periodically preconditioned conjugate gradient–restoration algorithm. Therefore, there is no limitation on the solution to this problem and more general problem is solved by the present approach, compared with the problem to have been solved so far. A simple space–robot model is used to demonstrate the validity of the present approach.

SYSTEM MODEL AND OPTIMAL CONTROL

Figure 1 shows the model of the space robot treated in this paper. The robot contains a main body, a bar, and two point–masses. The bar is installed at the center of mass of the main body and rotates around the point in the two–dimensional plane. Moreover, two point–masses move on the bar symmetrically around the attached point; each point–mass exists apart from the rotational center on the bar by the distance l. We wish to reorient the body of the robot without any external force while we bring the bar rotation and the point–mass movement to the desired value. The kinematics equations of the robot can be derived by conserving the angular momentum around the center of mass at zero:

$$I_0\dot{\theta}_0 + \left(I_1 + ml^2\right)\left(\dot{\theta}_0 + \dot{\varphi}_1\right) = 0 \tag{1}$$

where I_0 denotes the inertia moment of the main body around the center of mass, I_1 the inertia moment of the bar around the center of mass, m: the mass of each point–mass, l: the distance of the point–mass apart from the rotational center, φ the attitude angle of the main body, and θ the relative rotational angle of the bar to the main body; we assume that the robot is initially at rest. Equation (1) may be rewritten by

$$\dot{\theta}_0 = I(l)\dot{\varphi}_1 \tag{2}$$

with

$$I(l) = -\frac{I_1 + ml^2}{I_0 + I_1 + ml^2} \tag{3}$$

Now we would like to achieve the attitude reorientation of the robot with the minimum control cost; the variables $\dot{\varphi}$ and $\dot{l}$ are selected as the present control variables. Therefore, we might introduce the performance index to achieve such reorientation as follows:

$$J = (1/2)\int_0^{T_f} u^T u dt \tag{4}$$

with the boundary conditions

$$x(0) = x_1^* \tag{5}$$

$$x\left(T_f\right) = x_N^* \tag{6}$$

where the state vector is $x = (\theta \quad \varphi \quad l)^T$ and $u = (p \quad q)^T = (\dot{\varphi} \quad \dot{l})^T$. Using these notation, Eq. (2) is rewritten by

$$\dot{x} = \begin{pmatrix} I & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} p \\ q \end{pmatrix} = A(l)u \tag{7}$$

The problem is to find the optimal control input u to minimize the performance index J subject to the constraints, Eqs. (5)–(7).

SOLUTION TECHNIQUE AND NUMERICAL RESULTS

Direct transcription via a high–order integration converts the preceding problem as follows:

Minimize

$$F(z) = (1/2)\sum_{i=1}^{N} \beta_i u_i^T u_i \Delta t \tag{8}$$

with respect to z subject to

$$\Phi(z) = \begin{bmatrix} x_1 - x_1^* \\ x_2 - x_1 - \Gamma(x_1, u_1, u_2)\Delta t \\ x_3 - x_2 - \Gamma(x_2, u_2, u_3)\Delta t \\ \vdots \\ x_N - x_{N-1} - \Gamma(x_{N-1}, u_{N-1}, u_N)\Delta t \\ x_N - x_N^* \end{bmatrix} = 0 \tag{9}$$

where

$$z = \begin{bmatrix} x_1^T & x_2^T & \cdots & x_N^T & u_1^T & u_2^T & \cdots & u_N^T \end{bmatrix}^T$$

and the integration weightings β_i's are determined by the Simpson's rule in this paper. We select the fourth–order Runge-Kutta formula as a high–order integration method to give

$$\Gamma(x_{i-1}, u_{i-1}, u_i) = \frac{1}{6}\begin{bmatrix} p_{i-1}J_{i-1} + p_i K_{i-1} \\ 3(p_{i-1} + p_i) \\ 3(q_{i-1} + q_i) \end{bmatrix} \qquad i = 2, 3, \cdots, N \tag{10}$$

where

$$J_{i-1} = I(l_{i-1}) + I(l_{i-1} + \frac{\Delta t}{2} q_{i-1}) + I[l_{i-1} + \frac{\Delta t}{4}(q_{i-1} + q_i)] \tag{11}$$

$$K_{i-1} = I(l_{i-1} + \frac{\Delta t}{2} q_{i-1}) + 2I[l_{i-1} + \frac{\Delta t}{4}(q_{i-1} + q_i)] \tag{12}$$

with $x_i = x(t_i)$ and $u_i = u(t_i)$ and the time t_i's satisfy the relation:

$$t_1 = 0, \quad t_N = T_f, \quad t_{i+1} = t_i + \Delta t \tag{13}$$

It may be noted that the second and third components of Γ lead to the linear interpolation for the control variables. Moreover, the augmented function J for the present problem is given by

$$J(z, \lambda) = F(z) + \lambda^T \Phi(z) \tag{14}$$

where

$$\lambda = \begin{bmatrix} \lambda_1^T & \lambda_2^T & \cdots & \lambda_{N+1}^T \end{bmatrix}^T \tag{15}$$

Under this setting, the optimal solution must satisfy simultaneously the equations

$$J_z(z, \lambda) = 0 \tag{16}$$

$$\Phi(z) = 0 \tag{17}$$

where

$$J_z(z, \lambda) = F_z(z) + \Phi_z \lambda \tag{18}$$

and the subscript z is the differentiation with respect to z. Furthermore, the Hessian of the transcribed problem is given by

$$\mathrm{H}(z, \lambda) = J_{zz}(z, \lambda) = F_{zz}(z) + \sum_{j=1}^{N+1} \Phi_{jzz} \lambda_j \tag{19}$$

where Φ_j and λ_j are the j–th scalar components of Φ and λ, respectively. An optimal preconditioner is periodically applied to the conjugate gradient–restoration algorithm to accelerate the convergence. The preconditioner is the positive–definite reflection of the Hessian matrix evaluated at the beginning of each conjugate gradient sequences. The preconditoner is efficiently constructed using a singular–value decomposition of the Hessian.

Now we solve this mathematical programming problem via the periodically preconditioned conjugate gradient–restoration algorithm. The algorithm is implemented in MATLAB, and the one dimensional search is conducted using a golden section search with the tolerance of 10^{-6}, and the least positive singular value allowed in computing the preconditioner is 10^{-12}. We determine the convergence of the algorithm using the following relations:

$$\|\Phi(z)\| \le 10^{-6} \text{ and } \|J_z(z, \lambda)\| \le 10^{-3} \tag{20}$$

Moreover, the restoration phase of the algorithm is terminated using the first of Eq. (20).

Now we would like to show the results of the numerical optimization. The physical parameters are given as $m = 1$, $I_0 = 2$, $I_1 = 1$, N=100, and T_f=100. Figure 2 shows the optimal time histories of the state variables with the boundary conditions, $x_1^* = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}^T$ and $x_N^* = \begin{bmatrix} \pi/8 & \pi/8 & 0.1 \end{bmatrix}^T$. There exists no periodical behavior in the histories. For comparison, we show the optimal states' histories when the initial and final postures of the bar and the point–masses are the same, that is, $x_1^* = [0 \quad 0 \quad 0]^T$ and $x_N^* = [\pi/8 \quad 0 \quad 0]^T$.The histories of φ and l are shown to be periodical.

CONCLUSION

Owing to the nonholonomic structure, a free–flying space robot can reorient its attitude only by actuating the manipulators. We formulate the approach to plan the optimal path of the manipulator movement to achieve such reorientation. This formulation has been done as an optimal control problem in a more general way than that of the problem to have been solved so far. That is, the approach can afford to solve the case where the initial and final posture of the manipulators are different and where the optimal solution is not a smooth and continuous function of time. The optimal control problem is converted via discretization with high–order integration methods into a finite dimensional problem and solved by the periodically preconditioned conjugate gradient–restoration algorithm. A simple space–robot is used to exemplify the present approach and to demonstrate the validity.

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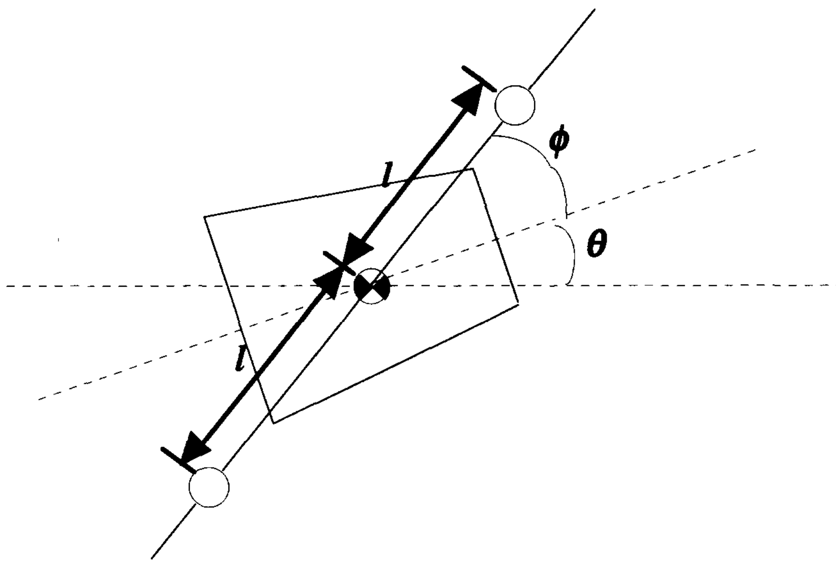

Fig.1 System Model

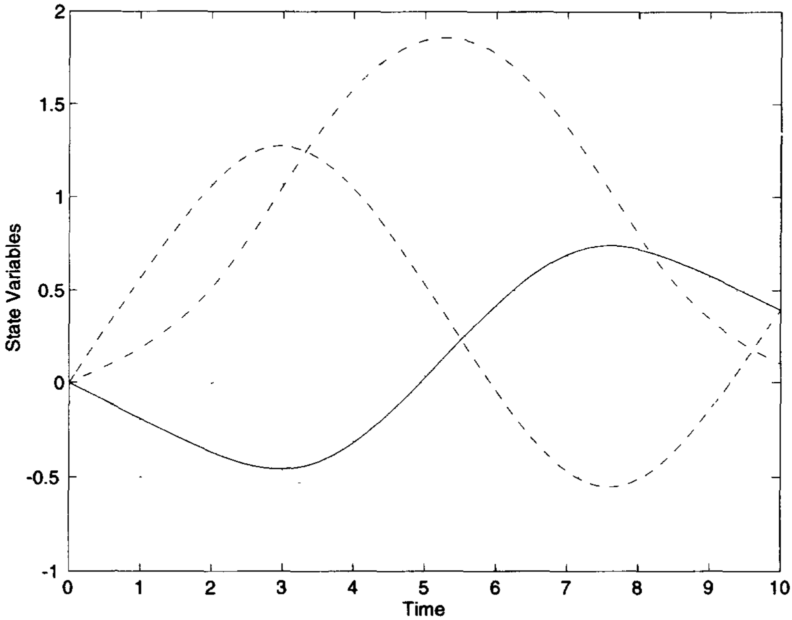

Fig.2 The Optimal Time Histories of θ (Solid), φ (Dashed), and l (Dash–Dotted) without Periodicity

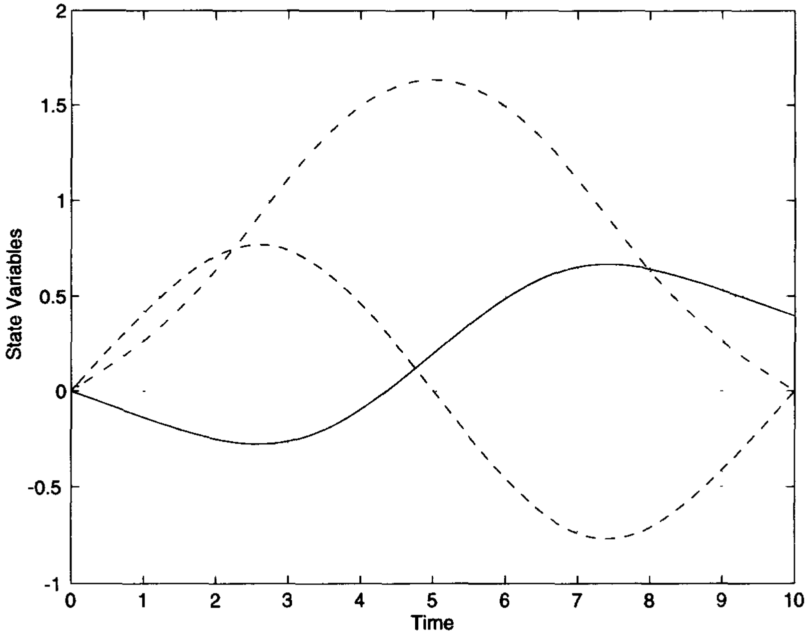

Fig.3 The Optimal Time Histories of θ (Solid), φ (Dashed), and l (Dash–Dotted) with Periodicity