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NONLINEAR RADIATION OF A PISTON VIBRATING WITH A HIGH AMPLITUDE

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Nonlinear wave physics deals basically with effects connected with evolution of disturbances of finite amplitude. A slow distortion of time-spatial and spectral characteristics is observed with wave propagation. But nonlinear effects may be already pronounced at small wave distances from a radiator and they can have a significant influence on the process of radiation of itself. In this paper a radiation of a flat piston vibrating with high amplitude is studied theoretically. At a first step a solution for a piston motion driven by external force is determined. It is shown that a piston subjected to the harmonic force of large amplitude can radiate not only the fundamental frequency but high order harmonics as well. A form of particle vibration near a piston face is disturbed. A nonlinear reaction to radiation is arisen also.

In the classical problems of nonlinear acoustics well known nonlinear effects - wave profile distortion, shock front formation, harmonics generation - are developed at the spatial scales which are considerably larger then the characteristic wave scales [1]. But in some interesting cases nonlinear effects can be well pronounced closely to sound source and they can have a significant influence on the process of radiation of itself. . An excitation of sonic boom waves by airplane moving with hypersonic or transonic speed is an example of a such situation in gas mechanics. This wave is strongly distorted in a near wave zone. Reaction to radiation offers an additional nonlinear resistance to a plane motion [2].

A similar phenomenon is also essential for the radiation of intensive ultrasonic waves. But until recent time a problem of nonlinear radiation of the periodical waves seemed to be far from reality. The point is that a speed of vibration of a radiating surface in the liquids does not exceed several of meters per second even for the most effective ultrasound transducers; these values are of three order less than the sound speed. A different situation emerges in two-phase liquid-bubble media, where the sound speed is decreased sharply with the bubble concentration increasing and its magnitude may equate to the speed of radiator face vibration. Therefore one would expect of pronounced development of nonlinear effects near a transducer face. It seems probable that a similar situation has been realized in the experiments on the study of the second threshold of cavitation [3].

Let us consider the problem in a linear approximation. An equation of piston motion when it is subjected to harmonic external force $F(t)$, can be generally written as:

$$m\frac{d^2X}{dt^2} = F(t) - p_{ac}S, \tag{1}$$

where m is the mass of a piston, S - is its surface area, p_{ac} - is acoustic wave pressure, X is the displacement of a piston from the equilibrium state. In the case of a plane wave

$$p_{ac} = c_0\rho_0 u(t - x/c_0), \tag{2}$$

where c_0 - is the sound speed, ρ_0 - is the medium density, u - is the velocity of medium particles. At the boundary of a piston the speed of its surface dX/dt must be equal to the speed of medium particles:

$$\frac{dX}{dt} = u(t)\Big|_{x=X}. \tag{3}$$

Using boundary condition (3) we can repeatedly write Eq.(1) as follows,

$$\frac{d^2X}{dt^2} + \alpha\frac{dX}{dt} = F(t)/m, \tag{4}$$

where $\alpha = c_0 \rho_0 S/m$.

When the external force is a harmonic one $F = F_0 \cos(\omega t)$, the piston displacement in a steady state of vibration can be determined from:

$$X = -\frac{F_0}{m\omega^2} \frac{1}{\sqrt{1+\alpha^2/\omega^2}} \cos(\omega t + arctg(\alpha/\omega)). \tag{5}$$

Eq.(5) shows that the piston is vibrated at the same frequency as the external force, but with some phase delay which is arisen due to the load onto the medium. Amplitude of vibration is straightly proportional to the value of external force amplitude F_0 - this is a pure linear relationship.

However, even in this linear approximation a spectrum of radiating wave will have a number of high order harmonics. The waveform of a disturbance propagating from a piston is determined by following inexplicit relationships [1,2]:

$$\begin{aligned} \frac{u}{u_0} &= \Phi(\omega t + \varphi) = \sin[\omega\xi(t) + \varphi], \\ \omega t &= \omega\xi + \frac{u_0}{c_0}\cos(\omega\xi) = \omega\xi + M\cos(\omega\xi) \end{aligned} \tag{6}$$

where $u_0 = \frac{F_0}{m\omega}\left(1+\frac{\alpha^2}{\omega^2}\right)^{-0.5}$, $\varphi = arctg(\alpha/\omega)$, $M = u_0/c_0$ - is a Mach number.

The form of particles vibration near a piston face $\Phi(t)$ can be obtained by eliminating variable ξ in the relations (6). A dashed line at a Fig. 1 presents one period of a sinusoidal vibration. It shows a form of vibration for the case of very small amplitudes F_0 of the external force. Solid curves show the distortion of vibration form with the growth of a driven force. It is seen that a duration of the positive half-period (compression) is reduced and a duration of a negative one (rarefaction) is induced. This effect becomes more pronounced with growth of Mach number.

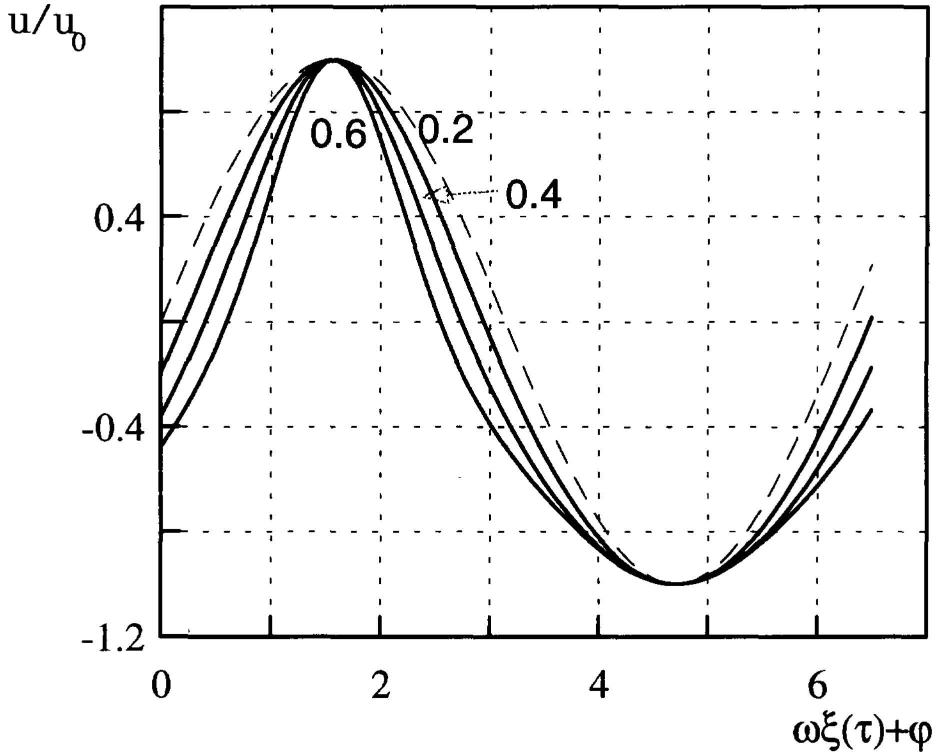

Figure 1. The profiles of a piston surface vibration. Numbers near the curves correspond to Mach numbers. Dashed line shows the form of vibration for $M << 1$.

The spectrum of vibration , described by Eq.(6) can be evaluated by use of Bessel-Fubini approach [1]. Function $u(t)$ can be expanded by its Fourier series expansion:

$$\frac{u}{u_0} = \sum_{n=1}^{\infty}(A_n \cos(n\omega t) + B_n \sin(n\omega t)).$$

The amplitude of a fundamental harmonic B_1 can be shown to be

$$\frac{B_1}{c_0} = M[J_0(M) - J_2(M)], \tag{7}$$

$(A_1 = 0)$. In the above J_n is the Bessel function of the first kind of order n. The second harmonic has phase shift and contains only cosine component with amplitude:

$$\frac{A_2}{c_0} = -0.5M[J_1(2M) - J_3(2M)]. \tag{8}$$

The complex amplitudes of high order harmonics C_n can be determined from general relation

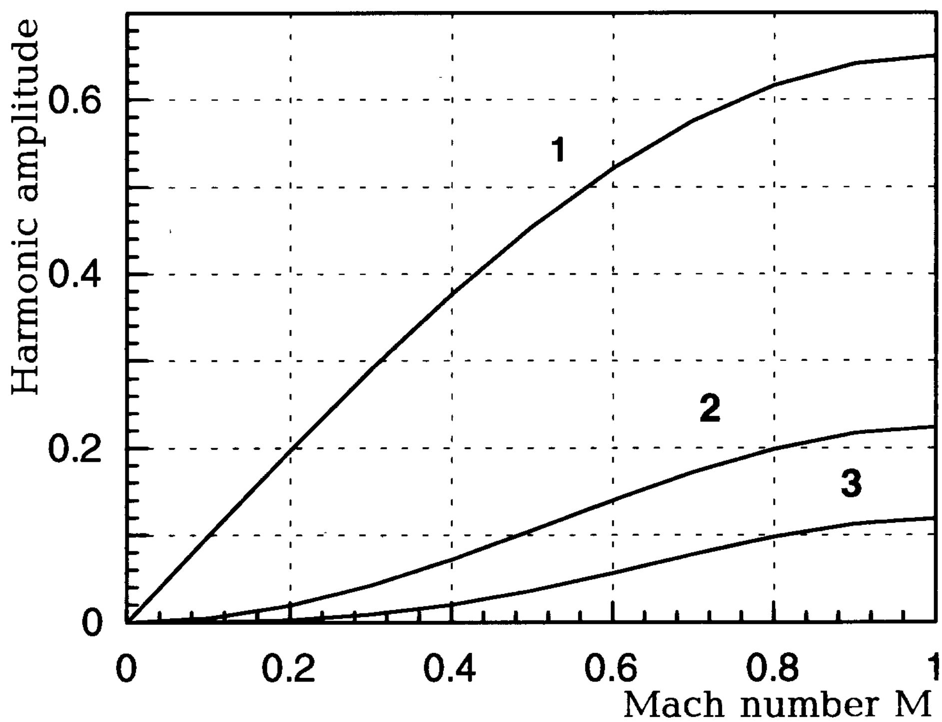

Figure 2. Normalized harmonic amplitudes of the velocity of particle vibrations near the piston surface.

$$\frac{C_n}{c_0} = \frac{M}{i^n n} J_n'(nM). \tag{9}$$

It is necessary to note that a regular component $\overline{u} = -0.5\, u_0^2 / c_0$ is a result of one-dimensional approximation we used. If a piston is supposed to be bounded then a regular component is vanished due to medium flow inwards of acoustic beam [2].

Fig.2 shows the amplitude of fundamental 1, the second 2, and the third 3 harmonics as a function of Mach number M, using Eqs. (7) - (9).

From this figure we note the following. For approximately $M \leq 0.2$ the piston vibrations can be considered as linear ones, and starting from $M \approx 0.3$ the nonlinear effects become significant. A relationship $B_1(M)$ falls from linear one, amplitudes of the second and the third harmonics quickly grow with increasing of Mach number.

A consistent account of nonlinear phenomena within the scope of a continuous medium approach can be done if both geometrical nonlinearity, described by Eq.(6) and physical one in dependence of $p_{ac}(u)$ will be taken into account. Assume that a piston can radiate not only acoustic wave but a Riemann wave also. In this case a general differential equation of a piston motion is written as a follows:

$$\frac{d^2X}{dt^2} + \frac{Sp_0}{m}\left\{\left[1 + \frac{\gamma - 1}{2c_0}\frac{dX}{dt}\right]^{\frac{2\gamma}{\gamma-1}} - 1\right\} = \frac{F(t)}{m}, \tag{10}$$

where γ is a specific heat ratio for the gas medium. In the case of liquids γ is an empirical constant determined from experiment [2]. Equation of motion (10) is valid up to transonic speeds of a piston motion. It is violated for the high negative velocities when a layer of vacuum at the boundary of a piston surface may be produced, and also at high positive speeds when a piston formally «catch up» shock fronts moving in front of it.

At oral presentation we also count on discussing a problem of effective ultrasound radiation in the liquid with cavitive bubbles. In a such medium the third kind of nonlinearity is appeared [3]. It is a structural nonlinearity which is developed due to two-phase composition of medium. A value of structural nonlinearity depends on bubbles concentration and bubble size distribution, therefore it is necessary to take into account the process of a cavitation development.

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