FIFTH INTERNATIONAL CONGRESS ON SOUND AND VIBRATION
DECEMBER 15-18, 1997
ADELAIDE, SOUTH AUSTRALIA

SMART NONLINEAR STRUCTURE OF VIBRATION ISOLATION

V. I. BABITSKY, A. M. VEPRIK

Department of Mechanical Engineering, Loughborough University,
Loughborough, Leicestershire LE11 3TU, UK

Ricor, En Harod Ihud, 18960, Israel

ABSTRACT

Modern condensed designs of mechanical systems often do not permit the effective use of a soft undamped flexural suspension as a universal solution for the limitation of vibratory energy transmission from the machine to its foundation and vice versa. As soon as an application involves exposure to a gross vibration disturbance of a shock or random broadband nature, containing dangerous low-frequency components, excessive mobility of the machine can overstress the flexural suspension and damage the machine or its surrounding. Therefore the motion limiters - bumpers - are usually the integral component of a vibration isolation arrangement. The presence of bumpers turns the vibration isolator into a potential vibro-impact structure where the possibility of strongly nonlinear resonance behaviour and the appearance of gross impact accelerations becomes the subject of serious concern.

The most typical method of vibration control consists of adding a considerable amount of damping to the machine-to-foundation resilient coupling and by sufficient increase of the free oscillation space. This reduces partially a low-frequency resonant response but originates the degradation of vibration isolation in the high-frequency band and does not eliminate completely the possibility of rare and intensive collisions due to the gross preimpact velocities in the increased sway space.

The idea of this novel vibration protection arrangement is based on co-operative use of low-frequency, undamped vibration isolation and soft, but heavily damped bumpers installed with minor oscillation space in such a way that they provide the following:

- impactless operation of machine at any space orientation with the highest possible vibration isolation ratio,
- safe and soft limiting of any defined by design excessive deflections of machine originated by ambient vibration,
- impossibility of excitation of self-sustained resonance at any casual ambient vibration disturbance.

The study of the dynamics and optimisation of a bumpered system of vibration isolation was conducted and applied to a linear compressor of a split Stirling cryocooler Ricor model

K529H for a focal plane cooling of an electro-optical device. The criterion for the choice of optimal bumpers was formulated.
As a result, the adequately sized bumpered vibration protection with the 93% vibration isolation at the frequency of four times more than the natural frequency arrangement was designed. It provided also the soft and safe restriction of excessive motion originated by ambient vibration and shock.
The vibration isolation arrangement of cryocooler met the requirement of low self-induced force transmission and passed the shock and broadband random vibration environmental testing in accordance with MIL STD-810E without degradation in performance.

INTRODUCTION

An universal solution for the limitation of vibratory energy transmission from machine to its foundation and vice versa is the proper usage of flexural suspension. If the natural frequencies of the resiliently-suspended machine are located well below the lowest frequency in the excitation spectra, a significant suppression of vibration transmission may be achieved in a particular high-frequency band. In this case, it is recommended to use a soft and undamped flexural suspension.
In fact, such a simple vibration isolation arrangement is feasible in a limited number of "quiet environment" applications. As soon as an application involves exposure to high levels of environmental vibration disturbance of a shock or broadband random nature, the excitation spectra contains dangerous low frequency components. As a result of possible large dynamic response appearance, the flexible suspension must be supplied with considerable free oscillation space to avert the internal impacts against the machine surroundings [1]. Modern condensed mechanical design, however, often does not permit such a possibility. The demands of the limited space and resilient element integrity obligates the close control of vibration amplification at quasi-resonance or at shock.
The most popular solution is to increase the safe free travel and suspension loss factor as much as is permitted by transmissibility ratio at the operational frequency and by design. It is considered that a loss factor value of 0.3 is an optimal one, providing an amplification at the natural frequency of about 2 and a vibration isolation of 73% at the frequency of four times that of the natural frequency. Increased free travel minimises the probability of undesirable strike-through of vibration isolation and controls the existence of nonlinear resonances.
Such a design produces a spacious, heavily damped vibration protection arrangement, where a huge loss factor of resilient elements negates the quality of vibration isolation in a high-frequency band. Moreover, the increase of loss factor does not furnish in a full measure the duty of the limitation of the excessive deflections due to ambient shock and random vibration:

- In the quasi narrow-band dynamic response of the flexibly suspended machine exposed to the random broadband vibration, relatively rare but gross outbursts of displacement may appear.
- The amount of damping in the flexural elements has very little influence on the peak response value of the flexibly suspended machine exposed to eventual ambient shock.

In order to protect a machine and its environment from unavoidable excessive motions, the pre-designed deflection limiters - bumpers - are usually an integral component of vibration isolation. Having sufficient stiffness, bumpers can effectively trim machine deflections in an emergency. In the framework of traditional approach to the design of bumpered vibration isolation arrangement, the impact is considered as a harmful environmental factor only. The presence of such bumpers turns the vibration isolation arrangement into the potentially strongly nonlinear (vibro-impact) system with new, sometimes unfavourable, dynamic qualities [2]. Main or even sub-harmonic vibro-impact resonance at a frequency range located well above the system's natural frequency may arise after casual vibration disturbance or frequency pulling at machine start-up. With the increase in free oscillation space there exists the danger of the appearance of gross impulsive accelerations due to large pre-impact velocities.

In the present paper the authors have made an attempt to consider soft impact as a desirable tool of safe restriction of excessive mobility of a machine. In this context such parameters as free oscillation space, bumper stiffness and loss factor are treated as a possible key for the total improvement of quality of vibration protection system. The synthesised optimal structure demonstrates some smart features of controllable systems of vibration isolation [3].

NOVEL BUMPERED VIBRATION PROTECTION ARRANGEMENT

The main idea of the novel vibration isolation arrangement is based on the essential use of undamped low-frequency vibration isolation of a high-frequency machine in combination with optimally softened and damped bumpers. They are installed with minor free travel.

The minor sway space has to ensure low-powered collisions due to the subsequent decrease of preimpact velocities. On the other hand, the clearance value has to be sufficient to allow machine operation without impacts in any space orientation as long as ambient vibration is absent. Thus, the sway space has to be greater than at least the aggregate of machine's maximum static deflection and of operational amplitude.

This way the highest possible vibration isolation ratio typical for the undamped low-frequency vibration isolator may be maintain in the high-frequency band as long as the ambient vibration is absent.

Meanwhile, such a quiet dynamic behaviour is not only possible one [2]. As a result of occasional vibration disturbance initiating a machine collision with the bumper, the impactless operation may lose its stability and self-sustained vibro-impact resonance may be spring up. Depending upon the self-induced forcing power, the consequent oscillations of main or even sub-harmonic nonlinear resonant type may be supported and as a result, the vibration protection system will totally lose its vibration isolation quality.

The primary concern of these vibration isolation arrangement is the proper choice of the parameters of the bumper providing the vibration protection system with the ability to prevent a vibro-impact resonance arising from initial small, impactless oscillation within the minor free oscillation space.

The secondary concern of the design of vibration isolation arrangement is the control of peak accelerations under ambient broadband random vibration or gross shock.

The choice of the minor free travel of machine, of the proper bumper stiffness and of the loss factor may provide the additional softness and safety to motion; thus the damaging impulsive accelerations will be smoothed.

Relative to a typical high-frequency self-induced forcing, these "smart" vibration isolation scheme is lightly damped and highly responsive with the highest possible vibration isolation ratio; yet it achieves the necessary apparent damping and stiffness under gross ambient disturbances such as shock and broadband random vibration.

The result is that the considered vibration protection arrangement provides isolation superior to a theoretical low-frequency undamped linear isolator, yet softly eliminates almost all gross dynamic responses.

Mentioned advantages are obtained mainly due to the removal of motion limitation and damping duties from the resilient elements and by concentrating these duties in the bumpers.

The choice of optimal bumper is usually accomplished with the proper optimal criterion. In the framework of the present research the following criterion emphasising the indicated advantages will be used:

$$I = \sqrt{\frac{1}{2}\left[\left(\frac{\hat{X}}{\hat{X}_{lin}}\right)^2 + \left(\frac{\hat{A}}{\hat{A}_{ex}}\right)^2\right]}, \tag{1}$$

where $\hat{X}$ and $\hat{A}$ symbolise the peak values of vibration isolator deflection and of acceleration respectively, $\hat{X}_{lin}$ denotes the peak deflection of corresponding linear vibration isolator without bumpers and $\hat{A}_{ex}$ signifies the peak value of the excitation acceleration. It can be seen that for the machine rigid attachment $I \approx 0.71$

1. Figure 1 depicts schematically a machine of mass M supported by a resilient element presented as combination of springy member K and viscous damping member B. Machine vibration is originated by dynamic force $Q(t)$. In the case of machine kinematic excitation with the foundation disturbance given by acceleration $A_{ex}(t)$ the dynamic force will be considered as a inertial force $Q(t)=-A_{ex}(t)\cdot M$. Machine motion $X(t)$ relative to foundation is then limited symmetrically by bumpers modelled in turn as combination of viscous damping member B_1 and springy member K_1 being installed on the same foundation with the sway space Δ with respect to the static equilibrium position of the machine.

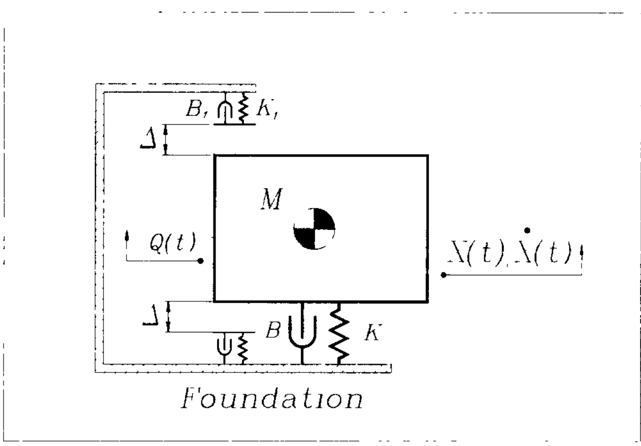


Figure 1.

The *d'Alambert* equation of motion may be obtained in the following form:

$$M\ddot{X} + B\dot{X} + KX + \Phi\left(X, \dot{X}\right) = Q(t) \tag{2}$$

where: $\ddot{X}, \dot{X}$ are machine acceleration and velocity respectively, $\Phi\left(X, \dot{X}\right)$ is the threshold-type force of impact interaction [2]:

$$\Phi\left(X,\dot{X}\right)=\begin{cases} K_1\left(|X|-\Delta\right)+B_1\dot{X} \;\; \forall \;\left(X\geq\Delta\wedge\dot{X}>0\vee X\leq-\Delta\wedge\dot{X}<0\right) \\ K_1\left(|X|-\Delta\right) \;\; \forall \;\left(X\geq\Delta\wedge\dot{X}<0\vee X\leq-\Delta\wedge\dot{X}>0\right) \\ 0 \;\; \forall \;|X|<\Delta \end{cases} \tag{3}$$

Dividing both parts of (1) by M we obtain:

$$\ddot{X} + 2\zeta\Omega\dot{X} + \Omega^2 X + f(X, \dot{X}) = \mathrm{Q}(t) / M \tag{4}$$

where the following notations are used:

$$\Omega = \sqrt{\frac{K}{M}}\ , \quad \Theta = \frac{\Omega}{2\pi}, \zeta = \frac{B}{2M\Omega}, \Omega_1 = \sqrt{\frac{K_1}{M}}, \Theta_1 = \frac{\Omega_1}{2\pi}, \ \zeta_1 = \frac{B_1}{2M\Omega_1},$$

$$f(X, \dot{X}) = \begin{cases} \Omega_1^2\left(|X| - \Delta\right) + 2\zeta_1\Omega_1\dot{X} \quad \forall \ |X| \geq \Delta \wedge \operatorname{sgn}(X)\cdot\operatorname{sgn}(\dot{X}) > 0 \\ \Omega_1^2\left(|X| - \Delta\right) \quad \forall \ |X| \geq \Delta \wedge \operatorname{sgn}(X)\cdot\operatorname{sgn}(\dot{X}) < 0 \\ 0 \quad \forall \ |X| < \Delta \end{cases} \tag{5}$$

where parameters Θ and ζ define the undamped natural frequency and loss factor of vibration isolator, Θ_1 and ζ_1 are the apparent undamped natural frequency and loss factor of bumper being related to machine mass.

2. In the present paper the typical case of bumpered vibration protection arrangement as applied for a split Stirling cryocooler will be discussed [4]. In considering the case of the Ricor model K529H cryocooler, the linear compressor with the self-induced force containing in its spectra a relatively gross pure-frequency component 14N RMS@60Hz was the dominant disturbing factor of the electro-optical device. It was established experimentally that quiet infra-red array operation can be achieved if the force export will be less than 1N RMS on the fundamental frequency and less then 0.1 N RMS on the higher-order harmonics. In order to achieve such an attenuation factor, the lightly-damped flexible machine suspension with the undamped natural frequency of Θ=15Hz (that is four times less that the operational one) and the loss factor less than z =0.05 has to be used (see, for instance, the curves of absolute transmissibility in [5]).

In the new arrangement (see Figure 2) the compressor unit of mass 0.5 kg (1) was suspended from its base (2) by all-metal undamped planar soft resilient springs (3). Excessive motion of the compressor, originated by possible base acceleration, was limited by elastomeric bumpers (4) being installed on the same base.

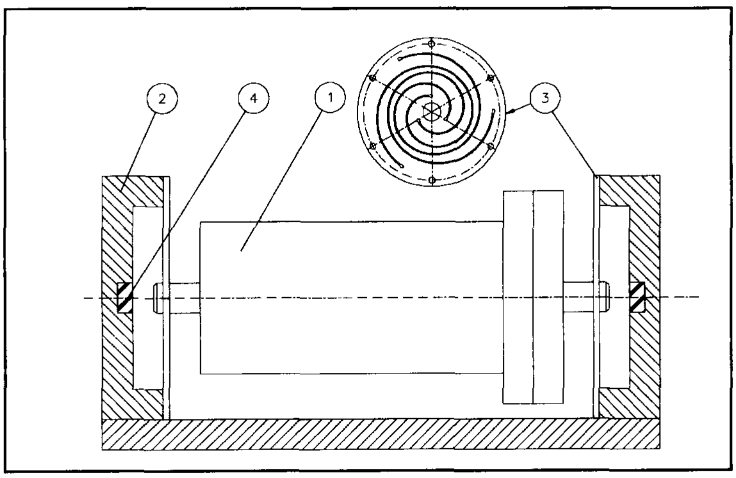
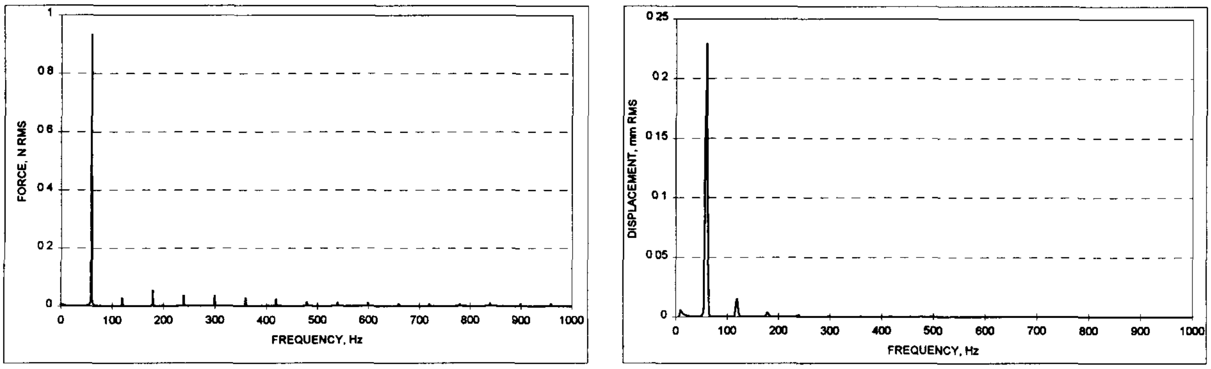


Figure 1

This arrangement provided the necessary undamped natural frequency and loss factor (Θ_1=15Hz and ζ_1=0.045). As a matter of low-frequency resilient suspension implementation, the self-induced force fundamental component was reduced up to 0.95 N RMS (see Figure 3,a); the higher-order harmonics were practically undetectable due to the sufficient mechanical low-pass filter features of designed vibration isolator. The fundamental component of dynamic deflection of the compressor housing was measured as 0.23 mm RMS (0.35 mm peak) with undetectable higher-order harmonics (see Figure 3,b).



(a) (b)

Figure 3

To allow compressor operation without impacts in any space orientation as long as environmental disturbances were absent, the clearance in between the compressor housing and the bumpers was chosen to be greater than the aggregate of the possible static deflection and the peak operational amplitude. Referring to the natural frequency of 15Hz we obtain for the maximum static deflection of 1mm. By adding the peak displacement of 0.35 mm (see Figure 3.b.) and rounding we choose finally Δ=1.5mm.

NUMERICAL SIMULATIONS

The scenario of the numerical optimisation was as follows:

- Firstly the “safety” domain was indicated.
  The criterion of vibration isolator “safety” was defined as the ability of the vibration protection arrangement to suppress the excitation of self-supporting vibro-impact resonances at any initial conditions (*dynamic safety* test).
- Secondly the parameters of the “optimal” bumper were found.
  The criterion of the vibration isolator “optimum” ***I*** reflects the effectiveness of excessive deflection restriction (as compared with the case of a linear vibration isolator under shock or broadband random vibration) without a generation of the damaging impact accelerations (*saw-tooth shock* and *broadband random vibration* tests).

The integration of (4) with the force of impact (5) was carried out in MatLab® with Simulink™ computational environment by using the fifth-order Runge-Kutta method with a variable discontinuity-sensitive integration step. The minimum and maximum time steps were 1 μs and 0.1 ms respectively, the computational tolerance was 10^{-10}.

1. In the *dynamic safety* test the response of the bumpered compressor vibration isolator on the ambient gross disturbance applied to the worked cryocooler was simulated. The test parameters were as follows: the self-induced sinusoidal force - 14N RMS @ 60Hz, external disturbance was classical saw-tooth shock 40g @ 6 ms. This test was provided for each natural frequency and loss factor of the bumper in the range Θ_1=60÷200Hz and ζ_1 =0÷1. In accordance with the vibration isolator behaviour after shock application, three different areas of the parameter map $\zeta_1 - \Theta_1$ were observed (see Figure 4)).

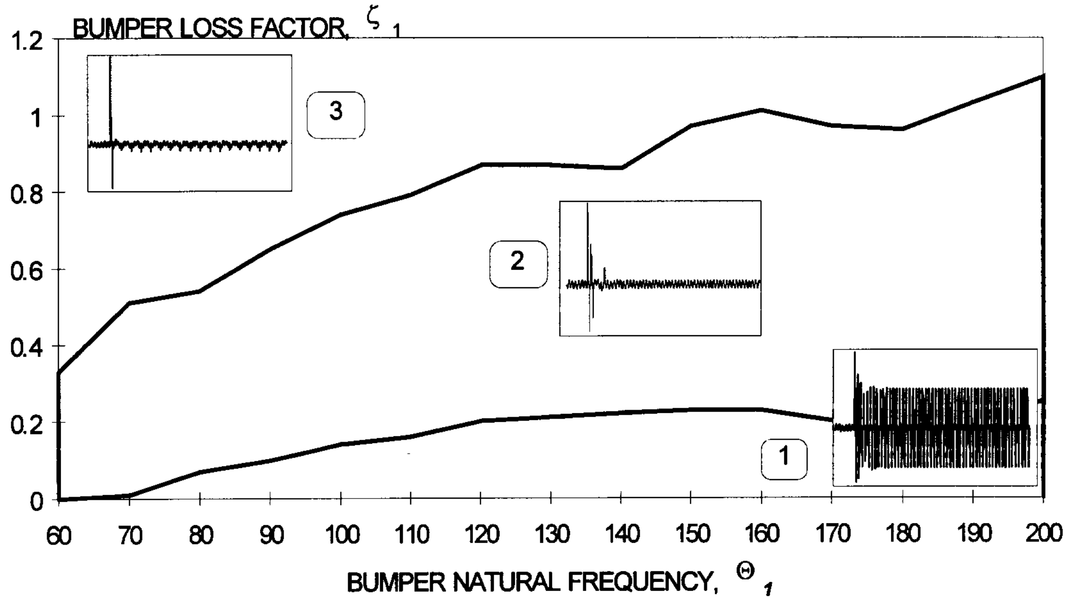


Figure 4

In zone ① of low loss factor the main vibro-impact resonance was observed after shock application. In zone ② (intermediate loss factor) no steady vibro-impact process was observed after shock application. In zone ③ (high loss factor) the low-powered sub-harmonic resonance of one third-order was observed (typical acceleration time histories are superimposed).

2. The goal of the saw-tooth shock test was to choose the parameters of the "safe" bumper, being located in the zone ② of Figure 4 and furnishing the minimum of criterion ***I***.

For that purpose corresponding simulations were carried out for different bumper parameters modelling the dynamic responses of the shut-off compressor with a saw-tooth shock 40g@6ms.

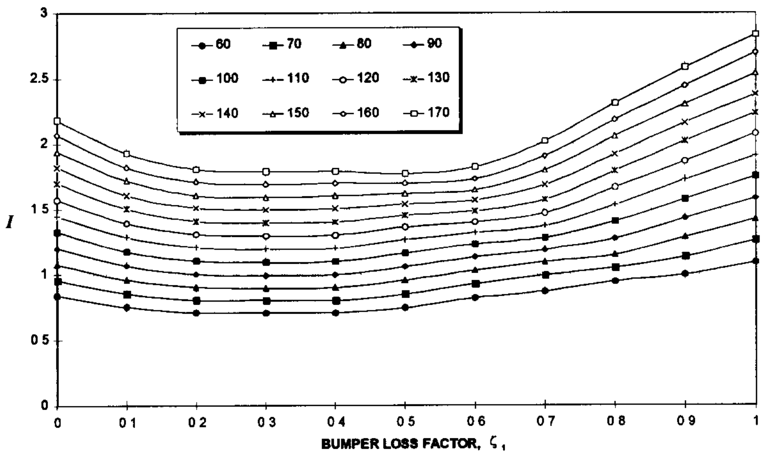


Figure 5

Figure 5 depicts the graphs of criterion ***I*** as a function of bumper loss factor for different bumper natural frequencies in the range of Θ_1=60÷170 Hz (see the legend). It can be easily seen that about the same value of bumper loss factor, namely 0.3, minimises the criterion. Considering the peak deflection of 3.5mm (bumper peak deformation of 2 mm) as permissible, we choose the optimal bumper with Θ_1=60 Hz and ζ_1=0.3.

3. The goal of the broadband random vibration test was to choose the bumper parameters that were located in the "safe" zone and minimising the criterion I. For that purpose, corresponding simulations were carried out for different bumper parameters by modelling the dynamic responses of shut-off compressor under broadband random excitation with overall level of 7.4 g RMS and with the peak acceleration value of 25 g (the graph of power spectral density of excitation acceleration is depicted by Figure 11.b).

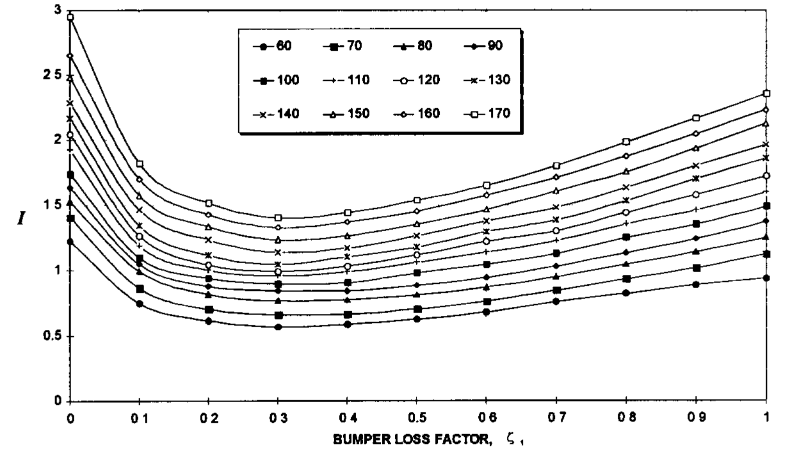
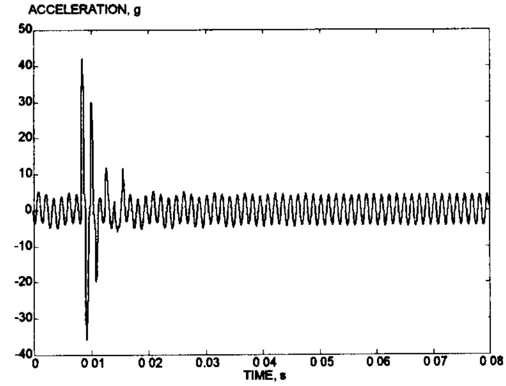


Figure 6

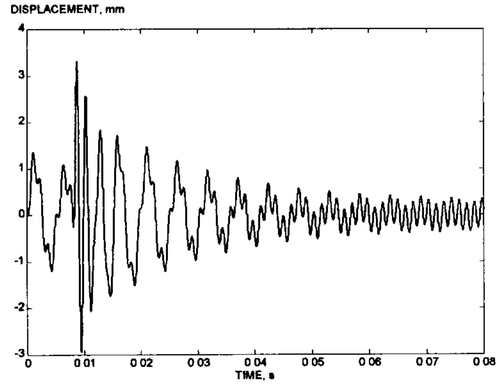
Figure 6 shows the graphs of criterion I as a function of bumper loss factor for different bumper natural frequencies in the range of Θ_I=60÷170 Hz (see the legend). The optimal loss factor of the bumper was about 0.3 (similar to that obtained before). The previously chosen bumper parameters Θ_1=60 Hz and ζ_1=0.3 will provide the following: acceleration levels - 4 g RMS and 20 g peak, displacement - 1.75 mm peak.

4. After the definition of the optimal bumper parameter, the corresponding simulations demonstrating the bumpered vibration isolator dynamics under specified excitation were carried out.

Figure 7 illustrates the acceleration (a) and displacement (b) of safe transients in the "*dynamic safety*" test conducted for the optimised bumpered vibration isolator of linear compressor.



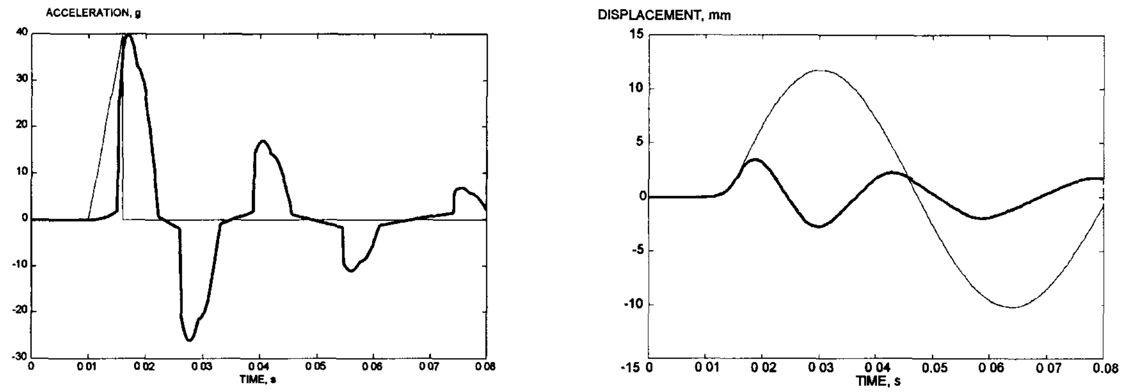
(a)



(b)

Figure 7

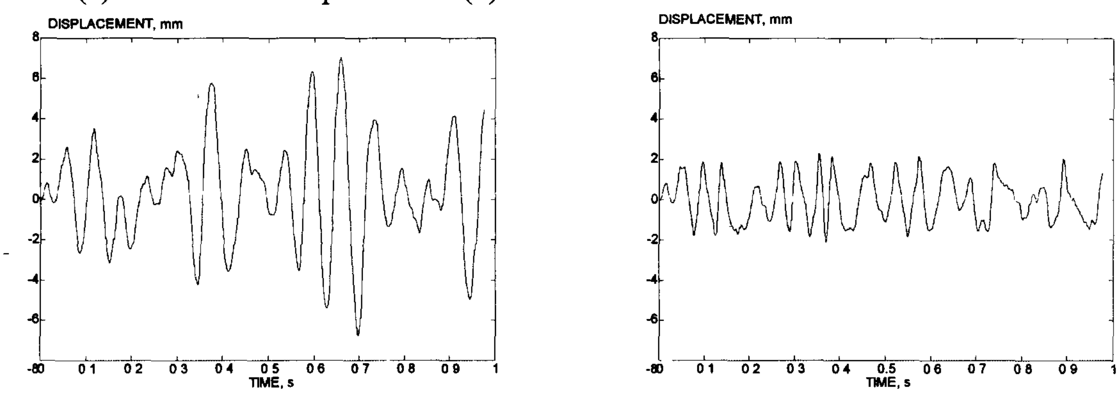
Figure 8.a illustrates the time histories of acceleration (thin line shows the excitation shape) for the optimised bumpered vibration isolator and Figure 8.b depicts the deflection time histories (thin line presents corresponding linear vibration isolator response). The value of optimal criterion I=0.71. It equals to the criteria value calculated for the rigid compressor attachment to its foundation and for the case of linear vibration isolator. But as it can be seen the maximum deflection of the vibration isolator was trimmed from 11.8 mm (linear case) to 3.5 mm, with a peak acceleration that does not exceed that of the excitation.



(a) (b)

Figure 8

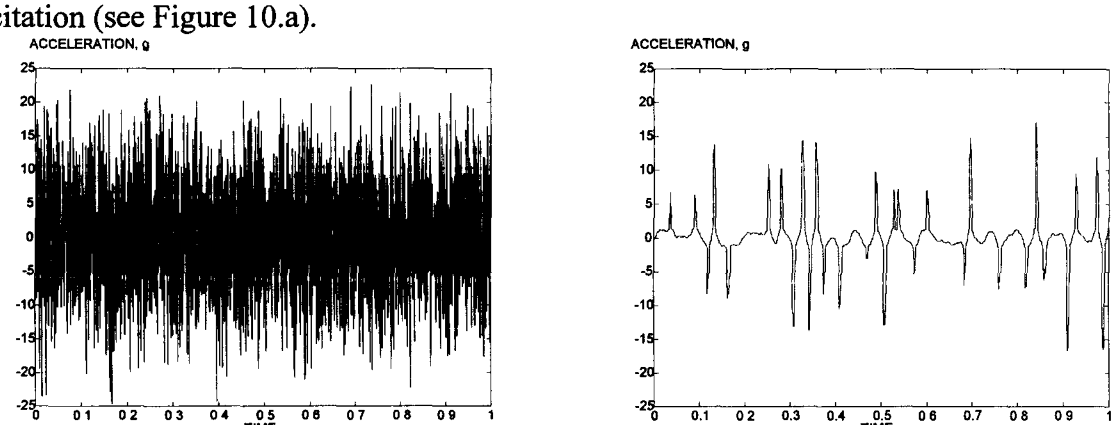
Figure 9 illustrates the 1s fragments of time histories of displacements of the linear vibration isolator (a) and that of bumpered one (b) in a broadband random vibration test.



(a) (b)

Figure 9

As it can be seen the excessive deflections were trimmed by a factor of about 4. The relatively rare and low peaks in the compressor acceleration response (see Figure 10.b) are of a characteristic quasi-periodic impact nature with peak values even lower than that of the excitation (see Figure 10.a).



(a) (b)

Figure 10

As a matter of fact the value of optimal criterion I=0.57 which is better than the criterion value calculated for the rigid compressor attachment (I=0.71) and much better than one for the case of linear vibration isolator (I=0.9).

Figure 11 depicts the normalized probability density distributions (a) and power spectral densities (b) of excitation and of the dynamic response.

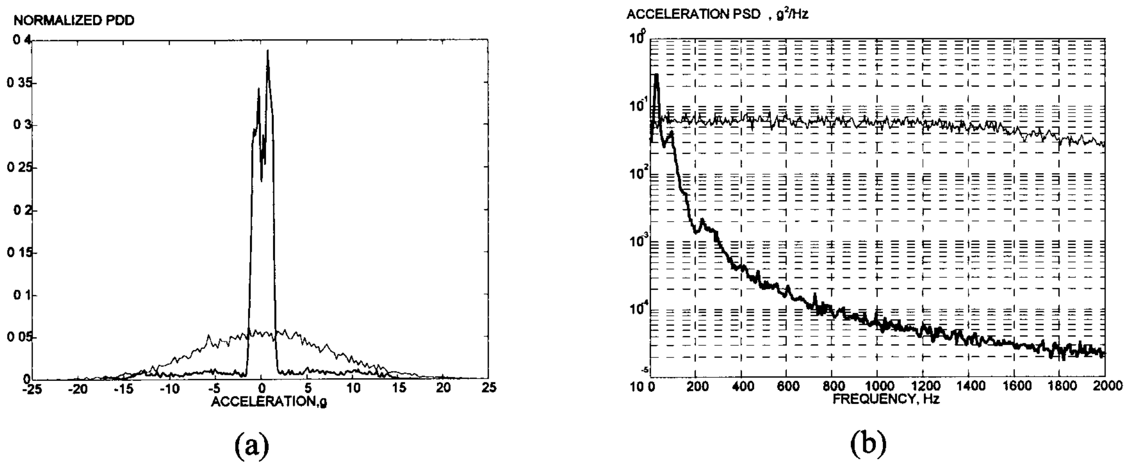


Figure 11

The analysis of the corresponding normalised probability density distributions of excitation (thin line) and that of the vibration isolator acceleration emphasises the conclusion: the dynamic response values are mainly located in a narrow path of ±2 g, the distribution tails (containing the dangerous high levels of acceleration) are much lower than that of the excitation.

The comparison of the corresponding power spectral densities of the excitation (thin line) and that of the response (thick line) illustrates the vibration attenuation in a very wide high-frequency band of 100÷2000Hz. The only narrow region of linear natural frequency of the vibration isolator (~15Hz) demonstrated an amplification factor of ~2 (typical for linear heavily damped vibration isolation arrangements with a loss factor of about 0.3).

EXPERIMENTAL TESTING

The bumpered vibration protection arrangement was designed in accordance with the results of the numerical optimisation. The characteristic parameters of the developed bumpers were measured as $\Theta_1 \approx 65$Hz and $\zeta_1 \approx 0.35$ at 23°C ambient temperature. The results of full mode random vibration and shock testing were very close to that of the numerical simulation.

The Ricor K529H cryocooler met the requirement of low self-induced force export and passed the full programme of shock and broadband random vibration environmental testing in accordance with MIL STD-810E without degradation in performance.

REFERENCES

1. M.Z. Kolovsky, Nonlinear theory of vibration protection systems. Nauka, Moscow, 1966, 318 pp.[in Russian]
2. V.I. Babitsky, The theory of vibro-impact systems. Nauka, Moscow, 1978, 352pp. [in Russian]
3. A. Veprik, A. Meromi, A. Leshecz, Novel technique of vibration control for split Stirling cryocooler with linear compressor. Proceedings of SPIE's 11th Annual International Symposium on Aerospace/Defence Sensing, Simulation and Controls "AeroSense", 20-25 April 1997, Orlando, Fl.
4. X. Wu, M.J. Griffin, A semi-active control policy to reduce the occurrence and severity of end-stop impacts in a suspension seat with an electrorheological fluid damper. Journal of Sound and Vibration, **203**(5), 1997, pp. 781-793.
5. Noise and Vibration Control. Edited by L.L. Beranek, McGraw Hill, 1971, 672 pp.