EXPERIMENTAL SPATIAL MATRIX IDENTIFICATION METHOD

(PRESENTATION OF CURRENT THEORY AND FUNDAMENTAL VERIFICATION)

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ABSTRACT

In this paper, the authors present a new experimental spatial matrix identification method that they have been developing. The method is to identify a set of the mass, damping and stiffness matrices that can represent the dynamic characteristics of an objective structure from experimental FRFs. The theory of the method is explained at first. Then, the result of an identification of a basic frame structure, which is made of L-shaped cross-sectional steel components, under the free-free boundary condition is presented. Both bending vibration modes and torsional vibration modes are located in the frequency range of the identification. The dynamic characteristics of the specimen under a different boundary condition are estimated from the previously identified set of spatial matrices, and compared with experimental results to verify the practical validity and usefulness of the method.

1. INTRODUCTION

Under the assumption of viscous damping, the equations of motion are expressed as

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = \{f\} \tag{1}$$

where $[M]$, $[C]$ and $[K]$ are the mass matrix, the viscous damping matrix and the stiffness matrix, respectively. And, $\{x\}$ and $\{f\}$ are the displacement vector and the applied force vector, respectively.

They say that it will be difficult to identify a set of spatial matrices, that can represent the dynamic characteristics up to a high frequency of interest with respect to mechanical structures, from experimental FRFs. Then, in the field of experimental approaches, modal parameters are identified from FRFs generally. Many methods [1] were proposed to identify modal parameters from FRFs. As examples of challenges to identify spatial matrices, there are papers by Leuridan [2], Roemer [3], Minas [4], Peterson [5], and Okuma [6], etc. From the practical viewpoint of structural dynamic analysis, it will be generally required to identify a set of spatial matrices whose number of degrees of freedom is much larger than the number of natural modes in the frequency range of identification. The set of spatial matrices is required the capability to represent the dynamic characteristics of the specimen under even different boundary conditions in the same frequency range. The theories of those papers will not be available for the identification under these practical requirements.

The theory of this paper can identify a set of spatial matrices that represent the dynamic characteristics of objective structures under the practical situation. In the next chapter, the theory is explained. In the 3rd chapter, a set of spatial matrices is identified with experimental FRFs of 22 measurement points with respect to a frame structure under the free-free boundary condition. The validity of the theory is verified by estimating FRFs between any two measurement points, which are not used in the identification, and by predictions of the dynamic characteristics under a different boundary condition.

2 . THEORY

The theory is explained briefly due to the limit of pages here. On vibration measurement, measure or determine the coordinates of all measurement points as well as FRFs. Because they are required as one of input data for the identification.

At first, create a physical connectivity , which is called "physical modeling", among the measurement points by yourself. By this definition, constraint equations can be computed with respect to the elements of spatial matrices. According to the principle that any mass matrix with any number of degrees of freedom must be transformed into a rigid body mass matrix, Eq.(2) can be formulated as

$$[\Psi]^T[M][\Psi] = [M_{rigid}] \tag{2}$$

where $[\Psi]$ is a set of mutually independent rigid motion modes that can be formulated with the coordinates of measurement points. The element formation of any rigid body mass matrix is well known as.

$$[M_{rigid}] = \begin{bmatrix} m & & & & & \\ 0 & m & & & sym. & \\ 0 & 0 & m & & & \\ 0 & -C & B & I_{xx} & & \\ C & 0 & -A & I_{yx} & I_{yy} & \\ -B & A & 0 & I_{zx} & I_{zy} & I_{zz} \end{bmatrix} \tag{3}$$

where m : mass quantity

I_{xx}, I_{yy}, I_{zz} : Inertia of moment around x-axis, y-axis and z-axis

I_{xy}, I_{yz}, I_{zx} : Product of Inertia of moment

$A = mX_g,\ B = mY_g,\ C = mZ_g$

(X_g, Y_g, Z_g) : Coordinates of Center of Gravity

Therefore, some constraint equations regarding the mass matrix will be formulated by Eq.(2).

With respect to the stiffness matrix, Eq.(4) can be formulated according to the principle that no stress is generated in all parts of a structure under any rigid body motion. So can the constraint equations regarding the viscous damping matrix be.

$$[K][\Psi] = [0] \tag{4}$$

where $[0]$: zero matrix

Next, determine an identification frequency range. The frequency range must include the resonant modes from the first order. Identify the undamped natural frequencies locating in the identification frequency range, the associating natural modes and modal damping ratios approximately by observing FRFs. It is useful to use MIF (Mode Indicator Function [7]) to estimate natural frequencies and modal damping ratios [8] in the cases of not so heavy damping.

Now, initial values of elements of spatial matrices are set by random numbers under subject to the constraint equations because of a very quick way. Then, the initial mass and stiffness matrices are improved to become a positive definite matrix and a semi-positive definite matrix respectively by the sensitivity analysis of eigenvalues with respect to the elements of the matrices. Negative eigenvalues should become positive numbers. The sensitivity analysis of diagonal elements of spatial matrices to become large positive numbers and non-diagonal elements to approach zero assists the achievement. Next, the undamped natural frequencies, calculated by Eq.(5), of interested orders in the identification frequency range are controlled to correspond with the targets by the sensitivity analysis of undamped natural frequencies.

$$\left([K] - \Omega^2[M]\right)\{\phi\} = \{0\} \tag{5}$$

where $\{\phi\}$: natural mode
Ω : natural angular frequency

The natural frequencies must be controlled satisfactorily. If not, the "physical modeling" should be considered unacceptable. When the correspondence of all natural frequencies of interest is satisfied, their associating natural modes are improved to correspond with the targets by the sensitivity analysis of natural modes. The feedback route is necessary for the computational process to keep the matrices positive definite ones here. When both natural frequencies and natural modes of interest correspond with their targets satisfactorily, move forward to improve the viscous damping matrix. If not, bad corresponding natural modes calculated by Eq.(5) are replaced by their target vectors coercively under the treatment of the normalization of the vectors with respect to the mass matrix. Let us denote the resultant natural mode matrix by $[\Phi']$. Then, the mass and stiffness matrices are modified by Eq.(6). The spatial matrices become full coefficient matrices, respectively in this case.

$$\begin{aligned} [M] &= \left([\Phi']^T\right)^{-1}[\,\mathrm{I}\,][\Phi']^{-1} \\ [K] &= \left([\Phi']^T\right)^{-1}[\Omega^2][\Phi']^{-1} \end{aligned} \tag{6}$$

where $[\Phi']$: natural mode matrix into suitable columns of which target mode vectors are substituted

Now, here is the process to improve the viscous damping matrix. At first, the viscous damping matrix is created by copying the resultant stiffness matrix. Then, a scalar coefficient is multiplied into the damping matrix to satisfy the equality of both sides of Eq.(6) as well as possible with respect to the eigenvalues of interested orders calculated by Eq.(8). Furthermore, by the sensitivity analysis of eigenvalues with respect to elements of the viscous damping matrix, the viscous damping matrix is improved to satisfy the relation of Eq.(7) better regarding the interested orders of eigenvalues of Eq.(8) under the constraint that the associating natural modes keep correspondence well with those of Eq.(5).

$$\lambda_r = 2\zeta_r \Omega_r \ \left(r = 1 \sim n\right) \tag{7}$$

where n: the number of resonant modes of interest in the identification frequency range

$$\left([C] - \lambda[M]\right)\{\phi\} = \{0\} \tag{8}$$

When the control can be achieved, go forward to the last process. If not, bad corresponding eigenvalues of interested orders are replaced by the values calculated by the equation in the right hand side of Eq.(7). In this case, the viscous damping matrix will become a full coefficient matrix.

On the last process, the most suitable real number is multiplied to all spatial matrices to make FRFs calculated from the set of the spatial matrices fit with experimental FRFs as well as possible at first. Then, the set of spatial matrices is improved to fit FRFs much better by a mathematical optimization method. The authors currently utilize the steepest descent method for this process basically.

It should be noted here that a set of spatial matrices identified is not the unique solution of the dynamics of an objective structure because of a system identification from experimental FRFs of a single point excitation and the limit of the frequency range. However, the set of spatial matrices can represent the dynamic characteristics of an objective structure in the identification frequency range even under changing the boundary condition and/or connecting some additional masses , etc., approximately. Therefore, they can be used for many kinds of practical analyses and simulations.

3 . BASIC VERIFICATION

The practical validity of the theory is verified basically by an application to a frame structure made of L-shaped cross-sectional steel components as shown in Fig.1. By applying a hammering force at the measurement point No.1, FRFs in the normal direction of the structure plane are measured at 22 measurement points (#1~#22) under the free-free boundary condition, which is realized by suspending the structure with 4 rubber strings. Then, the number of degrees of freedom of spatial matrices is 22. The identification frequency range is set from 5 Hz up to 180 Hz. There are locating the first four resonant modes in the frequency range. The resonant modes are the first torsional mode at about 14 Hz, the fist bending mode at about 86 Hz, the second torsional mode at about 115 Hz and the second bending mode at 177 Hz, respectively. It should be considered in this case that there are three independent rigid motion modes at zero Hz.

Fig.2 shows an example of fitting of FRFs between the experimental FRFs and the calculation from the set of spatial matrices identified. Table1 lists all the natural frequencies and their associating modal damping ratios obtained from the set of spatial matrices. The first three zero natural frequencies mean the rigid motion modes. The values in "Damping ratio" associating to those rigid motion modes do not mean the modal damping ratios but the real parts of eigenvalues. It is recognized that the three modes have no damping. The values from the fourth to the seventh orders are the parameters of the resonant modes locating in the identification frequency range. All other residual natural frequencies can be controlled to locate at the higher frequencies than the identification frequency range by the proposed method. Fig.3 shows the convergence of fitness of FRFs on the last process of the theory. The Solid line denotes the convergence by changing only the stiffness matrix, only the mass matrix, only the damping matrix and all of the matrices together one after the other on iterations. The dotted line denotes the convergence from the same initial matrices by changing all of the spatial matrices simultaneously on every iteration.

The first verification of the validity of the identification will be to show good fitness of FRFs that are not input in the identification. Fig.4 shows an example of them. The driving point and the response point are the measurement point NO.11 and NO.14, respectively. As shown in Fig.4, the FRFs between any two measurement points can be simulated from the set of the spatial matrices.

The second verification will be to show good predictions of the dynamics of the objective structure under a different boundary condition. Fig.5 shows an example of predicted FRFs of the structure under clamping four measurement points (#10, #11, #21, #22). Namely, the structure looks like a cantilever. The analysis is very easy because of using a set of spatial matrices. Four resonant vibration modes can be observed at about 12Hz, 20Hz, 90Hz and 112Hz on the experimental FRF. Table2 lists all natural frequencies calculated with the set of the spatial matrices under the boundary condition. From Fig.5 and Table2, it can be understood that the prediction is practically acceptable and that the method is practically valid. If you try to predict the dynamics of the same situation by the experimental modal analysis, you have to prepare three sets of FRFs by changing excitation locations [9], or prepare the rigid body properties by other means.

As the third verification, the rigid body properties calculated from the identified mass matrix are listed in Table 3 together with those by measurements and manually simple estimations. The quantity of mass and the center of gravity are identified from the mass matrix with the practically acceptable accuracy.

By the above mentioned verifications, it will be understood that the identification method presented in this paper possesses the practical validity and usefulness basically to identify a set of spatial matrices.

4 . CONCLUSIONS

The authors have presented the newest theory of the experimental spatial matrix identification method that they have been developing. By an identification and basic verifications regarding an actual frame structure, they have demonstrated the practical validity and usefulness of the method.

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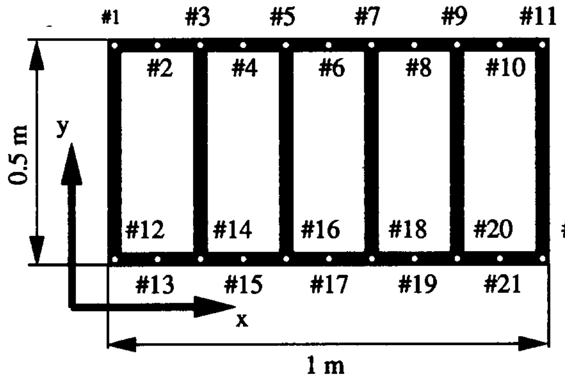

Fig.1 A Frame Structure as Specimen

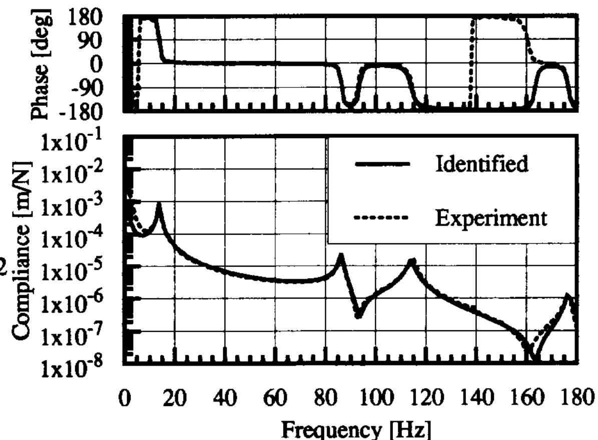

Fig.2 Fitting of FRFs

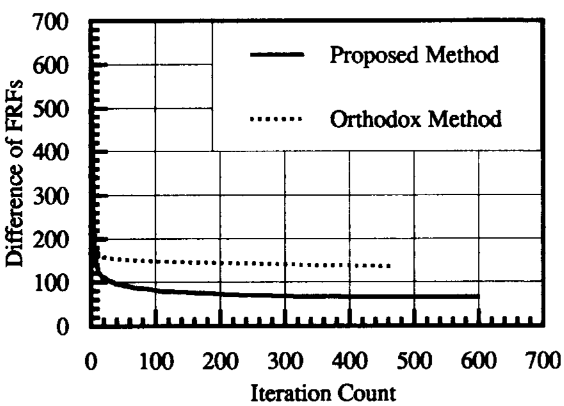

Fig.3 Convergence of Fitness of FRFs on ' the Second Improvement '

Table1 Natural Frequencies and Dampings calculated from Identified Matrices

Order	Natural Freq.s [Hz]	Damping Ratios	Order	Natural Freq.s [Hz]	Damping Ratios
1	0	0	12	477.3	0.0453
2	0.0	0	13	494.9	0.0416
3	0	-0.0001	14	523.0	0.0532
4	13.8	0.0288	15	535.1	0.0475
5	86.4	0.0071	16	566.5	0.0609
6	114.0	0.0126	17	586.8	0.0643
7	176.6	0.0048	18	594.7	0.0605
8	223.5	0.0348	19	618.1	0.0676
9	281.6	0.0211	20	626.8	0.0706
10	400.3	0.0451	21	674.8	0.0704
11	447.1	0.0393	22	737.1	0.0696

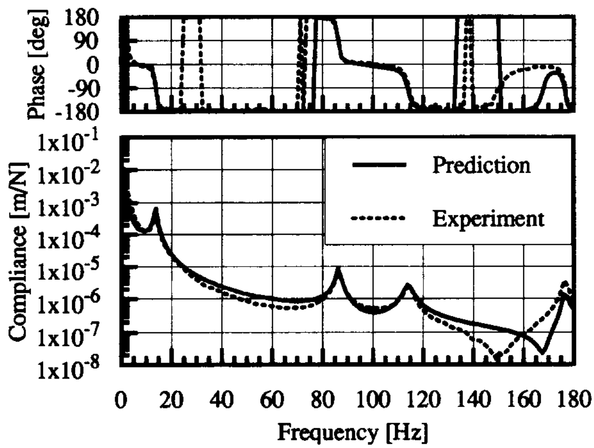

Fig.4 Predicted FRFs of Structure Excited at an Another Point

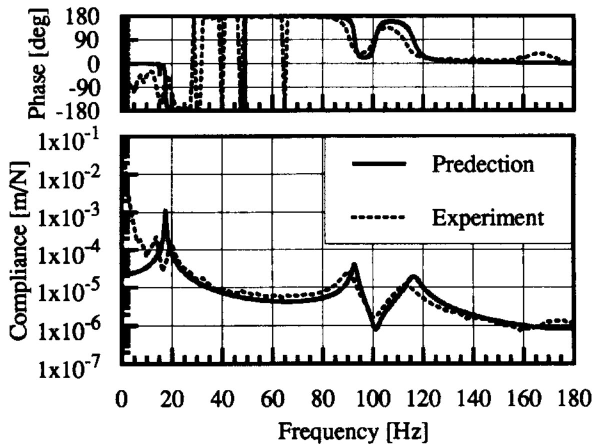

Fig.5 Predicted FRFs of Structure under a Different boundary Condition

Table2 Predicted Natural Frequencies and Dampings of Structure under a Different boundary Condition

Order	Natural Freq.s [Hz]	Damping Ratios	Order	Natural Freq.s [Hz]	Damping Ratios
1	15.1	0.0424	10	478.8	0.0503
2	17.5	0.0092	11	508.6	0.0541
3	92.8	0.0048	12	520.3	0.0633
4	116.2	0.0084	13	578.1	0.0610
5	188.7	0.0197	14	585.7	0.0418
6	265.8	0.0059	15	603.5	0.0696
7	290.2	0.0257	16	621.2	0.0701
8	418.8	0.0421	17	667.7	0.0697
9	444.5	0.0465	18	736.8	0.0644

Table3 Rigid Body Properties from Identified Mass Matrix

	From the mass matrix	Measurement	Approximate manual calculation
Mass[kg]	6.6	6.5	6.7
Center of Gravity [m]	Xg = 0.466 Yg = 0.242 Zg = 0.0	0.482 0.244 0.0	0.490 0.240 0.0
Principal Inertia of Moments [kgm^2]	I_1=0.374 I_2=0.709 I_3=0.0	0.26 ± 0.07 0.82 ± 0.15 0.0	0.24 0.63 0.0
Principal Axes of Inertia Moments	I_1–*axis* $\begin{Bmatrix} -0.99 \\ 0.013 \\ 0.0 \end{Bmatrix}$ I_2–*axis* $\begin{Bmatrix} -0.013 \\ -0.99 \\ 0.0 \end{Bmatrix}$ I_3–*axis* $\begin{Bmatrix} 0.0 \\ 0.0 \\ 1.0 \end{Bmatrix}$	Assumed as $\begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}$ $\begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$ $\begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}$	Assumed as $\begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix}$ $\begin{Bmatrix} 0 \\ 1 \\ 0 \end{Bmatrix}$ $\begin{Bmatrix} 0 \\ 0 \\ 1 \end{Bmatrix}$