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A TRAVELING WAVE APPROACH TO ACTIVE NOISE CONTROL IN DUCTS

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Abstract

This paper presents a new scheme for active noise control (ANC) in ducts. It uses three pressure sensors to measure and separate the far field wave components in a duct. These are the incident wave from the primary noise source, the reflection wave from the duct outlet, and the anti-sound wave generated by the loudspeaker. Use of traveling waves makes it easier to design ANC schemes since the transfer functions of traveling waves in a duct are simple delay factors with possible attenuation. When the "filtered-x" LMS is applied, the adaptation process is simplified since the error path is equivalent to a delay factor. The new scheme achieves optimal performance without using a pseudo random signal to identify the error path.

1 INTRODUCTION

There are several different approaches to active noise control (ANC) in ducts. These may be summarized as the feedback control approach [1]-[5]; the transfer function approach [6, 7]; and the acoustic standing wave approach [8].

The feedback control approach either decomposes the sound field into modal functions [1, 2], or derive state-space matrices [3, 4, 5] to manage feedback from the pressure sensors. The controller then synthesizes the anti-sound signal to suppress the noise field. Its performance is subject to the accuracy of the system model matrices, which depend on the accuracy of either the modal functions or the impedances of the two duct ends. Some feedback controller requires measurement of the time derivative of the pressure signal, which is more expensive and susceptible to measurement and numerical errors.

The other two approaches synthesize feedforward cancellation signals according to a reference signal. The reference is either measured directly from the primary source [7]; or measured between the primary and secondary (anti-sound) sources [6, 8]. In the first case, the reference is independent of the anti-sound signal. The only objective of the ANC is to match the transfer function of the duct and cancel the far field of the primary source [7]. In the second case, however, the reference contains signals from both sources. The ANC should avoid acoustic feedback from the secondary source as it attempts to cancel the effect of primary one [6, 8].

An important advantage of a feedforward ANC is its ability to improve performance by adaptation [9, 10]. The well-known "filtered-x" LMS algorithm [12] is a popular way to implement an adaptive ANC. It uses the transfer function of the error path to get the filtered-x signal. The accuracy of the adaptation hence depends on the accuracy of the transfer function of the error path that is, unfortunately, not conveniently available in many applications. While it is possible to identify the transfer function of the error path on-line, the side effect is adding a pseudo random signal to the anti-sound that is not cancelled.

The pseudo random signal excites the impulse response of the error path that, in turn, joins the cancellation error to form a propagating mixture. The adaptive law has to process the signal from the error sensor in order to separate the impulse response from the cancellation error. It is impossible to separate two unknown signals sharing the same frequency range without any error. Such error causes inaccuracy in the estimated transfer function of the error path that, in turn, causes inaccuracy to the ANC transfer function.

This paper presents a new approach to feedforward ANC in ducts. It places an additional pressure sensor near the primary error sensor, enabling the separation of traveling waves from the acoustic pressure signals. For one dimensional wave propagation problems, the transfer function of traveling waves are simple delay factors with possible attenuation. The separation of traveling waves simplifies problems associated with transfer functions of duct sections. The adaptation process becomes simpler. It becomes possible for an ANC to achieve optimal performance without the annoying pseudo random signal.

2 TRAVELING WAVE SEPARATION

The proposed system is very similar to those ANC's presented in [7]-[10], except for the additional pressure sensor placed near the downstream primary error sensor. Figure 1 illustrates the location of the sensors and the traveling wave components.

A loudspeaker is placed at $x_s = 0.0$ as the secondary source. The primary source is sufficiently far away from the zone such that the noise can be considered as coming from negative infinity. Three pressure sensors, labeled as s_1, s_2 and s_3 respectively, are placed at coordinates $x_1 = -l_1$, $x_2 = l_2$ and $x_3 = l_2 + l_3$ respectively. The upstream sensor s_1 should be placed in such a way that $x_1 = -l_1$ is not a node of the standing waves. This restriction is required to avoid a potential situation where some standing waves exist before the ANC adapts to its optimal state. There are generally no other restrictions on the coordinates of the sensors.

It is assumed that the duct is a linear wave propagating system. The incident wave comes from negative infinity and travels in the positive direction. It is a function of time and space. If one uses w_p to denote the incident wave observed at x_1, then the

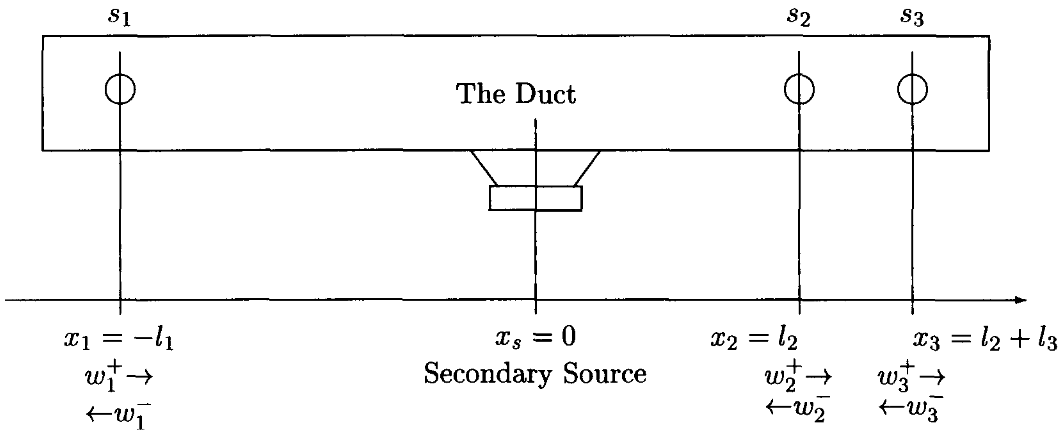

Figure 1: Traveling waves in a duct.

incident wave is $w_p e^{-jk(l_1+l_2)}$ when observed at x_2 and $w_p e^{-jk(l_1+l_2+l_3)}$ at x_3. Here subscript "p" indicates "wave from the *primary* source"; $e^{-jk(l_1+l_2)}$ is a delay factor with possible attenuation depending on whether the wave number k is real or complex.

The reflection wave is denoted by w_r, where subscript "r" indicates that the wave is due to "*reflection*". This signal, observed at x_2, travels in the negative direction. It becomes $w_r e^{-jk(l_1+l_2)}$ when observed at x_1.

The anti-sound wave, generated by the secondary source (loudspeaker), travels in both directions. Only the far field effects of the anti-sound are considered here. Let $p_s(x,t)$ and $v_s(x,t)$ denote, respectively, the pressure and velocity signals due to the *secondary* source. It is shown in [12] that p_s is spatially symmetric and v_s spatially anti-symmetric with respect to the secondary source, or mathematically,

$$\begin{aligned} p_s(x - x_s, t) &= p_s(x_s - x, t) && (1) \\ v_s(x - x_s, t) &= -v_s(x_s - x, t) && (2) \end{aligned}$$

where x_s is the coordinate of the secondary source. In this study, $x_s = 0.0$ for simplicity, hence p_s is spatially an even function and v_s spatially an odd function.

Introduce $Y_o = \frac{c_o}{S}$, where c_o is the speed of sound and S the area of cross-section of the duct. Then the forward and backward waves of the secondary source can be expressed as

$$\begin{aligned} w_s^+(x,t) &= Y_o v_s(x,t) + p_s(x,t) \\ w_s^-(x,t) &= Y_o v_s(x,t) - p_s(x,t) \end{aligned}$$

respectively. It is not difficult to see, from (1) and (2), that w_s^+ and w_s^- are spatially anti-symmetric with respect to the secondary source. With $x_s = 0.0$, one can verify that

$$w_s^-(-l_2, t) = -w_s^+(l_2, t) = -w_s \tag{3}$$

where w_s represents the forward traveling anti-sound wave observed at $x_2 = l_2$. The backward anti-sound travels in the negative direction. It is $-w_s$ when observed at $x = -l_2$ and

$$w_s^-(x_1, t) = w_s^-(-l_1, t) = -w_s e^{jk(l_2-l_1)} \tag{4}$$

at $x_1 = -l_1$. In the above equation, $e^{jk(l_2-l_1)}$ reflects the spatial difference between coordinates $-l_1$ and l_2. It is a delay factor if $l_1 > l_2$ or a lead factor if $l_2 \geq l_1$.

The focus can now be directed to the three spots where the pressure sensors are mounted. Let w_1^+, w_2^+ and w_3^+ denote, respectively, the forward traveling waves observed at these spots; and let w_1^-, w_2^- and w_3 denote the corresponding backward traveling waves. At $x_1 = -l_1$, the incident wave from the primary source is the only wave that travels in the positive direction when the coordinate of primary source is negative infinity. In a real application where the duct length is finite, the incident wave may include the wave from the primary source and the reflection from the negative-end of the duct. No matter what the components of the incident wave, its total effect must be cancelled by the anti-sound. For this reason, the upstream incident wave is considered to be from one single noise source defined as w_p at x_1, or

$$w_1^+ = w_p. \tag{5}$$

The reflection wave from the duct outlet and the secondary source constitute the backward traveling wave,

$$w_1^- = w_r e^{-jk(l_1+l_2)} - w_s e^{jk(l_2-l_1)} \tag{6}$$

where (4) has been used to replace $w_s^-(-l_1, t)$.

The forward traveling wave at $x_2 = l_2$, denoted as w_2^+, has two components: the incident wave of the noise and the anti-sound wave intended to cancel the noise. This is actually the error signal. On the other hand, there is only one component in w_2^-, that is the reflection from the duct outlet. The two waves are given by

$$w_2^+ = w_p e^{-jk(l_1+l_2)} + w_s \quad \text{and} \quad w_2^- = w_r. \tag{7}$$

Similarly, the two traveling waves at $x_3 = l_2 + l_3$ can be written as

$$w_3^+ = w_p e^{-jk(l_1+l_2+l_3)} + w_s e^{-jkl_3} \tag{8}$$

and

$$w_3^- = w_r e^{jkl_3}. \tag{9}$$

The sensors are assumed to be omni-directional microphones that measure the acoustic pressure of the sound field. According to (5), (6), (7), (8) and (9), the pressure signals have the following expressions

$$p_1 = \frac{1}{2}(w_p - w_r e^{-jk(l_1+l_2)} + w_s e^{jk(l_2-l_1)}); \tag{10}$$

$$p_2 = \frac{1}{2}(w_p e^{-jk(l_1+l_2)} + w_s - w_r); \tag{11}$$

$$p_3 = \frac{1}{2}(w_p e^{-jk(l_1+l_2+l_3)} + w_s e^{-jkl_3} - w_r e^{jkl_3}). \tag{12}$$

The traveling waves can be solved from the above equations. First, one obtains

$$w_2^+ = \frac{2}{1 - e^{-j2kl_3}}(p_2 - p_3 e^{-jkl_3}) \tag{13}$$

and

$$w_r = \frac{2}{1 - e^{-j2kl_3}}(p_2 e^{-j2kl_3} - p_3 e^{-jkl_3}) \tag{14}$$

according to (7), (11) and (12). Next, the incident and anti-sound waves are solved as

$$w_p = \frac{2p_1 - 2p_2 e^{jk(l_2-l_1)} - w_r e^{jk(l_2-l_1)}(1 - e^{-j2kl_2})}{1 - e^{-j2kl_1}} \tag{15}$$

and

$$w_s = \frac{2p_2 - 2p_1 e^{-jk(l_1+l_2)} + w_r(1 - e^{-j2k(l_1+l_2)})}{1 - e^{-j2kl_1}} \tag{16}$$

respectively from (10), (11) and (14). Whenever possible, it is recommended to choose $l_1 = l_2 = l$. Such a choice would simplify (15) and (16) substantially. The two traveling waves can then be computed respectively, as

$$w_p = 2\frac{p_1 - p_2}{1 - e^{-j2kl}} - w_r; \tag{17}$$

$$w_s = 2\frac{p_2 - p_1 e^{-j2kl}}{1 - e^{-j2kl}} + w_r(1 + e^{-j2kl}). \tag{18}$$

This choice will be assumed through out the rest of this paper for sake of simplicity.

3 ACTIVE NOISE CONTROL AND ADAPTATION

The objective of this study is to make $w_2^+ \to 0$. Since w_2^+ is the forward traveling wave observed at $x_2 = l_2$, its convergence to zero means no more noise propagating towards the outlet starting from x_2. When that happens, w_r only represents the environment noise from outside of the duct outlet. There is no need for the ANC to deal with it. The ANC synthesizes a signal s_a that passes through the power amplifier and the loudspeaker to excite the sound field. After a delay of e^{-jkl_2} and neglecting the far field effects, the anti-sound wave reaches $x_2 = l_2$ as w_s, *ie.*

$$w_s = C(z) s_a e^{-jkl_2} \tag{19}$$

where $C(z)$ represents the transfer process through the power amplifier, loudspeaker to the anti-sound. In this study, $C(z)$ is called the transfer function of the excitation path. It can be either measured off-line or identified on-line, since w_s and s_a are available in the present system.

Let $H(z) = \frac{s_a}{w_p}$ denote the transfer function of the ANC. Substituting into (19), one obtains

$$w_s = C(z)H(z)w_p e^{-jkl_2}. \tag{20}$$

This equation describes the passage of w_p, through the ANC $H(z)$, the excitation path $C(z)$ and a delay of e^{-jkl_2}, to coordinate x_2. According to (7), $w_2^+ \to 0$ implies $w_s \to -w_p e^{-jk(l_1+l_2)}$ and hence

$$C(z)H(z)w_p e^{-jk(l_1+l_2)} \to -w_p e^{-jk(l_1+l_2)}.$$

Obviously, the transfer function of ANC should be chosen in such a way that, at convergence, the last expression becomes an equality, or

$$H(z) = -T(z)e^{-jkl_1} \quad \text{and} \quad T(z) = \frac{1}{C(z)}. \tag{21}$$

A further substitution of (14) and (17) suggests that the ANC should synthesize s_a by

$$s_a = -\frac{e^{-jkl_1}}{\hat{C}(z)} w_p \tag{22}$$

where $\hat{C}(z)$ is an estimation of $C(z)$ either from an on-line adaptation algorithm, or from the exact form of $C(z)$ if available. This section considers a practical case where the exact form of $C(z)$ is not available. The focus here is how to obtain $\hat{C}(z)$. The adaptive technique provides an effective solution to this problem. The proposed ANC applies the "filtered-x" LMS algorithm to adjust $\hat{T}(z)$ and minimize

$$\begin{aligned} \min_{\hat{T}(z)} \|w_2^+\| &= \min_{\hat{T}(z)} \|w_p e^{-jk(l_1+l_2)} + w_s\| = \min_{\hat{T}(z)} \|[1 - C(z)\hat{T}(z)] w_p e^{-jk(l_1+l_2)}\| \\ &= \min_{\hat{T}(z)} \|[1 - \hat{T}(z)C(z)] w_p e^{-jk(l_1+l_2)}\| = \min_{\hat{T}(z)} \|w_p e^{-jk(l_1+l_2)} - \hat{T}(z) y\| \end{aligned} \tag{23}$$

where $y = C(z) w_p e^{-jk(l_1+l_2)}$ is the filtered signal. The two transfer functions $\hat{T}(z)$ and $C(z)$ are commutable since they are linear time-invariant. The minimization of (23) is not possible without exact knowledge of $C(z)$ that is needed for the synthesis of y. One possible way to get around is to substitute

$$\hat{y} = \hat{C}(z) w_p e^{-jk(l_1+l_2)}$$

that uses the estimated $\hat{C}(z)$ obtained by a different LMS process. The second LMS uses w_s and $s_a e^{-jkl_2}$ as the input/output pair, which are available by (18) and (22) respectively. It minimizes

$$\min_{\hat{C}(z)} \|w_s - \hat{C}(z) s_a e^{-jkl_2}\| = \min_{\hat{C}(z)} \|[C(z) - \hat{C}(z)] s_a e^{-jkl_2}\|, \tag{24}$$

obtains $\Delta\hat{C}(z) = C(z) - \hat{C}(z)$ and then updates $\hat{C}(z)$. This process does not affect $\hat{T}(z)$ which is substituted into (22) to compute $s_a e^{-jkl_2}$. During the adaptation, $\hat{T}(z)$ is not necessarily $\frac{1}{\hat{C}(z)}$, though the final objective is $\hat{T}(z) \rightarrow \frac{1}{\hat{C}(z)}$. Identification of $\hat{T}(z)$ is the job of the "filtered-x" LMS algorithm that minimizes

$$\min_{\hat{T}(z)} \|w_p e^{-jk(l-1+l_2)} - \hat{T}(z)\hat{y}\|. \tag{25}$$

The above expression differs from (23) only in that $\hat{y}$ substitutes y, since $C(z)$ is not available *a-priori*. The two LMS processes, (24) and (25), combine to form an upper bound on the intended objective (23) as follows

$$\begin{aligned} \min_{\hat{T}(z)} \|w_2^+\| &= \min_{\hat{T}(z)} \|w_p e^{-jk(l_1+l_2)} - \hat{T}(z)\hat{y} + \hat{T}(z)\hat{y} + w_s\| \\ &\leq \min_{\hat{T}(z)} \|w_p(x_2, t) - \hat{T}\hat{y}\| + \min_{\hat{T}(z)} \|\hat{T}\hat{y} + C s_a e^{-jkl_2}\| \\ &\leq \min_{\hat{T}(z)} \|w_p(x_2, t) - \hat{T}\hat{y}\| + \min_{\hat{C}(z)} \|[C - \hat{C}] s_a e^{-jkl_2}\| \end{aligned} \tag{26}$$

where (16) has been substituted for w_s while

$$\hat{T}(z)\hat{y} = \hat{T}(z)\hat{C}(z) w_p e^{-jk(l_1+l_2)} = -\hat{C}(z) s_a e^{-jkl_2}$$

because the linear transfer functions $\hat{T}(z)$ and $\hat{C}(z)$ are also commutable.

Since $\hat{T}(z)$ is not necessarily $\frac{1}{\hat{C}(z)}$, the adaptation processes allow certain estimation errors in both functions. There are no interactions between the estimation errors of the two LMS processes. Their joint effect achieves the final objective of minimizing $||w_2^+||$ as suggested by (26).

4 SIMULATION AND CONCLUSIONS

A simulation is conducted using the compact wave solver [11] to test the proposed ANC. It simulates the acoustic field of a duct section. The spatial variable x is normalized such that the sound travels 10 units per second. The simulated length of duct is 0.1 units. In the simulation, the primary source is placed at $x_p = -0.075$ units. It generates a noise signal

$$n(t) = \sin(2\pi ft) + 0.3\sin(2.5\pi ft).$$

The anti-sound loudspeaker is placed at $x_s = 0.0$ units while the ANC parameters are chosen to be $l_1 = l_2 = 0.0025$ and $l_3 = 0.000125$ units.

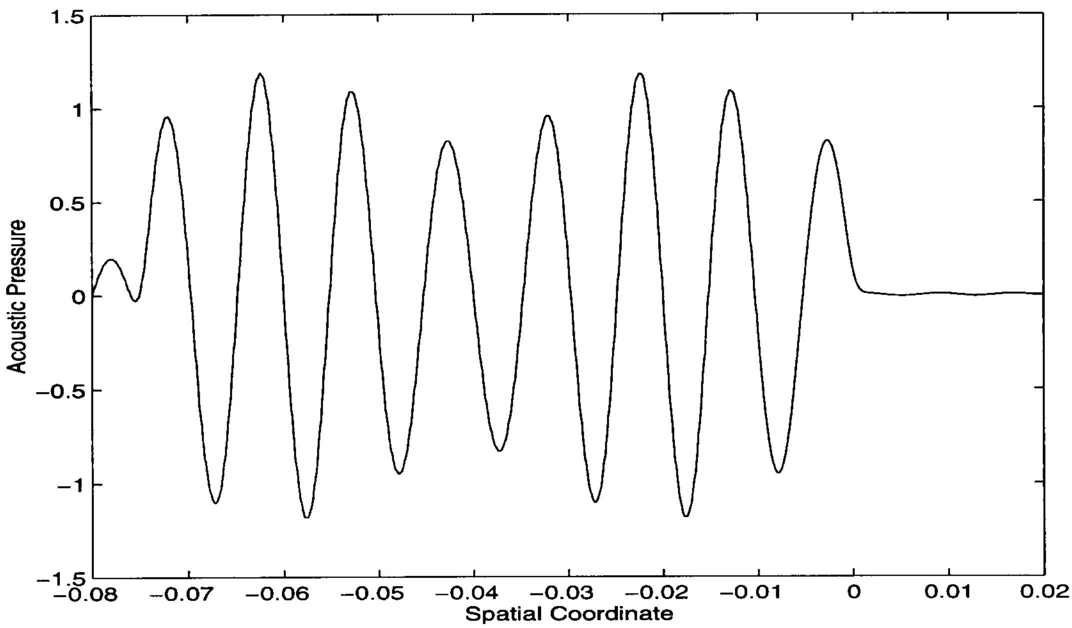

Figure 2: A snapshot of acoustic pressure in the duct.

In the simulation, the transfer function of the excitation path $C(z)$ is not available to the ANC system. Two separate LMS processes identify $\hat{C}(z)$ and $\hat{T}(z)$, respectively, as two FIR filters. Figure 2 shows a snapshot of the acoustic pressure signals in the duct when $f = 1000Hz$. The two adaptive LMS processes converge very quickly. The noise signal is well cancelled for $x \geq 0$ that confirms the theory of the proposed ANC.

In summary, this research proposes a new method for feedforward ANC in ducts. It separates the traveling waves in a duct; enables the ANC to obtain the incident wave of noise, the reflection wave from the duct outlet, the cancellation error wave and the anti-sound wave. This approach simplifies the problems of transfer functions since traveling waves propagate with simple delays and small amount of attenuation. A new adaptive

ANC is proposed on this ground that avoids dealing with the error path. Good performance of the proposed method is demonstrated by analytical and simulation studies.

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