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ACOUSTIC SEABED MODELS OBTAINED BY INVERTING EXPLOSIVE SHALLOW WATER TRANSMISSION LOSS DATA

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ABSTRACT

Acoustic transmission losses (TL) measured at 16 Hz along a track in shallow water have been inverted to obtain seabed acoustic properties. The data are represented by an analytic function of range with two free parameters. The seabed is modelled as an equivalent uniform half-space. A uniform space has 5 unknowns: sound and shear speeds, sound and shear absorption coefficients, and density. The number of unknowns is reduced to three by using regression equations to relate density and shear absorption to sound-speed. By holding shear-speed fixed for subsets of the process, the number of unknowns is reduced to two (sound-speed and absorption). The parameters of the TL analytic function are computed over intervals of these unknowns by fitting outputs of the "Oases" mathematical model, and solutions of the resulting pair of simultaneous equations are sought. Finally, these solutions are expressed as functions of shear-speed, and criteria are presented for selecting the optimum results. The equivalent half-space is not necessarily related to the actual properties of the seabed and is liable to vary with frequency. It should however, produce correct results when an appropriate TL model is run at the original frequency, for arbitrary source and receiver depths and water-column conditions.

1. INTRODUCTION

To predict the propagation of low-frequency sound in shallow water, it is necessary to have a geo-acoustic model (GAM) of the seabed. Such a model will depend on geological properties such as porosity, grain shape, degree of cementation, and the chemistry of the grains. These properties vary with depth beneath the seafloor, and also with horizontal position. In principle it should be possible to estimate a GAM from measured geological properties, but in practice there are two important limitations on proceeding in this way: (i) the relationships between geology and acoustic properties are available only as regression equations around which there are significant empirical fluctuations; and (ii) most regions of the seabed have not been adequately sampled with cores (many shallow water areas have had their seafloor properties surveyed, but this is not adequate for estimating the sub-bottom properties).

An alternative method is to measure acoustic transmission loss (TL) along a particular track, and to determine what GAM(s) would yield the measured data. To progress such a procedure, it is necessary to assume that the GAM does not vary with position along the track. Analysis is made easier if the acoustic properties of the water layer are also independent of horizontal position.

At first glance it might seem unnecessary to derive a GAM if the variable of primary interest, transmission loss, has already been measured. There are 3 reasons why it is necessary to obtain a GAM:

(i) the properties of the water layer vary with time, and the measured TL depends on the properties of the water as well as those of the seabed. Obtaining a GAM allows for the effect of the water, so that TL along that track can be predicted for occasions when the water has different properties.

(ii) the measurement was made with source and receiver at particular depths. Once a GAM is obtained, TL can be computed for other sensor depths.

(iii) in considering the spread of data for shallow water TL over a geographical area, it is important to isolate the contribution due to spatial variation of the seabed. Otherwise, the spread tends to be attributed to the well documented seasonal variability of the water layer. A useful parameter by which to rate seabeds is their reflectivity, which is easily computed from their GAM's.

This paper describes an inverse technique and its application to derive an effective geoacoustic model from a set of explosive propagation measurements. The TL data to which it is applied were obtained along a track on a continental shelf. This track, which was 24 km in length, was selected because the seafloor depth was approximately constant (106 ± 1 m), and there was little difference between the temperature profiles measured at each end of it. A sonobuoy with a hydrophone 18-m deep had been deployed at the start of the track, and 27 small explosive charges were fired (18 m deep) at ranges from 1 to 24 km. The signals of the shots were recorded, digitised, and converted into energy spectra. The energies in the third-octave band centred on 16 Hz were computed, converted to decibels, and subtracted from the estimated source strength of the explosive in that band. This procedure yielded TL as a function of range.

II METHOD

The data for TL at 16 Hz were fitted with the analytic function:

$$TL\ (r) = P + 10 \log (r) + Q\, r / 10^4, \qquad (1)$$

where r is the range in metres. The results obtained for the 2 free parameters, to be denoted by P(empirical) and Q(empirical), were 32.5 dB re m and 8.5 dB /10 km respectively. P can be considered as 10 log (p), where p is the transition range between spherical and cylindrical spreading (here, p = 1780 m). The criterion for fitting was to minimise the standard deviation of the data around this function (the value of which was 2.3 dB). Attempts were made to decrease the standard deviation by increasing the number of free parameters, such as by replacing the coefficient of log (r) with another free parameter. It was found however that it was not possible to reduce the standard deviation of the data with feasible functions.

Since the TL data have been described with only two parameters, the number of seabed acoustic unknowns that can be independently manipulated to achieve a fit is also only two. The geoacoustic model of a uniform half-space contains 5 unknowns: the sound and shear speeds (Cp and Cs), sound and shear absorptions (Ap and As), and relative density (ρ). If a uniform layer were also included, then there would be 11 unknowns: 5 each for the layer and half-space, plus 1 for the layer thickness[1]. The approach has therefore been to obtain an "equivalent uniform half-space" for the track, and to determine by trial and error which pair of unknowns (if any) should be regarded as the primary variables.

The model used in the inversion process was the "Oases" mathematical TL model (Schmidt, 1996). Oases was selected since it is an accurate model, and requires only that the environment be range-independent. It is also robust, since it computes straightforward integrals (rather than, for example, searching for modes in the complex plane, as accurate normal-mode models need to do). For the present scenario and frequency, the dimensions of the range-depth plane are 250 wavelengths in range by 1 wavelength in depth, and for these comparatively small dimensions an Oases run is completed within a couple of seconds. Each Oases output (TL versus range) was smoothed and Eq. (1) fitted to it to yield "model" values for P and Q, to be compared with their empirical values. There will be cases where, even if P = P(empirical) and Q = Q(empirical), the Oases TL function will not have the same range dependence as Eq. (1), and in such cases there will be differences between the smoothed Oases function and Eq. (1). This effect has been monitored by computing the RMS difference between the two functions.

A. Treatment of the unknown variables

The next step was to select the pair of unknowns to be used as independent variables. Model values for P and Q would be regarded as functions of those variables. After some trials, it was found that P tended to vary with Cp, while Q tended to vary with Ap. Cp and Ap were therefore selected as the independent variables, since this would give the best chance of the contours of P = P(empirical) and Q = Q(empirical) being orthogonal, which in turn would decrease the error in the estimation of their point of intersection (the solution). On the basis of Hamilton's (1980) summary of sound absorption, Ap has been constrained to not exceed 2 dB /cycle.

The shear speed was selected as the third independent unknown, and an important milestone in the analysis is to obtain a functional relation between solutions (Cp, Ap) and Cs. This relation is expected to depend on the remaining unknowns As and ρ. These "additional" unknowns were treated as follows:

(1) Shear absorption

It was found in trial runs that Ap and As complemented each other: if As was held at a low value, then the solution for Ap would be high, and vice versa. There was therefore no benefit in treating As as an independent unknown, and the feasibility of a relation between As and Ap was examined. Data for sound and shear absorption measured in different sediment types have been summarised by Hamilton (1980, pp. 1329 - 1331). It is found that As /Ap ≈ 2 for sand, 0.7 for chalk, and 0.5 for limestone[2]. A function based on these results would be somewhat complicated, so other approaches were examined. For the bulk modulus of the seabed to have a positive imaginary part (otherwise a sample would vibrate with increasing amplitude following a dilatational excitation), it is necessary that the maximum value of As satisfy the following condition:

$$As = 0.75\ Ap\ (Cp\ /Cs)^2 \qquad (2)$$

For convenience, this expression has been used for As, since (i) it defines As in terms of variables that will have been given values, and (ii) if an independent expression were used, As would have to be amended whenever it exceeded the value given by Eq. (2). When Cs << Cp, Eq. (2) would often yield As ≥ 10 whereupon it would become contiguous with the preceding variable in the data file. To avoid this problem, As was limited to a maximum of 9.9. This artifice was considered to be a minor issue, since when Cs << Cp the shear wave has little effect on reflectivity, regardless of As.

(2) Relative Density

Results obtained from further trial runs indicated that the solutions for Cp and Ap varied noticeably but slowly with density. On reviewing Hamilton (1980) it was concluded that ρ is best estimated using the regression curve between it and sound-speed (Hamilton's Fig. 24). For the range of Cp to be covered in this analysis [1900 to 3000 m/s], the appropriate regression equation given by Hamilton (1978) is:

$$\rho = 2.351 - 7.497\ (1000\ /Cp)^{4.656} \qquad (3)$$

The standard deviation of the ρ data relative to this function is approximately 0.1.

B. Precision

It is necessary to choose a tolerance for transmission loss, so that the precision to which the unknowns need to be computed may be determined. To be compatible with the standard deviation of the data, a tolerance of ±1 dB at the mid-range of 10 km has been selected, to be achieved by prescribing a tolerance of ± 0.5 dB for both P and Q. For an example GAM with (Cp, Cs, Ap, As, ρ) = (2170, 1000, 0.12, 0.4, 2), the approximate 1-way precision to which each of the 5 variables need to specified to yield this tolerance, are listed in Table 1. These values are liable to be quite different in other regions of the Cp-Ap plane.

C. Running the TL model

For the first set of runs, Cs was set to 0, and Cp and Ap were each incremented to cover a large rectangle in the Cp-Ap plane (say $1900 \leq Cp \leq 2400$, and $0 \leq Ap \leq 0.3$). Typically Cp would be given 6 or 7 values and Ap given 4 or 5 values, so a set would consist of around 30 Oases runs. The separate contours of P and Q would be examined to determine whether a point in the Cp-Ap rectangle existed for which the target-contours intersected. If not, then a set of runs would be conducted over a more promising rectangle. When it came to localising the contours, it was found that the variations of P and Q were sufficiently non-linear that the increments in Cp and Ap generally had to be reduced to 5 m/s and 0.01 dB/cycle respectively before the contouring process would yield reproducible results.

For subsequent sets of runs, Cs was increased (in steps of 100 m/s), and the above process repeated until no further solutions could be obtained.

D. Summary

(i) express As and ρ as functions of Cp [Eqs. (2) and (3)];

(ii) hold Cs fixed for a subset of the analysis;

(iii) run the TL model over rectangles in the Cp-Ap plane in order to find solutions to the simultaneous equations:

$$P\,(Cp, Ap) = P(\text{empirical}) \pm 0.5 \tag{4a}$$

and

$$Q\,(Cp, Ap) = Q(\text{empirical}) \pm 0.5 \tag{4b}$$

for the unknowns Cp and Ap;

(iv) repeat for a sequence of values for Cs.

	Cp	Cs	Ap	As	ρ
P	5	14	large	large	0.09
Q	≥ 5	large	0.01	0.4	large

Table 1: For a particular GAM, the approximate precision needed in each of the 5 variables to yield a tolerance of 0.5 dB in P and/or Q.

III RESULTS

The first finding was that no solution for (Cp, Ap) could be obtained until Cs reached 200 m/s, and no solution could be found for Cs = 500 or 600 m/s (taking into account that regions where Ap > 2 were not examined, and that the increment in Cs was 100 m/s). Similarly, no solution could be found for Cs greater than 1100 m/s. The solutions that were obtained for Cp and Ap, together with the corresponding values of As, ρ, and the RMS differences between TL(model) and Eq. (1), are listed in Table 2.

It can be seen from Table 2 that the reason no solution could be obtained for Cs < 200 m/s was that Ap would have to be greater than 2 dB /cycle. For Cs = 500, 600, or > 1100 m/s, Ap would have to be negative, which would be non-physical (the seabed would cool as its heat energy converted into sound!).

Shear-speed (m/s)	Sound-speed (m/s)	Sound absorption (dB/cycle)	Shear absorption (dB/cycle)	Relative Density	RMS {TL(model) - TL(empirical)} (dB)
0	-	(> 2)	-	-	-
100	-	(> 2)	-	-	-
200	2122	1.79	9.9	2.125	0.46
300	2070	0.350	9.9	2.098	0.48
400	2055	0.136	2.692	2.089	0.33
500	-	(< 0)	-	-	-
600	-	(< 0)	-	-	-
700	2107	0.043	0.292	2.118	0.92
800	2120	0.089	0.469	2.124	0.60
900	2140	0.124	0.526	2.134	0.39
1000	2169	0.122	0.430	2.147	0.16
1100	2209	0.049	0.148	2.164	0.27

Table 2: For a sequence of shear speeds, the solutions for Cp and Ap; the corresponding As and relative density; and the RMS differences between the empirical and model TL functions.

The minimum value of the RMS difference between TL(model) and Eq. (1) occurs at Cs = 1000 m/s, for which Ap = 0.122. This solution should therefore be regarded as superior to the others. The solution for Cp is consistent with a chalk sediment, for which Hamilton (1980, p. 1330) reports a value for Ap of around 0.17 dB /cycle. The result obtained here for Ap is 72% of the cited result.

IV CONCLUSIONS

Using a fit to transmission loss data with only two parameters, it has been possible to infer that the actual seabed is, for a frequency of 16 Hz, equivalent to a uniform half-space with the following geoacoustic model: (Cp, Cs, Ap, As, ρ) = (2169, 1000, 0.122, 0.430, 2.147). Cp and ρ are consistent with a chalk sediment, but the ratio Cp /Cs would be more typical if it were close to 1.9 (instead of 2.17), and Ap is 72% of a result cited in the literature.

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[1] If a non-uniform layer is allowed, then gradients of the variables are additional unknowns.

[2] Hamilton presented the data in units of dB /km /Hz. For chalk and limestone, the conversion to dB /cycle takes into account Hamilton's conclusion (p. 1324) that Cp /Cs = 1.9. For sand, it has been assumed that Cp /Cs ≈ 1800 /150 = 12. The ratio As /Ap is then obtained from ks /kp × Cs /Cp. There is a trend in the data for this ratio to vary inversely with Cs, as suggested by Eq. (2).