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Predictive Statistical Energy Analysis and equally spaced point connections

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Abstract

Using the wave approach, the theoretical prediction of Statistical Energy Analysis plate-to-plate coupling loss factors is based on calculating the random incidence transmission coefficient matrix associated with the equivalent infinite line junction. The 4x4 semi-infinite line wave dynamic stiffness of each plate is used to calculate the transmission coefficient for a particular angle of incidence; these angle transmission coefficients are then numerically integrated to obtain the required random incidence transmission coefficient.
Many line connections in real engineering structures are, in fact, a series of equally spaced point connections. The spacing of these points is often neither large nor small in comparison to structural wavelengths. Furthermore each connection point may itself exhibit dynamic behaviour such as when point isolators are employed. This paper presents a theoretical solution to this problem based on the Fourier decomposition of the connection line into a series of true line connections each with a different trace wavenumber. The cross transfer matrix of a single point connection is then incorporated into the theory to model its dynamic behaviour.
Experimental results are then compared with this theory using a two-plate assembly with an I-sectioned connecting beam and with various numbers and types of point connectors.

1 Introduction

Predictive SEA clearly depends on the calculation of the coupling loss factors between the chosen subsystems, and for plate assemblies these subsystems are usually chosen to correspond to the different travelling wavetypes that each plate can support. For a flat isotropic plate this implies three subsystems to model the bending waves, the in-plane compression waves and the in-plane shear waves.

By adopting the wave approach, as against the more traditional modal approach, it can be shown that for plates a given SEA coupling loss factor, η_{ab}, is related to the appropriate 'infinite' random incident transmission coefficient, $\tilde{\tau}_{ba}$, through the formula

$$\eta_{ab} = \frac{L k_a \tilde{\tau}_{ba}}{2\pi^2 \omega n_a} \tag{1}$$

where ω is the circular frequency, k_a and n_a are respectively the wavenumber and modal density of wavetype a, and L is the length of the line connection.

An 'infinite' transmission coefficient is simply that associated with the canonical junction of infinite extent (that is where all plates are assumed semi-infinite). The procedure to calculate $\tilde{\tau}_{ba}$ is first to calculate the angle dependent transmission coefficient $\tau_{ba}(\theta_a)$, where θ_a is the angle of incidence with respect to the normal, and then to numerically integrate this using

$$\tilde{\tau}_{ba} = \int_0^{\pi/2} \tau_{ba}(\theta_a)\cos(\theta_a)\,d\theta_a \tag{2}$$

Assuming all plates are thin, flat and isotropic and that all connecting beams can be modelled using Timoshenko beam theory the problem was solved for a general geometry junction by Langley and Heron (1990). Recently Heron (1997) used the concept of a linewave impedance, to extend the theory to include strip plates (to better model thin sectioned beams) and this extended theory has been shown to exhibit very good agreement with experimental results: Heron (1995), Monger *et al* (1997). The theory assumes a simple line connection between all the elements at the line junction (semi-infinite plates, infinite strip plates and infinite beams). Mathematically this implies, for a given frequency ω and incident trace wavenumber k, a dependency of $e^{-ikx}e^{i\omega t}$ on the space variable x (parallel to the line junction) and on the time variable t.

The aim herein is to extend this theory to the case where a plate is attached, possibly by isolators, to the other elements at the junction by a discrete set of points. From an SEA standpoint, such a theory could be developed based upon one of two assumptions. Either it could be assumed that the point connections are randomly spaced, or it could be assumed that that the point connections are equally spaced. The former is a simple extension of SEA point connection theory (multiplying by the number of point connections). Unfortunately randomly spaced connections are not normally used in the engineering world and so the more complicated assumption of equally spaced point connections must be studied. This case is fundamentally more complex because it has to be modelled using an extension to line, rather than point, connection theory. This extension to SEA theory is described below.

2 Point and line variables

Let the six point force variables be given by the column vector $\mathbf{f}$ such that

$$\mathbf{f}^{\mathrm{T}} = \left(f_x, f_y, f_z, m_x, m_y, m_z\right) \tag{3}$$

where the superscript T denotes transposition, and where $\left(f_x, f_y, f_z\right)$ represent the point forces and $\left(m_x, m_y, m_z\right)$ represent the point moments. Now let the four line force variables along a line parallel to the x axis be given by the column vector $\tilde{\mathbf{f}}$ such that

$$\tilde{\mathbf{f}}^{\mathrm{T}} = \left(f_1, f_2, f_3, f_4\right) \tag{4}$$

The vector $\tilde{\mathbf{f}}$ is a column vector of forces/moments per unit length. Assuming that over a small enough length δx the line forces do not significantly vary, and choosing the line velocity variables such that

$$\tilde{\mathbf{v}}^{\mathrm{T}} = \left(-ik^{-1}v_x, v_x, w_x\right) \tag{5}$$

then the force variables are related by

$$\tilde{\mathbf{f}}\delta x = \boldsymbol{\Psi}\mathbf{f} \tag{6}$$

where the 4 x 6 matrix $\boldsymbol{\Psi}$ is given by

$$\Psi = \begin{bmatrix} -ik & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & ik \\ 0 & 0 & 1 & 0 & -ik & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \end{bmatrix} \tag{7}$$

Furthermore the point velocity vector $\mathbf{v}$ and the line velocity vector $\tilde{\mathbf{v}}$ are related through the equation

$$\mathbf{v} = \boldsymbol{\Psi}^{\mathrm{H}}\tilde{\mathbf{v}} \tag{8}$$

where the superscript H denotes complex conjugate transposition.

3 Fourier force decomposition

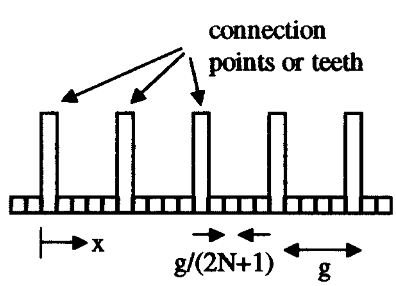

figure 1 A comb of teeth

For clarity of presentation the connections will be referred to as teeth and the assumed infinite and equally spaced line of teeth will be referred to as a comb.

In order to combine current line junction theory with a theory for these comb connections, it is necessary to model the point forces at the teeth in terms of line forces. Letting the incident wave (which generates the junction force) have a trace wavenumber k_{inc}, the point forces can be modelled using a function of the form

$$\mathbf{f}(x) = \frac{e^{-ik_{\text{inc}}x}}{(2N+1)} \sum_{l=-N}^{+N} e^{-2\pi il(x/g)} \tag{9}$$

where 2*N*+1 is the number of digitised points per tooth spacing (five in figure 1) and g is the tooth separation distance.

Equation 9 has been chosen because

$$\mathbf{f}(x) = \begin{matrix} e^{-ik_{\text{inc}}x} & \text{at a tooth} \\ 0 & \text{elsewhere} \end{matrix} \tag{10}$$

which meets the twin requirements of zero net force at points between the teeth and a net tooth force with the correct dependency on x.

Hence the component vector line force, $\tilde{\mathbf{f}}_l$ associated with a trace wavenumber k_l is related to the total tooth vector point force, $\mathbf{f}_\text{P}$, through equation 6, such that

$$g\,\tilde{\mathbf{f}}_l(k_l) = \Psi_l(k_l)\mathbf{f}_\text{P} \tag{11}$$

4 Governing equations for the line junction

With the exception of the comb connection, the line forces and velocities for all 2*N*+1 trace wavenumbers implied by equation 9 are independent from each other due to the form of equation 9. Thus, for each k_l ,say, it is possible to write

$$\tilde{\mathbf{Z}}_l(k_l)\tilde{\mathbf{v}}_l(k_l) = \tilde{\mathbf{e}}_l(k_0) - \tilde{\mathbf{f}}_l(k_l) \tag{12}$$

where $\tilde{\mathbf{Z}}_l(k_l)$ is the usual junction line wave impedance ignoring the comb, see Heron (1997). $\tilde{\mathbf{v}}_l(k_l)$ is the junction line wave velocity, $\tilde{\mathbf{f}}_l(k_l)$ is the junction line wave force due to the comb reactions, and $\tilde{\mathbf{e}}_l(k_0)$ is the external junction line wave force due to the incident wave; the latter will be zero for all $l \neq 0$.

All teeth are assumed to be identical. The velocity $\mathbf{v}_\text{P}$ and force $\mathbf{f}_\text{P}$ on the two ends of a single tooth are related through the standard 12 x 12 cross point impedance $\mathbf{Z}_\text{P}$ of the tooth such that

$$\mathbf{Z}_\text{P}\mathbf{v}_\text{P} = \mathbf{f}_\text{P} \tag{13}$$

The velocity at a tooth, $\mathbf{v}_\text{P}$, is related to the line wave velocity components $\tilde{\mathbf{v}}_l(k_l)$ through the equation

$$\mathbf{v}_\text{P} = \sum_{l=-N}^{+N} \Psi_l^\text{H}(k_l)\tilde{\mathbf{v}}_l(k_l) \tag{14}$$

because at the teeth all the components are in phase through equation 9.

Equations 11, 12, 13 and 14 form a closed set of equations for $\mathbf{v}_\text{P}$, $\mathbf{f}_\text{P}$, $\tilde{\mathbf{v}}_l(k_l)$ and $\tilde{\mathbf{f}}_l(k_l)$, given $\tilde{\mathbf{e}}_l(k_0)$. In practice it has been found preferable to use the cross transfer matrix for a tooth instead of the cross impedance matrix $\mathbf{Z}_\text{P}$. Furthermore it has been found that the equations are best solved by first computing $\mathbf{v}_P$ and $\mathbf{f}_P$ (by eliminating $\tilde{\mathbf{v}}_l(k_l)$ and $\tilde{\mathbf{f}}_l(k_l)$ .from equations 11, 12 and 14)and subsequently calculating $\tilde{\mathbf{v}}_l(k_l)$ and hence the line junction transmission matrix.

Bosmans and Vermeir (1996 and 1997) developed a more extensive theory to include the finite length of the teeth but only for a much simplified stiffness model for each tooth. However it would clearly be possible to merge these two closely related theories should this become necessary.

5 The experimental assembly

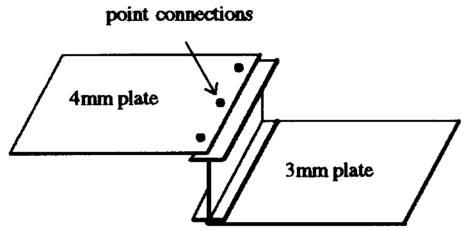

figure 2 Basic assembly

A diagram of the basic two plate assembly is shown in figure 2; two flat isotropic aluminium plates were connected together using an aluminium I-beam. The 3mm plate had a length of 1.5m and a width of 0.825m, and the 4mm plate had a length of 1m and a width of 0.825m. Damping treatment was applied to both plates to give an estimated bending energy damping value of 1.5% for both plates over all frequencies; this value was used in the theoretical models.

The 3mm plate was always directly connected to the I-beam using 33 bolts with a bolt spacing of 25mm. The 4mm plate was connected by a variable number of equally spaced 'sleeved' bolts. Each plate was mechanically excited at 5 randomly chosen positions and the response of each plate was measured using 5 randomly positioned miniature accelerometers. The input drive point acceleration level was measured.

6 Small spacer results

A single 'small spacer tooth' comprised a steel bolt, a washer, and an aluminium sleeve. This 'tooth' was modelled as two beams in parallel (an inner solid steel cylinder with a diameter of 5mm and a height of 7.5mm, and an outer hollow aluminium cylinder with an outer diameter of 8mm, an inner diameter of 5mm and a height of 7.5mm) sandwiched between the 4mm plate and the I-beam. The steel bolt head (diameter 8mm, height 5mm) and the steel washer (diameter 10mm, height 1mm) were modelled as rigid masses and 'added' to the tooth at a position just above the 4mm plate.

The results from the 9 point connection case, which corresponds to a tooth spacing of 100mm, are shown in figure 3. The mean square spatially averaged velocity response of the 4mm plate due to forcing on the 3mm plate has been normalised using the drive point velocity. The experimental results are shown as thin lines; the two outer dotted lines are the 90% confidence limits based on the five force and five response measurements. All the experimental data were obtained using a frequency bandwidth of 100Hz.

The theory is plotted for different values of the parameter N. The number of trace wavenumbers within a model is 2N+1; that is the sum over wavenumber components is taken from -N to +N. As can be seen the theory is fully convergent even at N=1, and the agreement between theory and experiment is good. All theoretical results have been calculated at 500Hz intervals and no frequency averaging has been used. The I-beam was modelled using infinite strip plate theory, see Heron (1997).

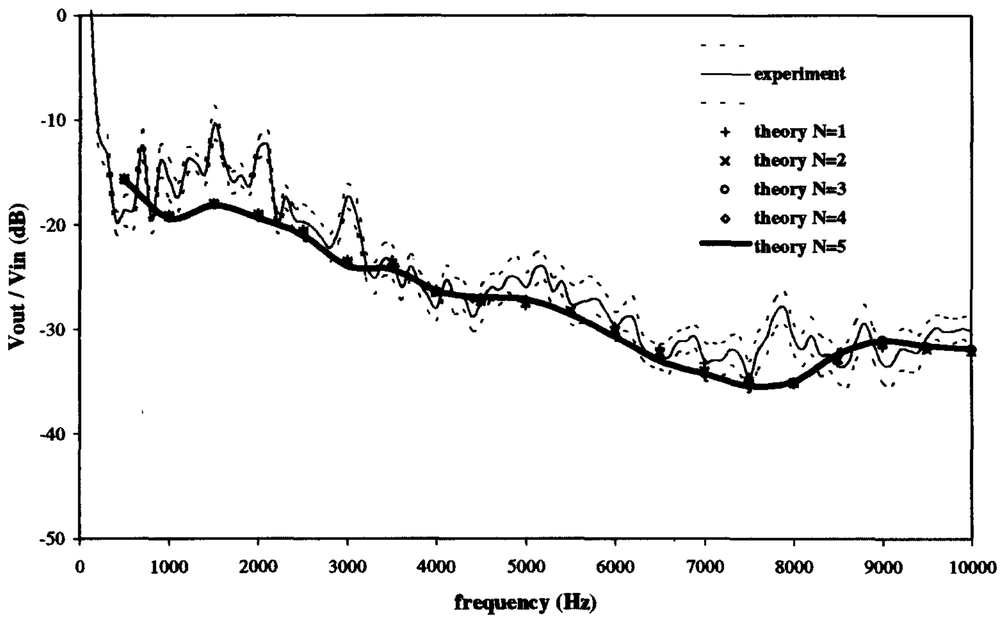

figure 3 Small spacer with 9 point connections

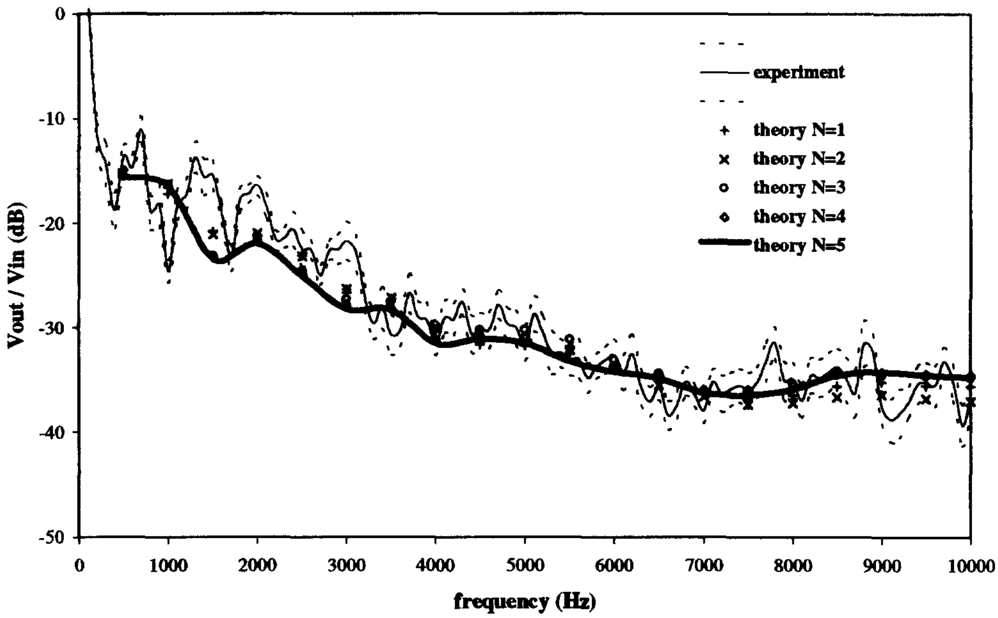

figure 4 Small spacer with 3 point connections

Figure 4 shows the results when the number of teeth is reduced from 9 to 3. In both cases the two outer teeth are kept at a distance of 12.5mm from the end of the connection line, thus end effects, ignored in the theory, could play a significant role at the lower frequencies; particularly with regard to the 3 point connection case.

Good agreement between the theoretical predictions and the experimental results is shown. Indeed the agreement is remarkably good here since intuitively the 3 point connection case was not expected to be modelled accurately by a theory based on an infinite line of equally spaced points.

7 Large spacer results

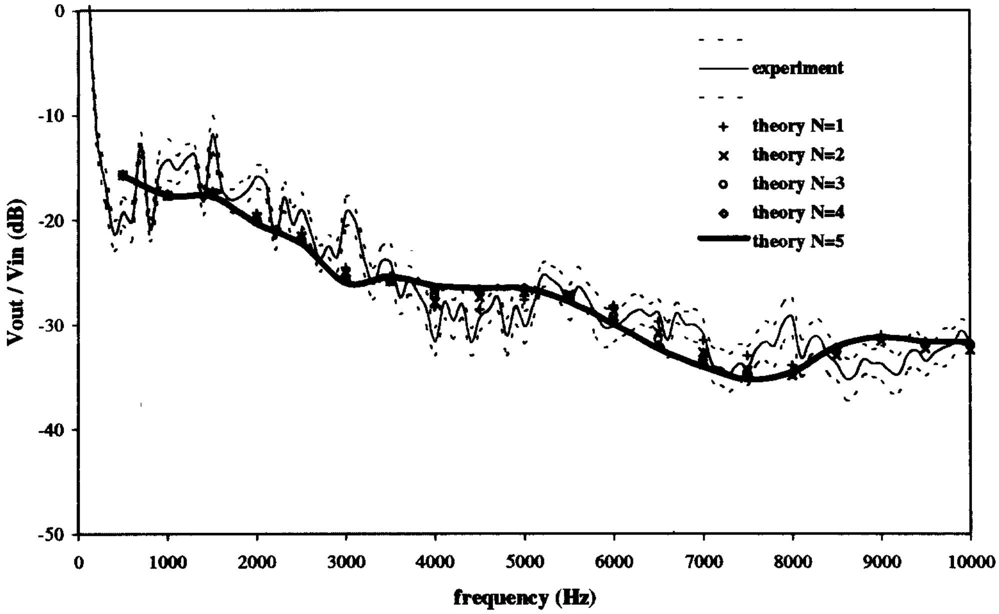

figure 5 Large spacer with 9 point connections

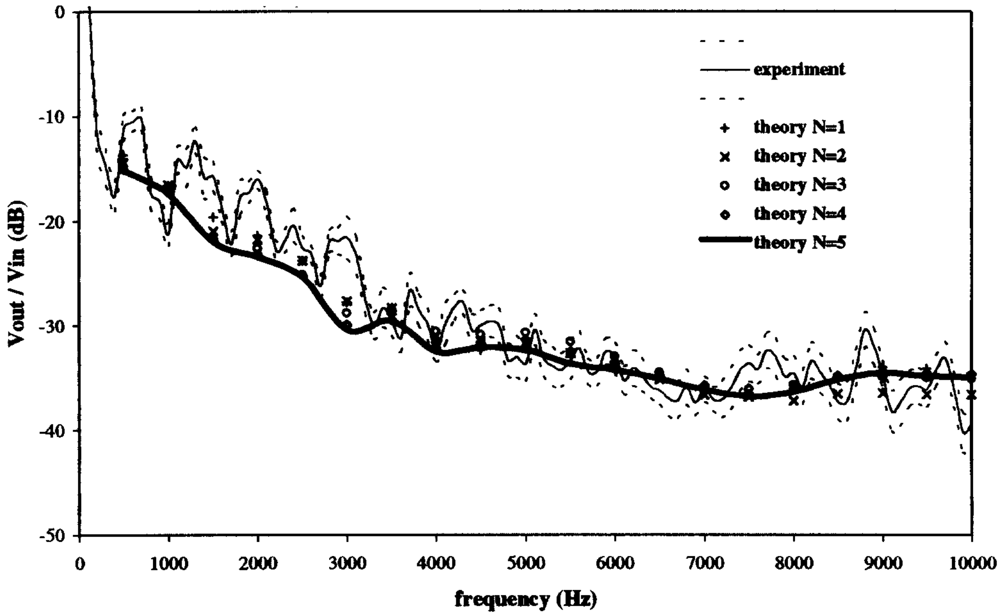

figure 6 Large spacer with 3 point connections

Figures 5 and 6 are the results for a large spacer, with 9 and 3 point connections. The only difference between the small and large spacers was the height and outer diameter of the sleeves which were both increased to 15mm. As with the small spacer results, good agreement has been obtained between the theoretical predictions and the experimental results.

8 Discussion

Overall, the comparison between the experimental results and the theoretical predictions is very good. The small discrepancies at the lower frequencies can be attributed to end effects which are not modelled within the theory.

This is an important step forward for predictive SEA because many apparent line junctions are often, in reality, a series of point connections and this new theory can be used to model such junctions accurately. Examples are a line of rivets in an aircraft fuselage assembly, or a line of nails connecting a plaster board to a wooden batten. The theory could even be used when there are just two connections, such as at a door hinge line; here we would be modelling the infinite extension of the door and hinges with the hinge spacing fixed, a not unreasonable model. There is also some evidence, see Monger *et al* (1997), that when a single plate is riveted across a centre line to create two plates, simple line connection theory is inaccurate at the higher frequencies and the theory developed herein should solve this problem.

9 Conclusions

A mathematical model has been developed for the complex joints associated with line connections which are in fact a series of equally spaced point connections, where each point or 'tooth' is a fully dynamic component. The theory makes no assumption about the ratio of the plate wavelength to the point separation distance, nor about the particular tooth design.

This development is a significant extension to existing SEA theory with the aim of allowing a general line junction comprising semi-infinite plates, infinite strip plates, beams and now equally spaced point connections to be modelled.

Very good agreement has been obtained between the experimental results from some specially designed two-plate assemblies and the predictions using this new theory.

10 References

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