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VIBRATION ANALYSIS OF ROTOR-BEARING-PEDESTAL SYSTEMS

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ABSTRACT

In analysing the vibration behaviour of turbomachinery, a problem exists in modelling foundation-pedestal support systems whose natural frequencies are within or close to the operating speed range. Often, a finite element model of the foundation-pedestal support is unavailable and it is impractical to determine its modal properties experimentally. As a first approximation, one could regard the foundation as rigid and replace the pedestals by equivalent supports having mass and stiffness properties which correctly reflect the system unbalance response over the operating range. This paper outlines a method for identifying such pedestal properties for general rotating machinery using measurements of the motion of the pedestals and of the rotor. The proposed technique assumes a knowledge of the stiffness and damping properties of the support bearings (which may be hydrodynamic) but does not require a knowledge of the rotor, nor of the unbalance excitation; but merely that it be sufficient to provide measurable motion data. Numerical experiments show that excellent identification of pedestals is feasible even with the two digit measurement accuracies attainable with field instrumentation, suggesting applicability to practical turbomachinery where there is significant vibration at the pedestals though relatively insignificant vibration in the foundation itself.

NOMENCLATURE

C	damping coefficient	$\mathbf{M}$	mass matrix
$\mathbf{C}$	damping matrix	n	number of speeds
F	force	X	response
K	stiffness coefficient	Ω	speed
$\mathbf{K}$	stiffness matrix	ω	natural frequency
M	mass		

SUPERSCRIPTS (IF NOT OTHERWISE DEFINED)	
i	imaginary part
r	real part
—	vector
^	amplitude
.	velocity
..	acceleration

SUBSCRIPTS (IF NOT OTHERWISE DEFINED)	
b	bearing
p	pedestal
r	relative
y	vertical direction
z	horizontal direction

1. INTRODUCTION

The vibration behaviour of rotating machinery can be significantly affected if any of the natural frequencies of the support structure are in the vicinity of the operating speed range. If available, the parameters of the structure, ie its mass, damping and stiffness coefficients, or its modal parameters, can be included in existing vibration analysis software [1]. If not, the structure would need to be modelled by some finite element analysis or identified by experimental modal analysis or, as is often the case if these techniques are impractical, some means for identifying the structure using the response measurement data due to rotor unbalance would need to be implemented [2,3]. Unfortunately, no satisfactory means has yet been developed, to the authors' knowledge, for identifying complex foundations, which have natural frequencies in the operating speed range. However, if the foundation can be assumed to comprise flexible housing or pedestal structures which are connected to a rigid base at each bearing support, a significant simplification results. Such a foundation is of practical relevance, and this paper summarises theoretical investigations on the identification of just such flexible pedestals using the direct K, C, M approach [2]. This approach, though found to be too sensitive to measurement error to be able to identify a coupled foundation [4], is expected to be applicable here because of the simplification afforded by isolating the response at one pedestal from that at another.

2. THEORY

Figure 1 is a schematic of a rotor-bearing-pedestal system. Assuming that the pedestals are flexibly supported on the ground (ie on a rigid foundation) and can be represented by their mass, direct damping and direct stiffness coefficients, their equations of motion are [1]:

$$\mathbf{M}_p\ddot{\overline{X}}_p + \mathbf{C}_p\dot{\overline{X}}_p + \mathbf{K}_p\overline{X}_p = -\overline{F}_p \ , \tag{1}$$

where the mass, damping and stiffness matrices are diagonal. The forces transmitted from the rotor to the pedestals are to be calculated using motion measurements in conjunction with assumed known dynamic bearing properties, ie:

$$\overline{F}_p = -\mathbf{C}_b\dot{\overline{X}}_r - \mathbf{K}_b\overline{X}_r \ . \tag{2}$$

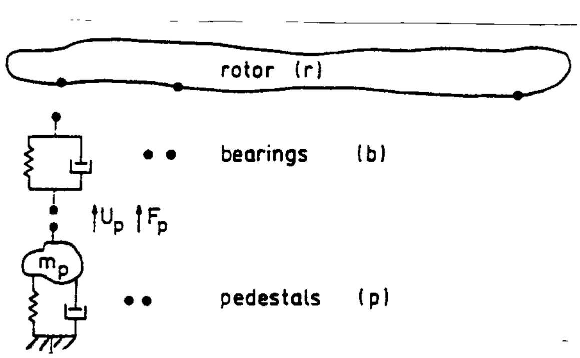

Fig. 1: Schematic of a rotor-bearing-pedestal system.

In the case of harmonic excitation, eqn (1) becomes:

$$\left(-\Omega^2\mathbf{M}_p + i\Omega\mathbf{C}_p + \mathbf{K}_p\right)\hat{\bar{X}}_p = -\hat{\bar{F}}_p \quad . \tag{3}$$

Since the motions of the pedestals are not coupled, and the unknown elements in $\mathbf{M}_p, \mathbf{C}_p$ and $\mathbf{K}_p$ are real quantities while the forces and responses are complex quantities, by separating the real and imaginary parts, eqn (3) can be written, for each pedestal as:

$$\begin{bmatrix} \hat{X}^r_{py} & 0 & -\Omega\hat{X}^i_{py} & 0 & -\Omega^2\hat{X}^r_{py} \\ 0 & \hat{X}^r_{pz} & 0 & -\Omega\hat{X}^i_{pz} & -\Omega^2\hat{X}^r_{pz} \\ \hat{X}^i_{py} & 0 & \Omega\hat{X}^r_{py} & 0 & -\Omega^2\hat{X}^i_{py} \\ 0 & \hat{X}^i_{pz} & 0 & \Omega\hat{X}^r_{pz} & -\Omega^2\hat{X}^i_{pz} \end{bmatrix} \begin{Bmatrix} K_{py} \\ K_{pz} \\ C_{py} \\ C_{pz} \\ M_p \end{Bmatrix} = -\begin{Bmatrix} \hat{F}^r_{py} \\ \hat{F}^r_{pz} \\ \hat{F}^i_{py} \\ \hat{F}^i_{pz} \end{Bmatrix} \quad . \tag{4}$$

If there is no damping in the pedestal, the first two equations in eqn (4) are equivalent to the second two. As a result, either the real or imaginary part of the forces and responses can be used. They can even be superimposed in order to smooth the results. However, if some damping exists, as will always be the case in practice, all the equations should be used to avoid the introduction of unnecessary errors. On the other hand, it can be shown that regardless of whether the pedestal damping coefficients $\mathbf{C}_p$ are included in the parameters to be identified or not, the results for the identified $\mathbf{K}_p$ and $\mathbf{M}_p$ are unaffected. Thus, for each pedestal, eqn.(3) can also be written as:

$$\begin{bmatrix} \hat{X}_{py} & 0 \\ 0 & \hat{X}_{pz} \end{bmatrix} \begin{bmatrix} 1 & 0 & \Omega & 0 & -\Omega^2 \\ 0 & 1 & 0 & \Omega & -\Omega^2 \end{bmatrix} \begin{Bmatrix} K_{py} \\ K_{pz} \\ iC_{py} \\ iC_{pz} \\ M_p \end{Bmatrix} = -\begin{Bmatrix} \hat{F}_{py} \\ \hat{F}_{pz} \end{Bmatrix} \tag{5}$$

or

$$\begin{bmatrix} 1 & 0 & \Omega & 0 & -\Omega^2 \\ 0 & 1 & 0 & \Omega & -\Omega^2 \end{bmatrix} \begin{Bmatrix} K_{py} \\ K_{pz} \\ iC_{py} \\ iC_{pz} \\ M_p \end{Bmatrix} = -\begin{Bmatrix} \hat{F}_{py}/\hat{X}_{py} \\ \hat{F}_{pz}/\hat{X}_{pz} \end{Bmatrix} \quad . \tag{6}$$

Separating the real and imaginary parts of eqn (6) results in:

$$\begin{bmatrix} 1 & 0 & -\Omega^2 \\ 0 & 1 & -\Omega^2 \end{bmatrix} \begin{Bmatrix} K_{py} \\ K_{pz} \\ M_p \end{Bmatrix} = -\begin{Bmatrix} \left(F^r_{py}X^r_{py} + F^i_{py}X^i_{py}\right)\Big/\left(X^{r^2}_{py} + X^{i^2}_{py}\right) \\ \left(F^r_{pz}X^r_{pz} + F^i_{pz}X^i_{pz}\right)\Big/\left(X^{r^2}_{pz} + X^{i^2}_{pz}\right) \end{Bmatrix} \tag{7}$$

and

$$\begin{bmatrix} \Omega & 0 \\ 0 & \Omega \end{bmatrix} \begin{Bmatrix} C_{py} \\ C_{pz} \end{Bmatrix} = -\begin{Bmatrix} \left(-F^r_{py}X^i_{py} + F^i_{py}X^r_{py}\right)\Big/\left(X^{r^2}_{py} + X^{i^2}_{py}\right) \\ \left(-F^r_{pz}X^i_{pz} + F^i_{pz}X^r_{pz}\right)\Big/\left(X^{r^2}_{pz} + X^{i^2}_{pz}\right) \end{Bmatrix} \tag{8}$$

It is clearly seen that the identification of $\mathbf{K}_p$ and $\mathbf{M}_p$ is independent of $\mathbf{C}_p$.

Thus, at each speed, the responses at each pedestal location and the forces calculated from eqn (2) result in four equations for the five unknown pedestal parameters. Measurements at various speeds in the speed range form a set of 4 × n linear equations for the same five unknowns. The least squares method may be used to solve those equations for each pedestal to obtain the identified $\mathbf{M}_p, \mathbf{C}_p$ and $\mathbf{K}_p$ [2].

3. NUMERICAL IDENTIFICATION

To verify the identification procedure, numerical tests were conducted on a fictitious rotor-bearing-pedestal system with an arbitrarily chosen unbalance distribution as shown in Figure 2.

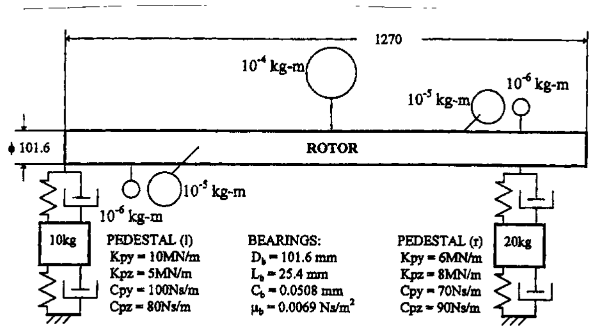

Fig. 2: Rotor-bearing-pedestal with unbalance distsribution.

The pedestal dampings were excluded in the first case. The unbalance response "measurements" were the calculated results of the in-house impedance matching software [1] which outputted both the responses of and the forces on the pedestals. Data were truncated to some specified number of significant digits to simulate measurement accuracy. The pedestals were so chosen that their resonances were in the speed range of interest. This approach was similar to that used in [2] when evaluating the direct identification procedure for a more general foundation.

Figure 3 shows the amplitudes of the responses at the measurement points, Figure 4 shows the corresponding phase angles and Figure 5 shows the phase differences between the pedestal responses and forces transmitted. While the phase changes in Figure 4 look complicated and were found to be unbalance distribution dependent, the phase differences shown in Figure 5 clearly indicate the pedestal resonances (when the angles jump from 0 to 180 degrees). Note that when the pedestals are considered as part of the system, these resonances do not

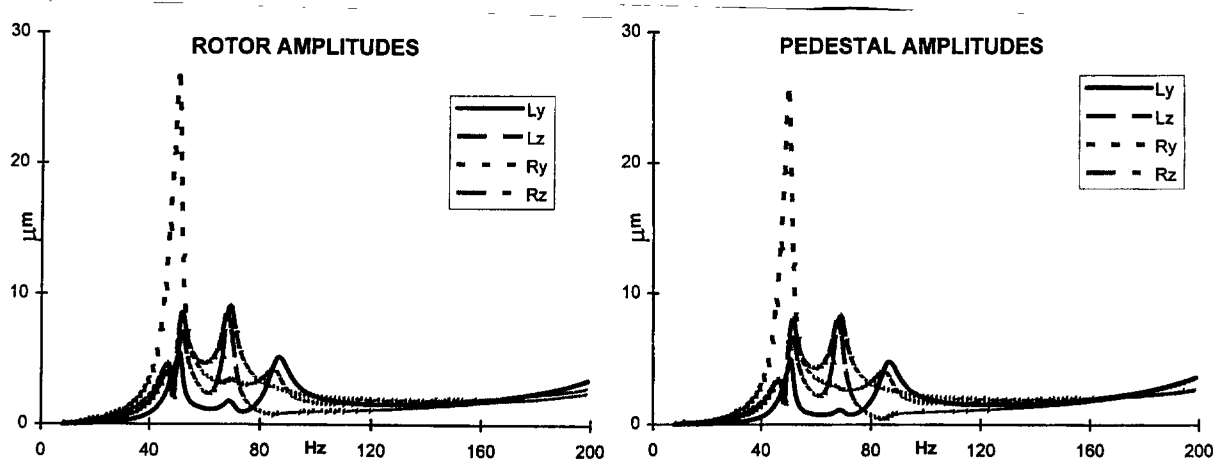

Fig. 3: Response amplitudes at measurement points (C_p = 0).

correspond to the amplitude peaks in Figure 3. Figure 5 can serve as an indication of the amount of pedestal damping (depending on how steeply the phases vary at resonance), and of the appropriateness of the evaluated transmitted forces (depending on how close the phases are to 0° before a resonance and to 180° after a resonance).

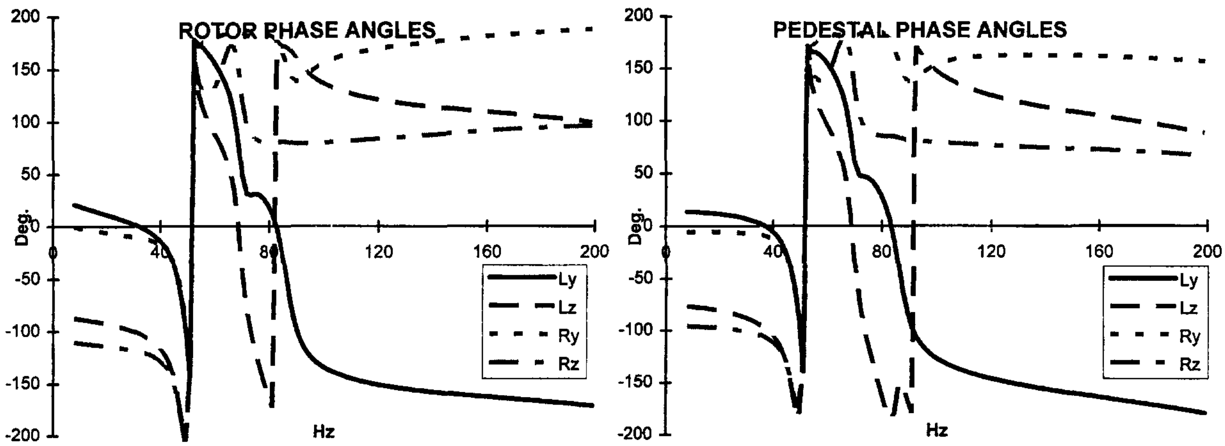

Fig. 4: Response phase angles at measurement points ($C_p = 0$).

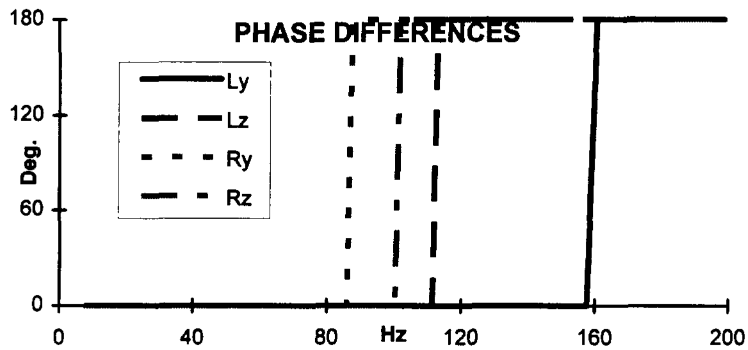

Fig. 5: Phase differences between responses at and forces transmitted to pedestals ($C_p = 0$).

4. RESULTS AND DISCUSSION

Table 1 shows the identified pedestal parameters using "measured" data with 12, 3 and 2 significant digit accuracies as input. Comparison with the actual values shows that even when the input data are truncated to two significant digits, corresponding to a maximum error of 5%, the identified parameters are still quite good, being correct to 2% or less for all parameters, ie the output error is not greater than the input error. Recalculating the response amplitudes, phase angles and phase differences displayed in Figures 3 to 5 but using the 2 digit identified pedestal parameters in Table 1 to represent the pedestals, resulted in negligible

Table 1: Identified Pedestal Parameters (C_p = 0)

	LEFT PEDESTAL			RIGHT PEDESTAL		
	K_{py} (MN/m)	K_{pz}	M_p (kg)	K_{py} (MN/m)	K_{pz}	M_p (kg)
Actual	10	5	10	6	8	20
12 digits	10.000	5.0000	10.000	6.0000	8.0000	20.000
3 digits	10.001	5.0005	10.000	5.9998	8.0007	20.003
2 digits	9.8968	4.9244	9.7653	5.9551	7.9284	19.711

change to Figures 3 to 5. When superimposed, the actual and identified response curves were indistinguishable to the naked eye. Figure 6, which shows the "measured" and predicted rotor responses in the y direction, is typical.

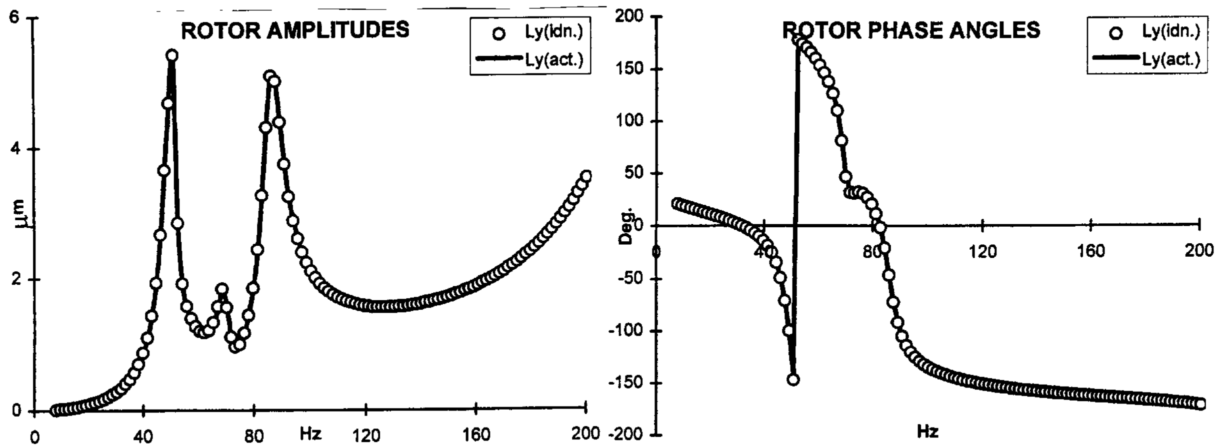

Fig. 6: Comparison between measured and predicted response amplitudes and phase angles.

Note that in the above example there was no damping in the pedestals. In such cases, either the real or the imaginary part, or both parts of the "measurements" could be used for parameter identification. However, should pedestal damping be present, using just the real or the imaginary part of the data, or their superimposition, could result in inaccurate results.

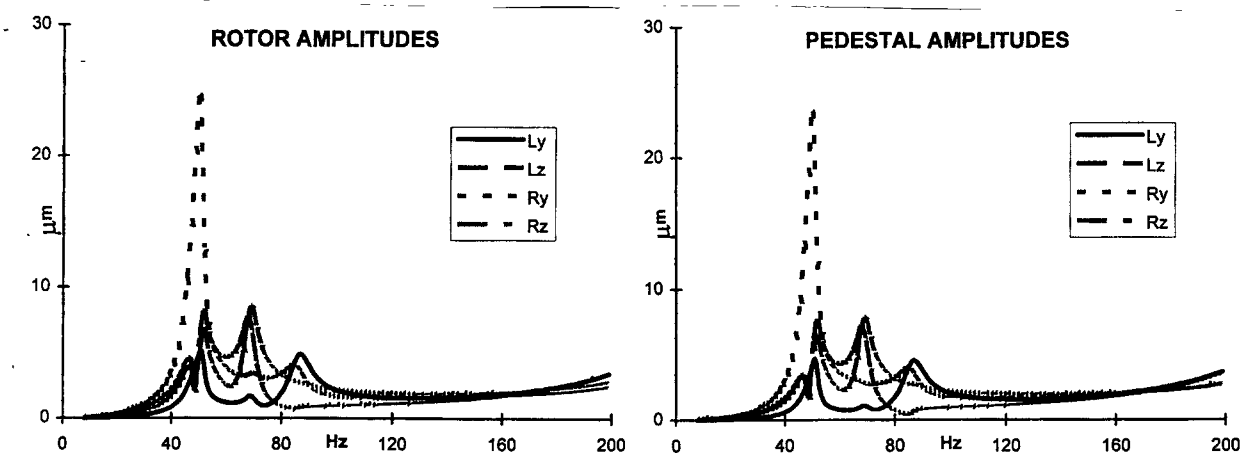

Fig. 7: Response amplitudes at measurement points ($C_p \neq 0$).

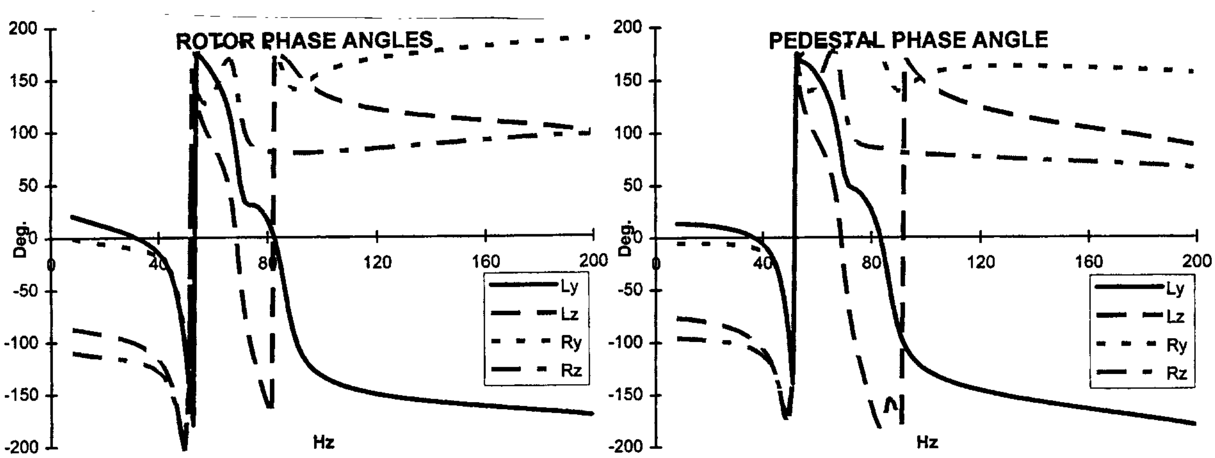

Fig. 8: Response amplitudes at measurement points ($C_p \neq 0$).

Figures 7 to 9 show the response amplitudes, the corresponding phase angles, as well as the phase differences between the pedestal responses and the forces transmitted when damping was introduced into the pedestals. They are similar to their counterparts in Figures 3 to 5 except instead of sudden changes in phase angles, the phase change around the resonances in Figure 9 is smooth.

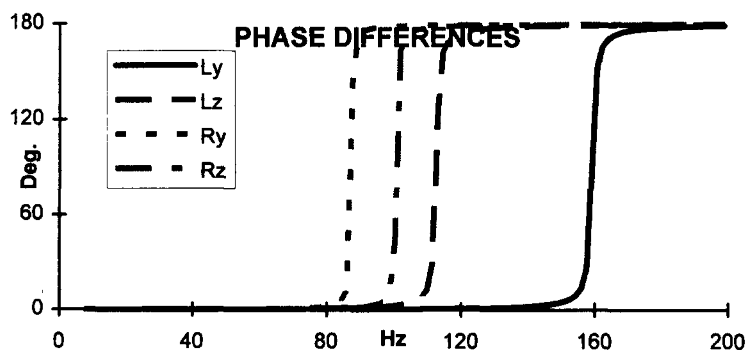

Fig. 9: Phase differences between responses at and forces transmitted to pedestals ($C_p \neq 0$).

Table 2 summarises the results obtained using 12 digit "measurement" accuracies when eqns (4) were utilised in the manner indicated. All the other parameters in the system remained unchanged.

Table 2: Identified Pedestal Parameters ($C_p \neq 0$, 12 digits)

R & I : All equations from Eqns (4) were used independently
R + I : Equations obtained from equating real and imaginary parts were superimposed
R : Only equations obtained from equating real parts were used
I : Only equations obtained from equating imaginary parts were used

	LEFT PEDESTAL			RIGHT PEDESTAL		
	K_{py} (MN/m)	K_{pz}	M_p (kg)	K_{py} (MN/m)	K_{pz}	M_p (kg)
Actual	10	5	10	6	8	20
R & I*	10.000	5.0000	10.000	6.0000	8.0000	20.000
R + I	10.047	4.9645	10.000	6.0052	7.9521	19.976
R	9.9686	5.0065	9.9829	6.0060	7.9783	20.008
I	10.032	4.9826	9.9835	5.9966	8.0134	20.004

* all the damping coefficients were correctly identified as well.

It is seen that inappropriate analysis of the data introduces error into the results even when the "measurement" data are accurate to 12 digits, ie in the presence of damping, all equations in eqn (4) should be used independently.

5. CONCLUSIONS

An approach for identifying pedestal parameters is presented here which appears feasible for flexible pedestals whose resonances are within or close to the operating speed range. Assuming the presence of some unbalance excitation and a knowledge of the bearing support stiffness and damping characteristics, the parameters can be identified from the measurements of motions of the rotor and pedestals at the supports for selected rotor speeds. Such measurements could be taken during a routine run-down procedure.

Identification of the pedestal mass and stiffnesses is unaffected by the pedestal damping (viscous) of the pedestals. However, both the real and imaginary parts of the signals need to be used to maximise identification accuracy.

Numerical tests show that good results are achievable even when the data are truncated to two significant digits, which is of the order of field measurement data accuracy, rendering the technique practically viable.

6. ACKNOWLEDGMENTS

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7. REFERENCES

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