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Parametrically Excited Vibrations of Plates Subjected to Pulsating Loads

BÜLLESBACH, JÜRGEN
Universität der Bundeswehr München
Institute for Mechanics and Statics

FISCHER, OLIVER
Bilfinger+Berger BauAG
Technical Service Center (SCT)

Abstract On the basis of a nonlinear dynamic stability theory for shell constructions of arbitrary geometries and loadings, the phenomenon of the parametrically excited vibration is treated in a general manner and an approach is offered to solve linear and nonlinear problems. For an example of application, instability regions for plates are determined with the help of the developed basic equations; various influences such as damping, initial static load and inertial interaction with the fundamental motion are shown and discussed.

1 Introduction

A particular phenomenon of the vibration and stability theory is the parametric excitation which, under certain excitation-eigenfrequency ratios of the system, cause the motion of the fundamental state (Fs) to become unstable and change into a different form of vibration with an often considerable amplitude[1]. The relevant differential equation systems are characterized by the fact that, in contrast to the forced vibration, the load is included as a parameter. According to the linear theory, these equations have solutions of unlimited growth for certain values of their coefficients, and thus whole regions of dynamic instability. The most dangerous ones, so-called main instability regions, meet the frequency axis where the excitation frequency is just twice the eigenfrequency of the loaded system. Within these regions, an arbitrary disturbance causes exponentially growing amplitudes. If, however, nonlinear terms are included the parametrically excited vibration remains limited (even without damping) and finally reaches a limit cycle with a stationary amplitude (small-scale instability, large-scale stability). The reason for this is the well-known

[1] For example, we would like to refer to recent experimental studies on cables [5] which show that very stable oscillations with related large amplitudes may occur in case of parametric resonances, in particular.

phenomenon of "protruding" resonance curves occuring in nonlinear vibrations. The limits of the instability regions, however, important to examine the stability of the Fs, can be determined with sufficient accuracy using nothing but linear differential equations; also see [1]. As an example for arches subjected to pulsating loads, Figure 1 shows the disturbance-induced, nonlinear parametrically excited vibration as a function of time as well as in the phase plane. The initial exponential increase of the amplitudes and the transition into the limit cycle are clearly visible (from [2]).

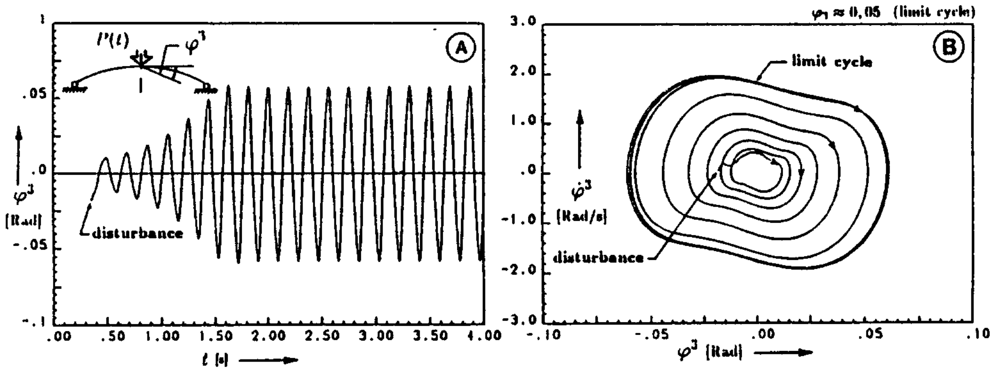

Figure 1 Nonlinear parametrically excited arch vibrations
apex rotation as a function of time (A) and corresponding phase diagram (B).

In the following, dynamic equations generally applicable to shell-type structures are derived allowing the examination of arbitrary vibration and instability phenomena (linear and nonlinear). The tensor formulation provides systematically structured and extremely compact relations which may be adopted to specific applications in a simple manner[2]. All basic equations, the constitutive relations, the dynamic boundary conditions and the stability equations comprize the nonlinear contributions of the disturbance-induced kinemates as well as the products with these and the deformations of the sf Fs. An imperfect initial geometry given, the imperfections important to the stability analysis of shells may be integrated into the overall concept, too.

2 Stability Equations for Shell Structures

To formulate stability equations for shell structures of arbitrary geometries, first the equations of motion for an undisturbed motion state (Fs) and for a neighboring motion state (Ns) disturbed in the initial displacement and velocity conditions are established[3]. For the Fs to be examined for its stability, the vector representation provides the following equations of motion in the region:

$$\begin{aligned} \overset{o}{\mathbf{n}}{}^{\alpha}_{,\alpha} + p^k \overset{+}{\mathbf{a}}_k \sqrt{\overset{+}{a}} + q^k \overset{o}{\mathbf{a}}_k \sqrt{\overset{+}{a}} &= \overset{o}{\mathbf{t}}_K + \overset{o}{\mathbf{d}}_K \\ \left(\overset{o}{\mathbf{a}}_3 \times \overset{o}{\mathbf{m}}{}^{\alpha}\right)_{,\alpha} + \overset{o}{\mathbf{a}}_{\alpha} \times \overset{o}{\mathbf{n}}{}^{\alpha} &= \overset{o}{\mathbf{a}}_3 \times \overset{o}{\mathbf{t}}_M + \overset{o}{\mathbf{a}}_3 \times \overset{o}{\mathbf{d}}_M \end{aligned} \tag{1}$$

[2] In analogy with the method shown here, general basic equations of a similar structure can be developed both for continua and rod constructions; see e.g. [2].

[3] compare [4].

In addition to the internal force and moment vectors ($\overset{o}{\mathbf{n}}^{\alpha}$, $\overset{o}{\mathbf{m}}^{\alpha}$), the left side of (1) contains directionally stable and unstable (p^k, q^k) load contributions, while the right side comprises inertial ($\overset{o}{\mathbf{t}}_K$, $\overset{o}{\mathbf{t}}_M$) and damping ($\overset{o}{\mathbf{d}}_k$, $\overset{o}{\mathbf{d}}_M$) forces and moments.

Both for the equation of motion of the **Ns** and that of the **Fs** the principle of virtual displacements can be utilized to obtain work equations the contributions of which can be summarized to portions of virtual internal work δW, kinetic energy δE, and virtual external work caused by directionally unstable loads in the region (δN_G) and at the boundaries (δN_R). Here, the work equation of the **Fs** is given as an example; the equation of the **Ns** has an analogous structure and differs only in that the superimposed "o" is substituted by a dash to indicate the state variables.

$$\begin{aligned}&\int_{(\overset{+}{A})} \overset{o}{\mathbf{n}}^{\alpha}_{,\alpha}\,\delta\mathbf{v}\,d\overset{+}{A} + \int_{(\overset{+}{A})} p^k\,\overset{+}{\mathbf{a}}_k\,\sqrt{\overset{+}{a}}\,\delta\mathbf{v}\,d\overset{+}{A} + \int_{(\overset{+}{A})} q^k\,\overset{o}{\mathbf{a}}_k\,\sqrt{\overset{+}{a}}\,\delta\mathbf{v}\,d\overset{+}{A} - \int_{(\overset{+}{A})} \overset{o}{\mathbf{t}}_K\,\delta\mathbf{v}\,d\overset{+}{A} - \\ &\int_{(\overset{+}{A})} \overset{o}{\mathbf{d}}_K\,\delta\mathbf{v}\,d\overset{+}{A} + \int_{(\overset{+}{A})} \left(\overset{o}{\mathbf{a}}_3 \times \overset{o}{m}^{\alpha}\right)_{,\alpha}\delta\boldsymbol{\varphi}\,d\overset{+}{A} + \int_{(\overset{+}{A})} \left(\overset{o}{\mathbf{a}}_{\alpha} \times \overset{o}{\mathbf{n}}^{\alpha}\right)\delta\boldsymbol{\varphi}\,d\overset{+}{A} = \mathbf{0}\end{aligned} \tag{2}$$

Further, the independent portions of the **Fs** can be separated from those of the **Ns**. The permissibility of this approach can be proven by directly deriving the relations from the 3D-continuum. In the work equation, the remaining portions can thus be given as

$$\delta W + \delta E = \delta N_G + \delta N_R \ . \tag{3}$$

Here, the contributions of internal work and kinetic energy are given as examples:

$$\begin{aligned}\delta W &= \int_{(\overset{+}{A})} s^{\alpha\beta}\,\delta\gamma_{\alpha\beta}\,d\overset{+}{A} + \int_{(\overset{+}{A})} m^{\alpha\beta}\,\delta\kappa_{\alpha\beta}\,d\overset{+}{A} + \\ &\quad \int_{(\overset{+}{A})} \overset{o}{s}^{\alpha\beta}\,\delta\Delta\gamma_{\alpha\beta}\,d\overset{+}{A} + \int_{(\overset{+}{A})} \overset{o}{n}^{\alpha 3}\,\delta\Delta\gamma_{\alpha}\,d\overset{+}{A} - \int_{(\overset{+}{A})} \overset{o}{m}^{\alpha\beta}\,\delta\Delta\varrho_{\alpha\beta}\,d\overset{+}{A} \\ \delta E &= \int_{(\overset{+}{A})} \overset{+}{\varrho}\,h\,\ddot{v}^k\,\delta v_k\,d\overset{+}{A} + \int_{(\overset{+}{A})} \overset{+}{\varrho}\,h\,d\,\dot{v}^k\,\delta v_k\,d\overset{+}{A} + \Delta\delta E\end{aligned} \tag{4}$$

The above relations apply also to geometrically nonlinear approaches, because they contain the disturbance-induced kinemates including square elements of v^k. δE shows the relevant terms from the inertial and damping forces, portions of a higher order of "h" are summarized in $\Delta\delta E$. With the last three of the integrals in δW the internal forces of the **Fs** - also including the portions due to bending - are introduced into the stability analysis. In addition, the first two integrals of the internal work contain those contributions caused by the disturbance-induced internal reaction increases $s^{\alpha\beta}$ and $m^{\alpha\beta}$. From the constitutive equations for the case of an elastic material behavior, so-called constitutive variables are assigned to these contributions which contain the strain and curvature increase:

$$\begin{aligned}s^{\alpha\beta} &= D\,G^{\alpha\beta\varrho\lambda}\cdot\gamma_{\varrho\lambda} + B\left(G^{\alpha\beta\gamma\lambda}\,\overset{+}{b}{}^{\varrho}_{\gamma} - G^{\alpha\beta\varrho\lambda}\,\overset{+}{b}{}^{\varepsilon}_{\varepsilon}\right)\cdot\kappa_{\varrho\lambda} \\ m^{\alpha\beta} &= B\,G^{\alpha\beta\varrho\lambda}\cdot\kappa_{\varrho\lambda} + B\left(G^{\gamma\beta\varrho\lambda}\,\overset{+}{b}{}^{\alpha}_{\gamma} - G^{\alpha\beta\varrho\lambda}\,\overset{+}{b}{}^{\varepsilon}_{\varepsilon}\right)\cdot\gamma_{\varrho\lambda} \\ \text{and:}\quad n^{\alpha\beta} &= s^{\alpha\beta} - \overset{+}{b}{}^{\alpha}_{\varepsilon}\,m^{\varepsilon\beta}\end{aligned} \tag{5}$$

In the constitutive variables linear contributions (L), products of v^k and Fs-deformations (F) as well as nonlinear portions in v^k (N) of can be grouped together as follows:

$$\begin{array}{lrcl} \text{strain increase:} & \gamma_{\varrho\lambda} & = & \overset{L}{\gamma}_{\varrho\lambda} + \overset{F}{\gamma}_{\varrho\lambda} + \overset{N}{\gamma}_{\varrho\lambda} \\ \text{curvature increase:} & \kappa_{\varrho\lambda} & = & \overset{L}{\kappa}_{\varrho\lambda} + \overset{F}{\kappa}_{\varrho\lambda} + \overset{N}{\kappa}_{\varrho\lambda} \end{array} \tag{6}$$

For the strain increase, the portions are given as examples:

$$\begin{array}{rcl} \overset{L}{\gamma}_{\varrho\lambda} & = & \frac{1}{2}\left(v^{\pi}|_{\lambda}\overset{+}{a}_{\varrho\pi} + v^{\pi}|_{\varrho}\overset{+}{a}_{\lambda\pi}\right) \\ \overset{F}{\gamma}_{\varrho\lambda} & = & \frac{1}{2}\left[\left(\overset{o}{v}^{\pi}|_{\lambda}v^{\tau}|_{\varrho} + \overset{o}{v}^{\tau}|_{\varrho}v^{\pi}|_{\lambda}\right)\overset{+}{a}_{\pi\tau} + \overset{o}{v}^{3}|_{\lambda}v^{3}|_{\varrho} + \overset{o}{v}^{3}|_{\varrho}v^{3}|_{\lambda}\right] \\ \overset{N}{\gamma}_{\varrho\lambda} & = & \frac{1}{2}\left(v^{\pi}|_{\lambda}v^{\tau}|_{\varrho}\overset{+}{a}_{\pi\tau} + v^{3}|_{\varrho}v^{3}|_{\lambda}\right) \end{array} \tag{7}$$

3 Approach to Linear and Nonlinear Stability Analyses

To solve the equation (3), the disturbance-induced displacements v^k and their virtual kinemates δv^k are described by product equations (k=1, 2, 3) as follows:

$$v^k(\Theta^1,\Theta^2,t) = \sum_{(n=1)}^{N} \overset{nk}{v}(\Theta^1,\Theta^2)\cdot T_n(t); \qquad \delta v^k(\Theta^1,\Theta^2,t) = \sum_{(n=1)}^{N} \overset{nk}{v}(\Theta^1,\Theta^2)\cdot \delta T_n(t) \tag{8}$$

In (8) the eigenmodes of the undamped vibration are used as functions $\overset{n}{v}^k$, since this considerably simplifies the solution due to the orthogonality charactaristics. The quantities T_n represent the unknown time functions of the disturbance-induced motion, while N stands for the number of eigenmodes used for the approximation. With regard to a systematic consideration of quadratic nonlinearities, the examination of high-order products in the disturbance-induced displacements and Fs quantities is not continued. After consideration of both the Fs and the orthogonality of the eigenmodes, (3) is given as:

$$\sum_{n=1}^{N} \underbrace{\left\{\ddot{T}_n + d\cdot\dot{T}_n + T_n\cdot\overset{n}{\omega}{}^2 + \sum_{k=1}^{N} T_k\cdot q_{nk}(t) + \sum_{k=1}^{N}\sum_{s=1}^{N} T_k\cdot T_s\cdot\alpha_{nks}\right\}}_{\Upsilon_n}\cdot\delta T_n = 0 \tag{9}$$

The nonlinear coefficients α_{nks} result from the internal work δW. Due to the component representation of the stability equations and the inertial and damping forces in the direction of the base $\overset{+}{\mathbf{a}}_k$ of the undeformed state, only rotational nonlinearities occur consisting of inertia and damping; because of minor importance they are not stated. The coefficients $q_{nk}(t)$ include reactions and deformations of the Fs as well as directionally unstable loads, while $\overset{n}{\omega}$ are the eigenfrequencies. Since the virtual changes δT_n are arbitrary and independent of each other, $\Upsilon_n\cdot\delta T_n = 0$ and $\Upsilon_n = 0$ can be postulated from (9) (n= 1, 2, ...N).

3.1 Linear Stability Analysis

In the case of a Fs constant in time the exitation term remains time-invariant and, with $q_{nk}(\kappa_o)$, depends from a load parameter κ_o alone. Thus, we obtain from (9) a linear time differential equation system with constant coefficients:

$$\ddot{T}_n + d \cdot \dot{T}_n + T_n \cdot \overset{n}{\omega}{}^2 + \sum_{k=1}^{N} T_k \cdot q_{nk}(\kappa_0) = 0 \quad ; \quad (n = 1, 2, \ldots, N) \tag{10}$$

With $T_n(t) = a_n \cdot e^{i\lambda t}, (i = \sqrt{-1})$ (10) is transformed into a linear homogeneous equation system with nontrivial solutions $a_n \neq 0$ if the denominator determinant D_N vanishes. With $\lambda_D^2 = \lambda^2 - i \cdot d\lambda$ and $\mathbf{Q}(\kappa_o) = [q_{nk}(\kappa_o)]$ this provides a special eigenvalue problem:

$$\det\left[\boldsymbol{D} + \boldsymbol{Q}(\kappa_0) - \lambda_D^2 \cdot \boldsymbol{E}\right] = 0 \qquad \text{mit:} \qquad \boldsymbol{D} = \begin{bmatrix} \overset{1}{\omega}{}^2 & & 0 \\ & \ddots & \\ 0 & & \overset{n}{\omega}{}^2 \end{bmatrix} \tag{11}$$

From the eigenvalues λ_D^2 the equation quantities $\lambda = \lambda_R + i \cdot \lambda_I$ can be calculated in a complex form. While the real part of the time functions describes a harmonic oscillation with $e^{i\lambda_R t}$, the imagenary part with $e^{-\lambda_I t}$ provides an exponential increase of the disturbance-induced motion in case of a negative λ_I, characterizing instability. With $R = \text{Re}(\lambda_D^2)$ and $I = \text{Im}(\lambda_D^2)$ this approach leads to the stability postulate:

$$R \cdot d^2 - I^2 \geq 0 \qquad \text{with the secandary condition:} \qquad 4R + d^2 \geq 0 \tag{12}$$

In the undamped case ($d = 0$) equation (12) is simplified reading $I = 0$ und $R \geq 0$. The smallest load value κ_0 which does not meet this stability requirement is critical.

For arbitrary fundamental motion states with periodically varying excitation term

$$q_{nk}(t) = q_{nk}(\kappa_0) + \kappa_t \cdot \sum_{m=1}^{M} [\overset{m}{a}_{nk} + \overset{m}{s}_{nk} \cdot \sin m\Omega t + \overset{m}{c}_{nk} \cdot \cos m\Omega t] \tag{13}$$

a system of extended HILL differential equations with periodic coefficients is obtained from (9) without nonlinear contributions. Here, κ_t is the parameter of the load depending on the time while Ω is a frequency magnitude. Now, in the plane (κ_t, Ω) there are whole regions of kinetic instability the boundaries of which can be determined using FOURIER series with the periods a) $T = 2\pi/\Omega$ and b) $2T$:

$$\text{a) } T_n = \tfrac{1}{2}\overset{n}{b}_0 + \sum_{j=2,4,..}^{J_1=\infty} [\overset{n}{a}_j \sin \tfrac{j\Omega t}{2} + \overset{n}{b}_j \cos \tfrac{j\Omega t}{2}] \, , \quad \text{b) } T_n = \sum_{j=1,3,..}^{J_2=\infty} [\overset{n}{a}_j \sin \tfrac{j\Omega t}{2} + \overset{n}{b}_j \cos \tfrac{j\Omega t}{2}] \tag{14}$$

By comparison of the coefficiants this method allows to give two infinite homogenous equation systems which can be reduced to finite systems (J_1, $J_2 \neq \infty$) [1]. With κ_o and Ω given, the postulation of $D_N = 0$ leads to eigenvalue problems of the form ($\alpha = 1, 2$):

$$\det\left[\boldsymbol{X}_\alpha(\kappa_0, \Omega) - \kappa_t \cdot \boldsymbol{Z}_\alpha(\Omega)\right] = 0 \quad , \quad \boldsymbol{X}_\alpha, \boldsymbol{Z}_\alpha : \text{ hyper matrices, see. e.g [4]} \tag{15}$$

The smallest κ_t provides the decisive boundary of the instability regions.

3.2 Nonlinear Stability Analysis

To perform a nonlinear stability analysis for arbitrary fundamental states, the nonlinear differential equations $\Upsilon_n = 0$ are first transferred into a first order system by introducing the new variables $\underset{\sim}{T}_k = T_k$, $\underset{\sim}{T}_{N+k} = \dot{T}_k$, $(k = 1, 2, .., N)$. When specifying a small disturbance $\underset{\sim}{T}_k(t_a)$, $(k = 1, 2, .., 2N)$ for an arbitrary time t_a, the displacement and velocity of the structure are disturbed - in accordance with LJAPUNOW - with scaled linear combinations of its eigenmodes. By an incremental numeric integration, $\underset{\sim}{T}_k(t)$ can be determined at any time t and the history of the disturbed motion can thus be approximated.

For a periodically varying Fs - similar to the linear analysis - a trigonometric approach may be used which, however, combines the periods T and $2T$ for determining the vibration amplitudes ($T_n = \frac{1}{2}\overset{n}{b}_0 + \sum_{j=1,2,..}^{J} [\overset{n}{a}_j \sin\frac{j\Omega t}{2} + \overset{n}{b}_j \cos\frac{j\Omega t}{2}]$). The functions δT_n are formed by changing the constants into the varied quantities $\delta\overset{n}{b}_0$, $\delta\overset{n}{a}_j$ and $\delta\overset{n}{b}_j$. By substitution into $\Upsilon_n \cdot \delta T_n$ and integration over the maximum period length $T_{max.} = 4\pi/\Omega$ a nonlinear algebraic equation system is obtained in $\overset{n}{b}_0$, $\overset{n}{a}_j$ and $\overset{n}{b}_j$ which can be solved numerically[4].

3.3 Evaluation of the Eigenmodes and the Fundamental State Quantities

To determine the eigenmodes and eigenfrequencies of arbitrary shell structures it is advisable to use an FE program[5]. To calculate the fundamental motion state its deformations are split up into a quasi-static (index S) and a dynamic (index D) portion according to:

$$\overset{o}{v}^k(\Theta^1, \Theta^2, t) = \overset{o}{v}^k_S(\Theta^1, \Theta^2, t) + \overset{o}{v}^k_D(\Theta^1, \Theta^2, t) \qquad (16)$$

In this way, all quantities of the Fs are split up. Similar to the approximation of v^k, see (8), the dynamic portion $\overset{o}{v}^k_D$ is represented by series approaches using the eigenmodes. The quantities $\overset{m}{q}_{nk}$ required to solve (9) and the nonlinear coefficients α_{nks} can then be determined by numeric differentation and integration at the nodal points of the FE-mesh.

If only the quasi-static solution is included, the dynamic interaction of the fundamental state vibration and the disturbance-induced motion is not covered. Thus, the analyses of parametrically excited vibrations do not provide instability regions originating from the resonance points of the fundamental state, but only those instability regions which are assigned to the known frequency ratios $\Omega = 2\overset{n}{\omega}/k$, $(k = 1, 2, \ldots, \infty)$ of the parametrically excited vibration. Moreover, phenomena like the main instability regions[6] merging with the regions of the fundamental vibration cannot be covered (also see [2] and [3]).

4 Example of Application - Parametrically Excited Vibrations of Plates

So far, the explanations given are applicable to arbitrary shell structures. The example of application given in the following implies a restriction to plates. This means that, with the use of the relations derived above, the covariant derivations $(...)|$ may be substituted by partial ones $(...)'$ and that contributions containing the initial curvature $\overset{+}{b}{}^\alpha_\beta$ disappear.

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[4] For detailed explanations of the described solution methods see [2].

[5] For the analyses of the examples given in section 4 the NASTRAN program was used.

[6] Such effects have been theoretically examined and experimentally confirmed in [1], too.

The numeric analyses are performed for rectangular steel plates with a length-width-ratio of $a/b = 2$ and a periodic load of $P(t) = P_o + P_t \cdot \cos(\Omega t)$ acting along one edge.

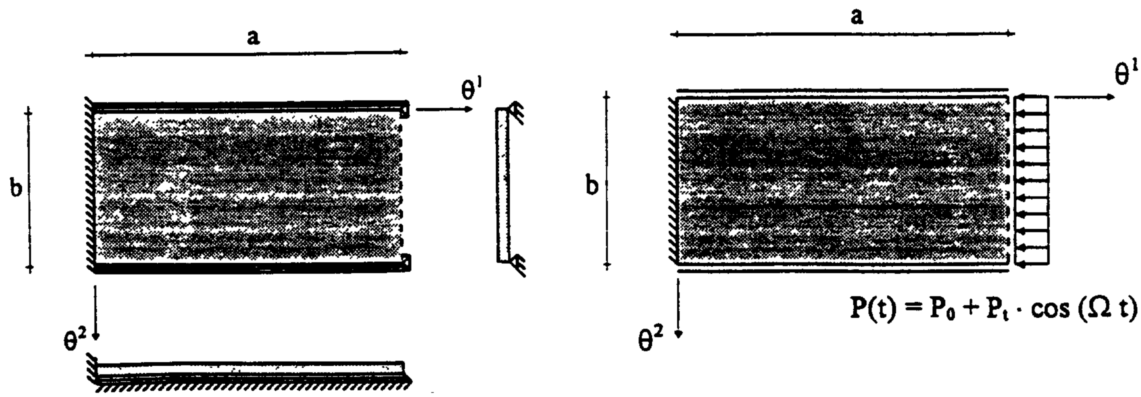

Figure 2 System, boundary conditions and loading.

In addition, it should be noted that at the boundaries $\Theta^2 = 0$ and $\Theta^2 = b$ there is no support in Θ^1-direction. After the quasi-static and dynamic solutions of the Fs have been found, a periodically varying excitation term q_{nk} is obtained from equation (13). The previous static calculation (solution according to 3.1) resulted in a linear buckling load of $\frac{P_{o,krit} \cdot b^2}{\pi^2 \cdot B} = 2.37$, which confirms the result given in [4]. Moreover, it could be demonstrated that, in case of a time-invariable load, the system undergoes a 'static failure', i.e. an intersection point of the load-frequency curve and the load axis provides the decisive critical load. Now, the instability regions for the pulsating load are determined by applying the method described in paragraph 3.1. For this purpose, the load parameters $\kappa_t = \frac{P_t}{P_{o,krit}}$ for the time-variable load portion and $\kappa_o = \frac{P_o}{P_{o,krit}}$ for the static preload are introduced.

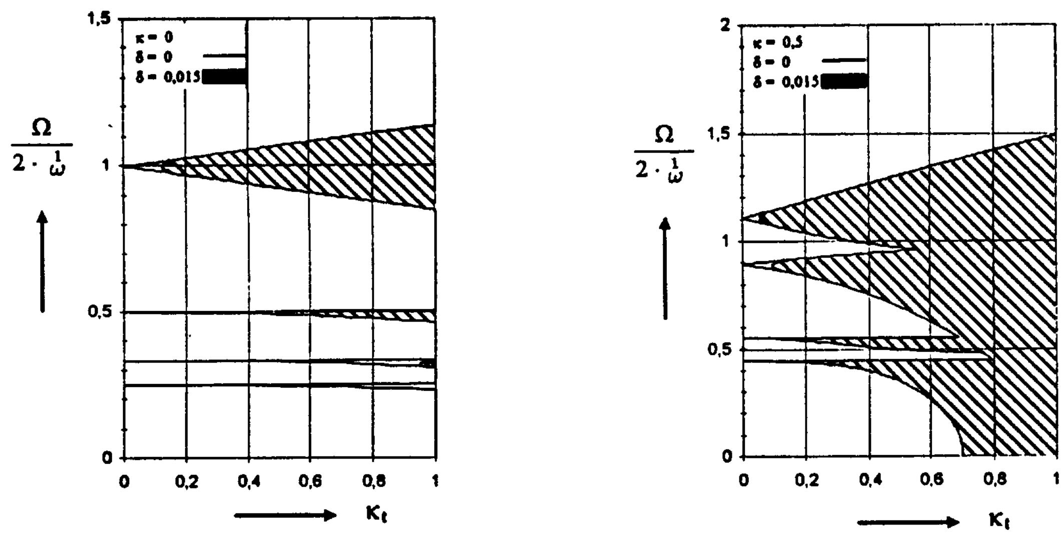

Figure 3 Instability regions for static preload $\kappa_o = 0.0$ (a) and $\kappa_o = 0.5$ (b).

For the incrementally increased excitation frequency Ω, the eigenvalue problems (15) for the periods T and $2T$ can now be solved. The resulting critical load values of κ_t can thus be plotted against the corresponding frequency ratio related to the value twice the first eigenfrequency of the plate vibration. Figure 3a shows the predominant influence of the main instability region for the first eigenmode. As expected, the width of the higher-order instability regions (in Fig. 3a depicted up to the 4th order) decreases with increasing orders. If damping influences are considered, these regions usually vanish completely (compare Fig. 3a: broken lines), while the main instability region clearly maintains its influence. Further, it can be recognized that the instability areas meet the frequency axis at $\Omega = \frac{2\overset{1}{\omega}}{k}$. Since the peaks of the regions are dependent on the eigenfrequencies of the loaded system, the instability regions are shifted downwards if the calculation is performed with a static preload (see Fig. 3b: $\kappa_o = 0.5$). Here, again, the results for a related damping of $\delta = \frac{d}{2\overset{1}{\omega}} = 0.015$ are plotted as a broken line.

Other calculations have shown that the dynamic interaction between the Fs and the disturbance-induced motion mentioned in paragraph 3.3 is small enough to be neglected, so that the inclusion of the quasi-static portion is sufficient. When dealing with parametric resonance of general shell structures, however, the interaction has always to be taken into consideration, because of the vicinity of the Fs and Ns eigenfrequencies. It was further investigated that the deformations of the Fs are of only minor influence on the width of the regions and can thus be neglected. Such deformations can always lead to a dangerous widening of the instability regions, if the Fs also contains a deformation component in the direction of the resulting disturbance-induced motion[7]. In general, however, for parametrically excited vibrations of plates it has to be taken into consideration that the adjacent eigenfrequencies cause wide instability regions which have to be considered.

Acknowledgements

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[7] This problem is treated in detail in [2].