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COUPLED STRESS DISTRIBUTION IN A VIBRATING ROD SUBJECTED TO VARIABLE TEMPERATURE AND MOISTURE†

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ABSTRACT

Thermal expansion, moisture expansion (swelling) and elastic displacements may be analyzed independently for static or quasi-static loads and slowly varying temperature or moisture inputs. In that case displacements produced by Fourier heat conduction and the analogous Fick moisture transport may be uncoupled from elastic displacements. On the other hand, in hygroscopic materials, due to the Soret and Dufour effects cross-coupling takes place when either of these inputs: thermal, hygroscopic and mechanical, are applied at a high rate.

Using the equations of Thermo-Hygro-Elasticity, the problem of a vibrating rod immersed in an environment with given temperature and mositure content is solved. For comparison the stress distributions due to the coupled inputs as well as due to uncoupled inputs will be calculated.

1. INTRODUCTION

The state of stress in hygroscopic elastic materials is affected by moisture expansion in addition to thermal and mechanical loads.

Under static and slowly varying temperature, moisture content, and load, the parameters, such as coefficients of expansions, conductivity, absorivity, moduli of elasticity, etc. are inter-dependent, but the equations of heat transfer, moisture transport and equilibrium are uncoupled.

† This paper is dedicated to Professor Franz Ziegler of the Technical University of Vienna, Austria, on the occasion of his 60th birthday.

Where either of these inputs is applied at a high rate, the parameters may be considered invarient, but the equations of motion are coupled.

The state of stress in an infinite rod immersed in water with a given temperature and subjected to axial vibrations will be analyzed. The problem has practical significance for bridge piers undergoing traffic induced stresses, off-shore drilling towers and drill strings, submerged piles, etc.

Because the effects are most significant in Thermo-Hygro-Elastic (THE) materials such as concrete, timber and composites, a uniaxially reinforced graphite epoxy rod is examined.

The indicated coupled effects require that Hook's law be combined with Fourier's heat conduction equations and Fick's, analogous, moisture diffusion relations with Soret's and Dufour's cross-coupling of the latter two.

As a result the generalized equations of motion become[1-5]:

$$\mu u_{,jj} + (\mu + \lambda) u_{k,ki} + \rho b_i - \beta_T T_{,i} - \beta_m m_{,i} = \rho \ddot{u}_i \tag{1.1}$$

$$D_m m_{,ii} + d_m^T T_{,ii} - \dot{m} - \frac{\beta_m m_o D_m}{k_m} \dot{u}_{j,j} = 0 \tag{1.2}$$

$$d_T^m m_{,ii} + D_T T_{,ii} - \dot{T} - \frac{\beta_T T_o}{\rho c} \dot{u}_{j,j} = 0 \tag{1.3}$$

Here, u_i, T and m are the displacement, temperature difference and moisture concentration; μ and λ Lamé constants; ρ the denisty; b_i the body force; D_m, D_T, d_m^T and d_T^m the moisture diffusivity, the temperature diffusivity and the cross-coupled diffusivities, respectively; $\beta = (3\lambda + 2\mu)\alpha_T$ and $\beta_m = (3\lambda + 2\mu)\alpha_m$ where α_T and α_m are coefficients of thermal and moisture expansions; T, c, k_m, T_o and m_o are the specific heat, moisture conductivity, temperature and moisture concentration in the natural state; the dot designates material time derivatives, and the comma space derivatives.

2. VIBRATIONS OF AN INFINITE ROD

An infinite rod, presented in Fig. 1, is subjected to oscillating loads with frequency, ω.

The generalized equations of motion, Eqs. 1.1-1.3 involve the cross-coupling effects of moisture and temperature. The solution of these equations is extremely complicated. To make the problem tractable, only the coupled effects of load and temperature or load and moisture concentration will here be considered.

Because of the analogy between heat-transfer and moisture transfer, either of these variables will be denoted by ϕ and the three generalized equations will be reduced to:

$$\mu u_{,jj} + (\mu + \lambda)\, u_{k,ki} - \eta \phi_{,i} = \rho \ddot{u}_i \tag{2.1}$$

$$\delta \phi_{,ii} - \dot{\phi} - \kappa \dot{u}_{j,j} = 0 \tag{2.3}$$

where the body force, b_i, is neglected, $\eta = (3\lambda + 2\mu)\alpha_\phi$. δ the diffusivity and $\kappa = T_o\eta/\rho c$ (ρ = density, c = heat or moisture capacity) with α_ϕ the coefficient of expansion, for either temperature or moisture.

The displacement components u_z and u_r will be harmonic functions with frequency ω and wave number ζ:

$$u_z(z,r,t) = f_z(r)e^{i(\zeta z-\omega t)} \tag{2.4}$$

$$u_r(z,r,t) = f_r(r)e^{i(\zeta z-\omega t)} \tag{2.5}$$

and due to axial symmetry, $u_\theta = 0$.

The coupled temperature or moisture is also a function of the frequency and wave number

$$\phi(z,r,t) = \phi_o + f_\phi(t)e^{i(\zeta z-\omega t)} \tag{2.6}$$

with ϕ_o the stress free temperature or moisture concentration of the porous rod. In the case of moisture concentration, under the application of the harmonic load, voids are periodically opened and closed resulting in variable moisture content, liquid being pumped into and out of the surrounding liquid.

The solution of the problem is in terms of complex Bessel functions with real and imaginary parts.

$$u_z = -e^{i(\zeta z-\omega t)}\alpha J_1(r\alpha)A - ie^{i(\zeta z-\omega t)}\zeta J_1(r\beta)B \tag{2.7}$$

$$u_r = ie^{i(\zeta z-\omega t)}\zeta J_o(r\alpha)A + e^{i(\zeta z-\omega t)}\beta J_o(r\beta)B \tag{2.8}$$

and

$$\phi = e^{i(\zeta z-\omega t)}J_o(r\alpha)C \tag{2.9}$$

where

$$\alpha^2 = \frac{w^2}{(c_1^u)^2} - \zeta^2 \ , \ \beta^2 = \frac{w^2}{c_2^2} - \zeta^2 \tag{2.10}$$

with c_1^u and c_2 the uncoupled dilatational and shear wave velocities, respectively[6]

$$(c_1^u)^2 = \frac{(\lambda + 2\mu)}{\rho} \ , \ c_2^2 = \frac{\mu}{\rho} \tag{2.11}$$

and C the amplitude of the harmonic temperature or moisture concentration.

$$C = -\frac{i(\alpha^2\omega\kappa + \zeta^2\omega\kappa)}{-i\omega + \alpha^2\delta + \zeta^2\delta}A \tag{2.12}$$

A and B are artibrary constants solved from boundary conditions on stresses.

The coupled dilatational wave velocity becomes

$$c_1^2 = \frac{\rho\omega^2}{(\lambda+2\mu)}\left\{\frac{1}{2}\left[1+i\left(\frac{\lambda+2\mu}{\delta\rho\omega}+\frac{\eta k}{\delta\rho\omega}\right)\right.\right.$$

$$\left.\left.+\sqrt{1-2i\left(\frac{\lambda+2\mu}{\delta\rho\omega}-\frac{\eta k}{\delta\rho\omega}\right)-\left(\frac{\lambda+2\mu}{\delta\rho\omega}+\frac{\eta k}{\delta\rho\omega}\right)^2}\;\right]\right\}-\zeta^2 \tag{2.13}$$

c_2 is not affected by coupling.

The stress components are derived from displacement equations. In the uncoupled case σ_{rr}, $\sigma_{\theta\theta}$, σ_{zz} and σ_{rz} are:

$$\sigma_{rr}^u = -A\left[\left((\lambda+2\mu)\,\alpha^2+\lambda\zeta^2\right)J_o(\alpha r)-2\alpha\mu\frac{1}{r}J_1(\alpha r)\right]$$

$$-\,iB\left[\mu\zeta(\beta J_o(\beta r)-\frac{1}{r}J_1(\beta r))\right] \tag{2.14}$$

$$\sigma_{\theta\theta}^u = -A\left[\lambda\left(\alpha^2+\zeta^2\right)J_o(\alpha r)+2\alpha\mu\frac{1}{r}J_1(\alpha r)\right]$$

$$-\,2iB\mu\zeta\frac{1}{r}J_1(\beta r) \tag{2.15}$$

$$\sigma_{zz}^u = -A\left[\alpha^2+(\lambda+2\mu)\zeta^2\right]J_o(\alpha r)+2iB\beta\mu\zeta J_o(\beta r) \tag{2.16}$$

$$\sigma_{rz}^u = -2iA\alpha\lambda\zeta J_1(\alpha r)-B(\beta^2-\zeta^2)\lambda J_1(Br) \tag{2.17}$$

with

$$B^u = -A\frac{C_{11}^u}{C_{12}^u} \tag{2.18}$$

$$C_{11}^u = -\left((\lambda+2\mu)\alpha^2+\lambda\zeta^2\right)J_o(\alpha r_o)+2\alpha\mu\frac{1}{r_o}J_1(\alpha r_o) \tag{2.19}$$

$$C_{12}^u = -2i\mu\zeta\left(\beta J_o(\beta r_o)-\frac{1}{r_o}J_1(\beta r_o)\right) \tag{2.20}$$

All other stress components are zero and on the boundary $\sigma_{rr}(r_o)=\sigma_{rz}(r_o)=0$.

For the coupled case the four non-zero stress components are given as

$$\sigma_{rr} = \sigma_{rr}^u+(3\lambda+2\mu)\alpha_\phi\phi \tag{2.21}$$

$$\sigma_{rr} = \sigma_{rr}^u+(3\lambda+2\mu)\alpha_\phi\phi \tag{2.22}$$

$$\sigma_{\theta\theta} = \sigma_{\theta\theta}^u+(3\lambda+2\mu)\alpha_\phi\phi \tag{2.23}$$

$$\sigma_{zz} = \sigma_{\theta\theta}^u+(3\lambda+2\mu)\alpha_\phi\phi \tag{2.24}$$

with

$$\phi = CJ_o(\alpha r) \tag{2.25}$$

and the amplitude C is the same as Eq. 2.12 using $B = -\frac{C_{11}}{C_{12}}A$ in the σ^u terms

$$C_{11} = C_{11}^u + C(3\lambda + 2\mu)\alpha_\phi J_o(\alpha r) \tag{2.25}$$

C_{12} is the same as in Eq. 2.14.

3. A NUMERICAL EXAMPLE

The coupled stress distribution in an isotropic rod with an outer radius $r_o = 0.1m$, subjected to harmonic oscillations and a given temperature field is presented. The relevant thermal and mechanical parameters are as follows:

Modulus of elasticity	$E = 6.4$ GPA
Poisson's ratio	$\nu = .23$
Density	$\rho = 1590$ kg/m^3
Thermal coeff. of expansion	$\alpha_T = 31.3 \times 10^{-6}$ 1/C°
Specific Heat	$c = 800$ m^2/sec^{2}K°
Diffusivity	$\delta = 2.57 \times 10^{-7}$ m^2/sec
Temperature	$\phi_o = 310K°$

While only a thermal problem is presented, an analogous hygro-elastic rod may be examined if the required hygroscopic parameters are used.

The stress components, as functions of the radial coordinates are plotted in Fig. 2. for $\omega = 200\pi$ rad/sec. They are presented in Fig. 3 versus excitation frequencies, ω varying between 0 and 200 π at the outer radius of the rod.

It is seen that coupling results in variations similar to those produced by damping. These effects are small in the present case but would be more significant for larger coefficients of thermal expansion.

4. CONCLUSIONS

The coupled equations of motion were presented for thermo-hygro-elastic materials. The general problem has been simplified by neglecting the coupling effects of temperature and moisture concentration. Only those of dynamic load and temperature or dynamic load and moisture were considered.

Displacements and stress components were analyzed in an infinity long vibrating rod subjected to a given environment. The effects of coupling on stresses were presented as functions of the radial coordinate and of frequency.

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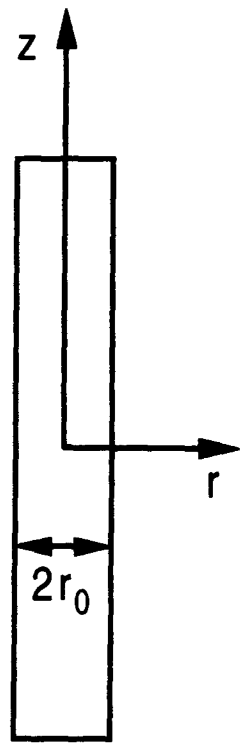

Fig. 1. Configuration of the rod.

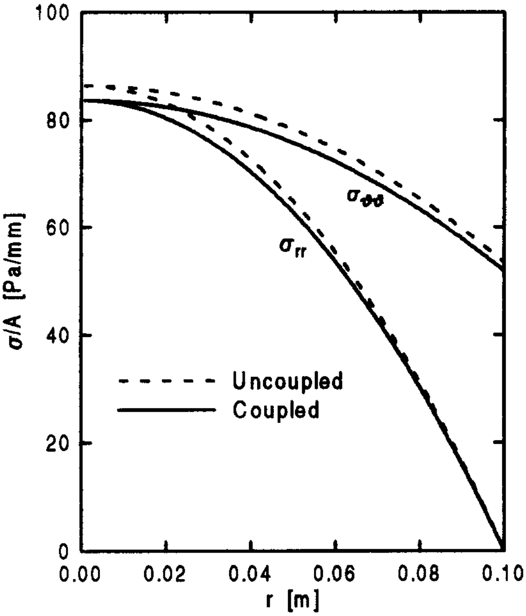

Fig. 2. Variations of uncoupled and coupled stress amplitudes as functions of radial distance.

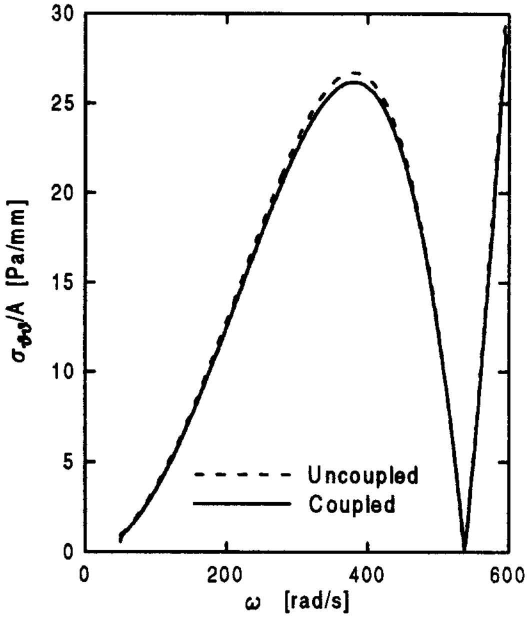

Fig. 4. Variations of uncoupled and coupled stress amplitudes as functions of loading frequency.

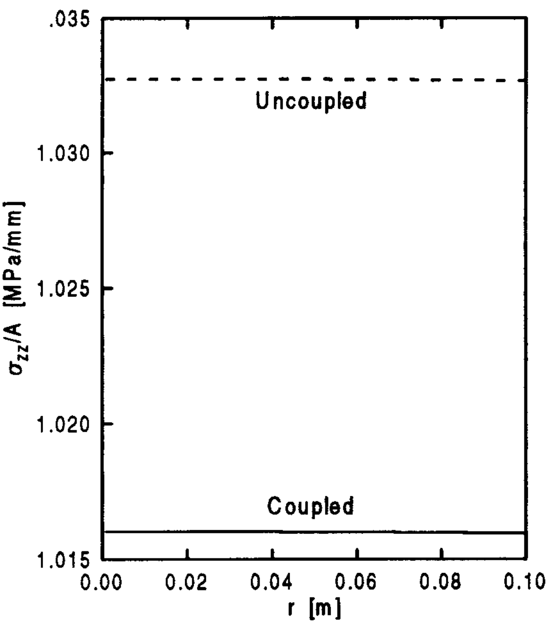

Fig. 3. Variations of uncoupled and coupled stress amplitudes as functions of radial distance.

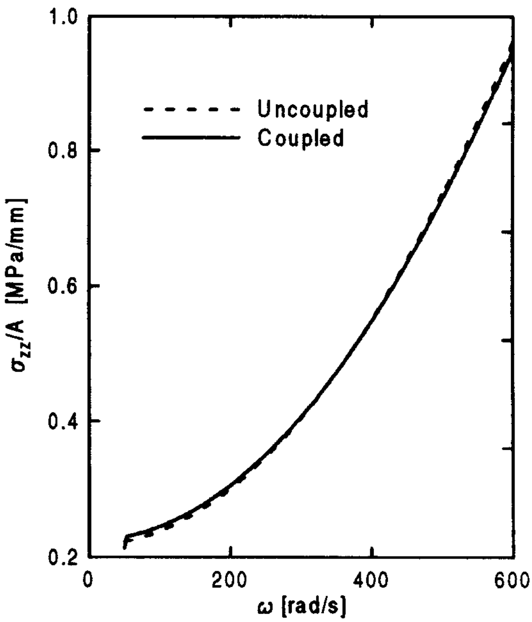

Fig. 5. Variations of uncoupled and coupled stress amplitudes as functions of loading frequency.