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ACTIVE CONTROL OF STRUCTURAL RADIATION USING WAVENUMBER SPECTRUM MEASUREMENTS

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ABSTRACT

There is a broad class of active control problems involving control of the acoustic power radiated from a vibrating structure. For many of these applications, it is desirable to implement the control using structural sensors, as opposed to far-field pressure sensors. It is known that the acoustic radiation corresponds to the supersonic wavenumber components of the vibration. Previous work by the authors has demonstrated that one can design distributed sensors that provide a direct measurement of the radiated power using a spatial Fourier transform of the sensor measurements. The sensors have been much more effective in predicting the radiated power than using a similar number of point sensors on the structure. The work reported here uses this wavenumber information as the basis for an active control system that attenuates the far-field radiation by means of minimizing the supersonic wavenumber components. The numerical results presented compare and contrast the far-field results obtained using this active control approach with results obtained when simply minimizing the vibration signal at the discrete locations. The results indicate that significant far-field control can be achieved by directly measuring the supersonic wavenumber vibration and minimizing that vibration, while ignoring the non-radiating subsonic wavenumber components.

INTRODUCTION

A number of current active control problems involve the task of minimizing the acoustic field radiated from a vibrating structure. Such structures include panels and shells, for example. It has been shown that one can minimize the far-field pressure to achieve good control results[1]. However, this has the disadvantage of requiring sensors in the far-field, which is typically not practical for these applications. As a result, control schemes have been developed that rely on structural measurements, which are available, as a means of controlling the far-field radiated pressure[2,3,4].

The most obvious approach to attempt to control structural radiation is to minimize the vibration of the structure at one or more sensor locations. However, it has been demonstrated that this can actually result in increased acoustic radiation in some cases, since reducing the vibration does not necessarily correspond to reducing the radiation. Thus, it is apparent that to be most successful, the approach chosen should be based on the mechanisms that lead to acoustic radiation. Another approach investigated involves controlling the far-field pressure using far-field sensors, and measuring the resulting vibration field at a number of sensor locations[2]. Then, with this information, the controller can be run again to minimize the difference between the vibration sensor response and the desired sensor response.

An alternative approach looks at the radiation from the structure in terms of the radiation resistance associated with the structure, and develops the control system to minimize the response of the associated "radiation modes," or "transformed modes[5,6]." This approach is attractive in that it isolates vibrational responses that lead directly to acoustic radiation, so that the control effort is focused on the energy that is radiated, and not on the energy that resides within the structure.

The previous approach is based in the spatial domain. One can also view the problem in the transform domain i.e. in the wavenumber domain[4,7]. Again it is known that only certain wavenumbers associated with a vibrating structure will radiate. As a result, if the control effort is focused on minimizing those wavenumber components that radiate, one can efficiently control the radiation from a structure in a more optimal fashion. The work reported in this paper focuses on using information obtained from the wavenumber domain as a basis for controlling the radiation from a vibrating structure. The results shown here are numerical results that have been obtained for a vibrating beam that matches an experimental beam used for previous work.

MEASURING ACOUSTIC RADIATION FROM STRUCTURES

To understand the active control approach used in this paper, it will first be useful to review concepts associated with acoustic radiation from structures, and measuring that radiation. Acoustic radiation can be viewed either in the spatial domain or in the transform, or wavenumber, domain. For the wavenumber domain, the spatial Fourier transform of the beam vibration results in the wavenumber spectrum that provides direct information regarding the radiation from the beam. It has been shown that the acoustic power radiated from a vibrating beam can be expressed as

$$\Pi = \frac{\omega \rho_f}{4\pi} \int_{-k_f}^{k_f} \frac{|V(k_x)|^2}{\sqrt{k_f^2 - k_x^2}} dk_x \,, \tag{1}$$

where ω is the angular frequency, ρ_f is the density of the fluid, $V(k_x)$ is the wavenumber spectrum of the beam velocity field, k_f is the acoustic wavenumber in the fluid, and k_x is the continuous structural wavenumber. It should be noted that this expression is for a beam of infinite width. If the beam has finite width, the square root in the denominator vanishes[8]. It can immediately be seen from Eq. (1) that the only structural wavenumbers that contribute to the acoustic radiation are those whose magnitude is less than the acoustic wavenumber. (These wavenumbers are referred to as supersonic.) Thus, if one can sense the supersonic wavenumbers, that would provide a good measurement of the acoustic radiation.

The difficulty in sensing the supersonic wavenumbers is that in many cases, the major lobe of the wavenumber spectrum lies in the subsonic region. As a result, if only the minimum number of sensors are used to sense the supersonic wavenumbers, spatial aliasing will occur[9]. There have been several approaches investigated to deal with this problem. The solution that has been investigated previously by the current authors involves using shaped PVDF sensors that are designed to act as low-pass filters in the wavenumber domain[7]. By using the shaped sensors, the higher wavenumber components can be attenuated to minimize the effects of aliasing. One example of this approach can be seen in Figure 1, for the third resonance of the beam. It is apparent that the wavenumber spectrum obtained using point sensors leads to significant aliasing errors in the supersonic region. (The solid vertical lines indicate the acoustic wavenumber.) While the spectrum obtained from the patch sensors is not perfect, it can be seen that the aliasing problem is largely overcome, and the estimate of the acoustic radiation would be expected to be much better. This is indeed the case, as can be seen in Figure 2, which shows the estimate of the radiated power (relative to the true value) for both the point sensors and the patch sensors. Over the design frequency range (550 Hz), the power estimate for the patches is within 3 dB of the true value, while the error in the estimate for the point sensors is typically greater than 40 dB. Thus, it appears that significant gains can be made in estimating the acoustic radiation using the patch sensors, rather than an equal number of point sensors.

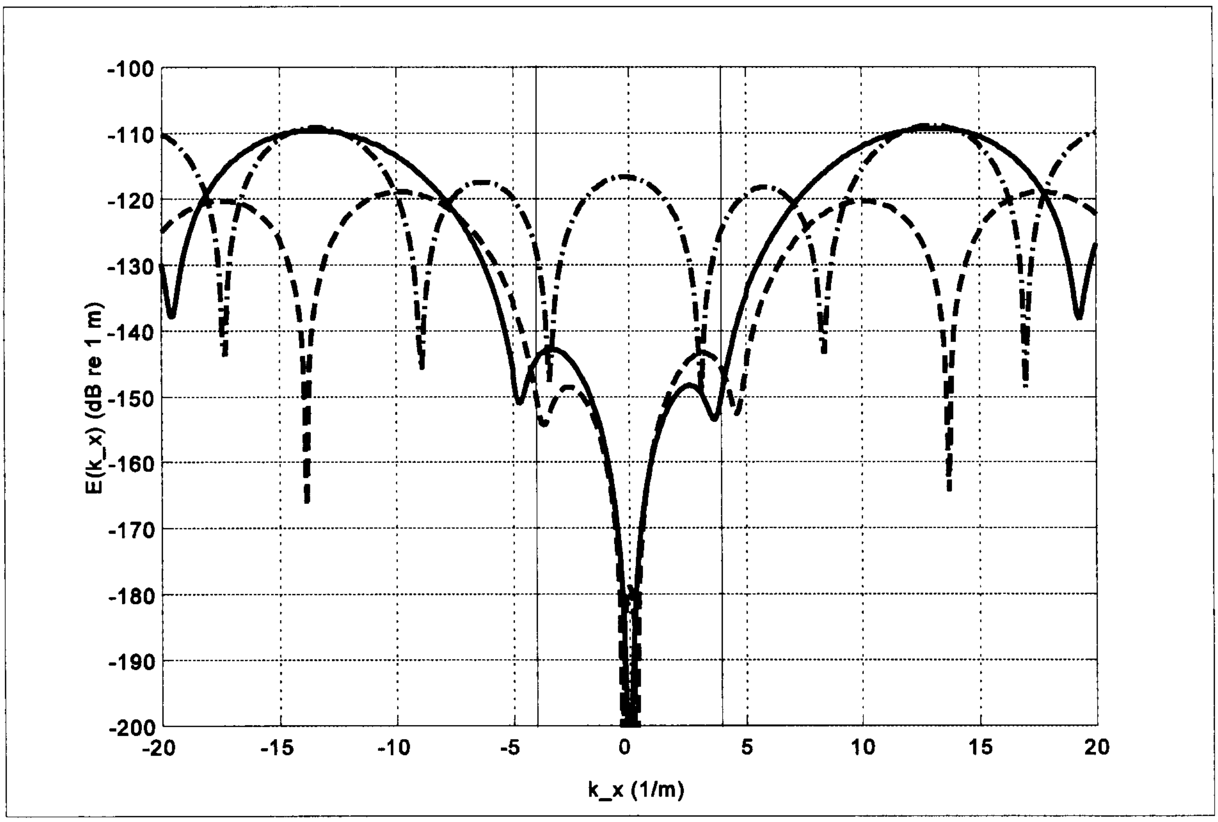

Figure 1. Fourier transform of strain response for resonance 3 (216.52 Hz). — theoretical response; --- patch response; -·-point response.

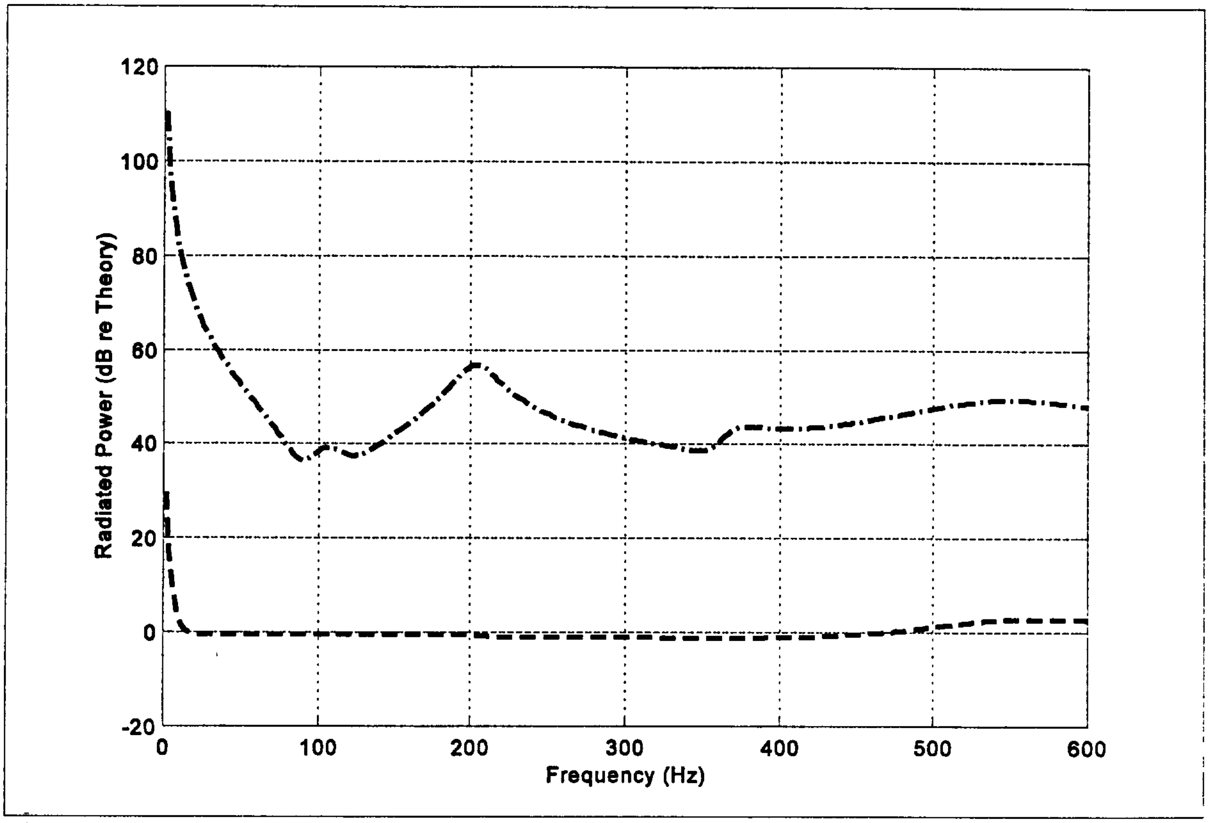

Figure 2. Estimated power of patch and point arrays. --- PVDF patch sensors; -·- point sensors.

APPLICATION TO ACTIVE CONTROL

Although it has been demonstrated that the acoustic power radiated can be estimated much more accurately using the patch sensors, it is not clear whether the patch sensors can be expected to lead to better performance in terms of active control of the structural radiation. For this reason, the control that can be expected using the two types of sensors has been investigated numerically. For this work, a simply-supported aluminum beam is assumed, with the beam parameters given in Table 1. It is assumed that six equally-spaced sensors are used to measure the response of the beam. This number is based on the design frequency range (550 Hz) chosen for this work.

To investigate how a wavenumber error criterion compares with the sum of squared errors from an array of discrete sensors, the response of a beam was numerically investigated using these two active control approaches. For a given point excitation location and frequency, the beam vibration response and radiated acoustic field were determined. The optimal control source strength for the given control source location was then determined, such that the supersonic

TABLE 1. Beam parameters used in the model.

Parameter	Symbol	Value
Length	L	0.914 m
Width	w	0.0508 m
Thickness	h	0.00635 m
Young's modulus	E	71 GPa
Mass density	ρ	2700 kg/m^3
Damping loss factor	η	0.05

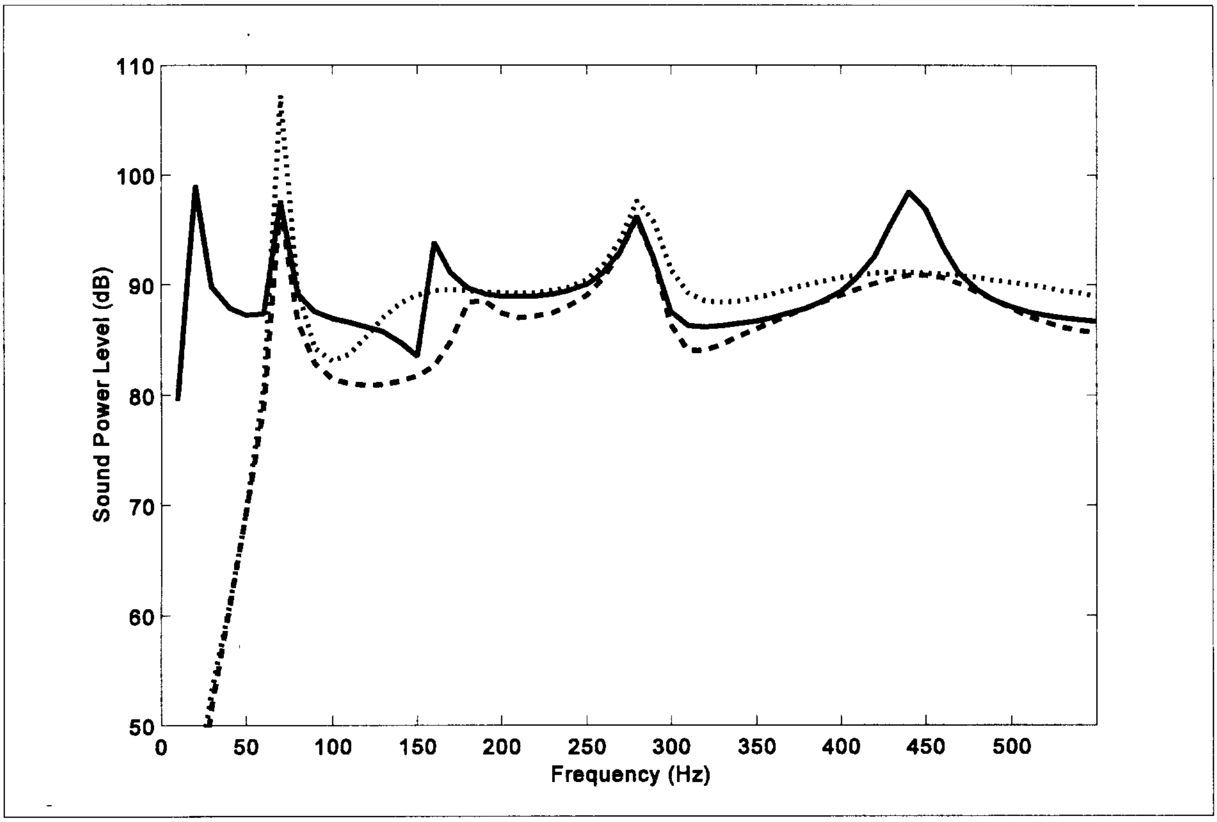

Figure 3. Sound power as a function of frequency (x_p = 0.64m, x_s = 0.45m). — no control; ···· minimization of point sensors; --- minimization of wavenumber spectrum.

wavenumber components were minimized, and the beam response and acoustic response were again determined. Finally, the optimal control source strength was determined for minimizing the vibration response of the six discrete point sensors, and the beam response and acoustic response determined.

The results for using the two control approaches are shown in Figure 3, in terms of the far-field radiated acoustic power. For these results, there is one primary control source assumed, which is positioned at x_p = 0.64m, and one secondary control source, positioned at x_s = 0.45m. It can be seen that the method of minimizing the supersonic wavenumber components yields the lowest radiated power at all frequencies, as one would expect. However, for much of the frequency range, the difference in the radiated power for minimizing the wavenumber components and the discrete error sensor signals is relatively small, differing by perhaps a few decibels or less over much of the frequency range. This behavior has been found to be typical with other actuator locations as well. This would tend to suggest that when a larger number of error sensors are used (six in this case), that one may not achieve large improvements in reducing the overall acoustic power by minimizing the wavenumber components. However, the results shown here do not give a complete picture of what is happening in the acoustic far-field. To gain further insight into the control mechanisms, it is also useful to look at the spatial response for a given frequency.

Figure 4 shows the far-field pressure response for the case of exciting the beam on-resonance. For these results, a single point force is assumed at x_p = 0.64m, and a single point control force is assumed at x_s = 0.2m. The excitation frequency is given by 160 Hz, which nearly corresponds to the third resonance frequency of 159.1 Hz. The radiated acoustic power with no control is 93.9 dB. When one minimizes the wavenumber components, the acoustic power is reduced to

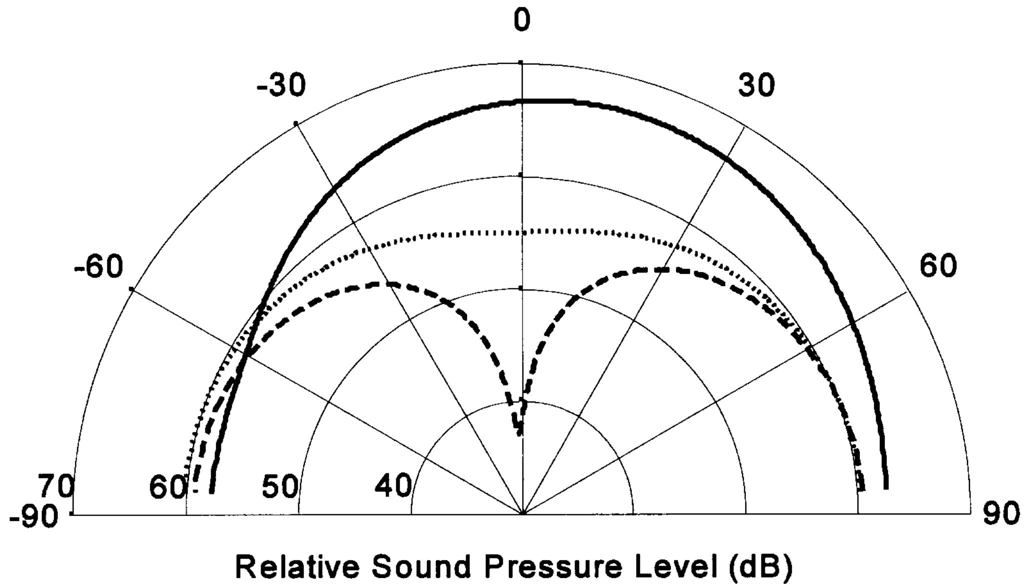

Figure 4. Far field acoustic pressure without and with control, for an excitation frequency of 160 Hz. —— no control; ···· minimization of point sensors; --- minimization of wavenumber spectrum.

84.3 dB, while it is reduced to 86.6 dB for the case of minimizing the discrete point sensors. Both approaches seem to give good reduction of the total radiated power, with the wavenumber minimization giving an additional 2 dB of reduction. However, when one looks at the far-field radiation patterns, it is apparent that minimizing the wavenumber components results in significantly improved attenuation at most angles in the far-field. The total radiated power is only similar for the two methods because of the response in the vicinity of ±90°. The reason for this far field radiation pattern can be seen by observing the wavenumber spectrum of the beam vibration, shown in Figure 5. Without control, there is a broad sidelobe of the wavenumber spectrum that spans the supersonic wavenumber region between ±2.96m^{-1}. When the vibration at the point sensors is minimized, the peak of the sidelobe is brought down by about 10 dB, and the spectral levels increase from there to their levels at the acoustic wavenumber. However, when minimizing the wavenumber spectrum, this dip at k_x = 0m^{-1} is much more pronounced. It can also be noted that the levels of the wavenumber spectrum are higher for values of k_x between about 3 and 18m^{-1}. However, since these wavenumber components do not affect the radiation, this has no negative impact on the acoustic far field, and the radiated field that results is significantly improved over using point sensors, particularly between about ±45°.

Similar results are obtained for off-resonance conditions. Figure 6 shows the far field acoustic pressure for the case of excitation at 120 Hz, which lies about midway between the second and third resonances of the beam. For the results shown here, the primary source is at x_p = 0.64m, and the secondary source is at x_s = 0.45m. For off-resonance conditions, the optimal solution for minimizing resonance is generally not to minimize vibration, since the beam response is not predominantly due to a single mode. This is apparent in the results shown in Figure 6, in that the far field pressure is not attenuated by minimizing the response of the point sensors. In fact, the total radiated acoustic power actually increases from 89.2 dB without control to 89.5 dB with control. However, if one chooses to minimize the supersonic wavenumber components, which

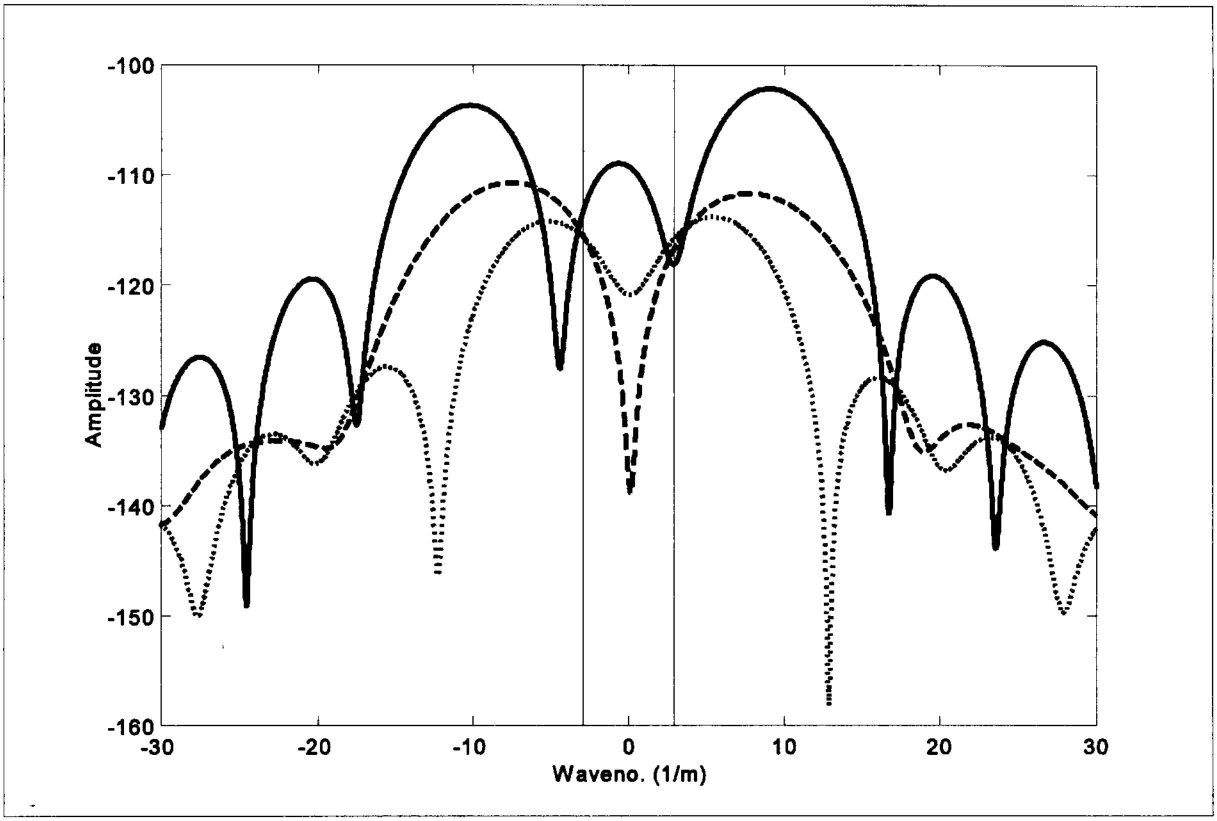

Figure 5. Wavenumber spectrum of the beam without and with control. — no control; ···· minimization of point sensors; --- minimization of (supersonic) wavenumber spectrum. (The acoustic wavenumber is represented by the vertical lines at $\pm 2.96 m^{-1}$.)

are responsible for acoustic radiation, the total power drops from 89.2 dB to 86.6 dB, and it is apparent that there is significant improvement in the far field pressure levels.

CONCLUSIONS

Previous work has indicated that there are large discrepancies in the estimations of the radiated acoustic power that are obtained from the use of a small number of point vibration sensors and an equal number of shaped PVDF sensors. The reason for this discrepancy has been traced to the problem of spatial aliasing. However, the work presented here indicates that the difference in the radiated acoustic power is not nearly as great when minimizing either the response of those point sensors or the supersonic wavenumber components. For many frequencies and configurations studied, the difference in the radiated acoustic power when using the two approaches was on the order of a few decibels or less.

In spite of the fact that the radiated acoustic power levels were comparable, there still appears to be a significant advantage to minimizing the supersonic wavenumber components. The similarity in the power levels is generally due to the similarity in the far field pressure levels over a relatively small angular region. However, typically there is a large difference in the two approaches over a large angular region, with the minimization of the wavenumber spectrum yielding significantly improved far field results in that region.

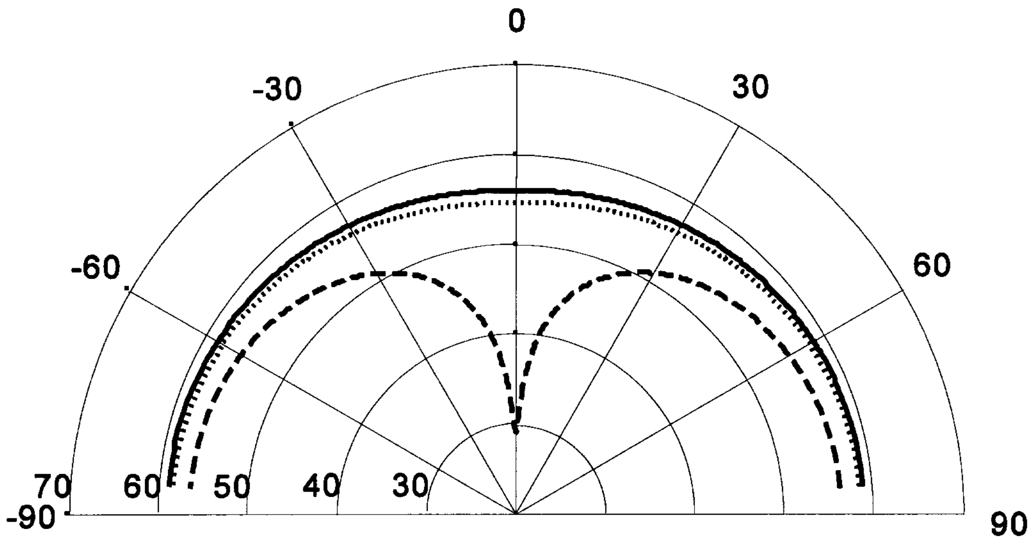

Figure 6. Far field acoustic pressure without and with control, for an excitation frequency of 120 Hz. — no control; ···· minimization of point sensors; --- minimization of wavenumber spectrum.

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