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Structural sensing of sound transmission into a cavity for active structural-acoustic control

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Abstract

The problem of actively minimising the transmission of harmonic sound through a curved panel into a contiguous cavity using structural sensors is investigated both numerically and experimentally. It is well known that a control system that simply aims to minimise the structural vibration does not necessarily lead to a reduction in sound transmission. However, by considering the dynamics of the coupled system, it is possible to derive an orthonormal set of structural "radiation modes" which are orthogonal with respect to their contribution to the acoustic potential energy of the internal coupled acoustic space. Minimisation of the amplitudes of these "radiation modes" is guaranteed to result in a reduction of the interior potential energy, in contrast to minimising the normal structural modes. Sensing of the orthogonal sets of "radiation modes" is accomplished by using "smart sensors" made from either shaped PVDF film or a linear combination of accelerometers, adjusted to sense a particular radiation mode.

1 INTRODUCTION

Over the past 10 years vibration control sources have been used extensively to minimise sound transmission through light-weight structures into coupled enclosures. Vibration control sources have two distinct advantages over more conventional acoustic sources: fewer secondary sources are generally required for global control of the interior noise field (Charette *et al.*, 1995); and surface mounted actuators are far less intrusive than bulky speaker/cabinet arrangements. However, the gains in system compactness are not necessarily realised in practice as microphones placed throughout the cavity are used to provide the controller error signals to achieve global control.

Surface mounted accelerometers offer an alternative to microphones and although a reduction in the vibration of the structure may be achieved, a reduction in the interior sound field does not necessarily follow, particularly at low frequencies where the acoustic modal density is low. However, it has been shown recently that is possible to calculate a quantity from the vibration of the structure, commonly referred to as a radiation mode, which is directly proportional to the sound radiated into the enclosed space (Snyder and Tanaka, 1993, Cazzolato and Hansen, 1997).

This paper uses the procedure developed by Cazzolato and Hansen (1997) where the sound transmission from a structure into an enclosed space can be measured directly using structural error sensors. The approach involves decomposing the surface vibration, usually via Singular Value Decomposition, into a number of velocity distributions which contribute independently to some cost function. It has been shown that for sound transmission problems only a very limited number of radiation modes contribute in most cases to the sound radiated from the vibrating structure and it is the number of efficient radiation modes, rather than the modal response of the structure, which defines the system dimensionality and subsequently, the control system order (Borgiotti, 1990).

Distributed parameter modal sensors called "smart sensors" have been employed to measure the modal amplitudes of the radiation modes and to provide inputs into an active control system. The use of independent (orthogonal) error signals for active noise control problems has been shown to offer a number of practical advantages, as it can: reduce convergence time for controllers; provide robustness to system parameter uncertainty; and minimise the number of sensors and actuators, and corresponding system dimensionality (Morgan, 1991).

The objective of the work described here was two fold: to investigate the feasibility of using structural sensors rather than microphones to control the sound transmission into cavities and in doing so, gain some understanding of the mechanisms of sound transmission into enclosures and the implications for active noise control systems; and to determine the most suitable means of sensing radiation modes using structural vibration measurements.

The active control of sound transmission into a rectangular cavity from a coupled curved panel excited by an external vibration source is investigated numerically. The structure and the acoustic space were modelled numerically using the commercially available finite element package ANSYS and coupled using modal coupling theory (Snyder and Hansen, 1994). Preliminary experiments using both discrete acceleration and continuous strain radiation modal sensors have been conducted with some success.

2 BACKGROUND THEORY

It has been assumed for the following analysis that the acoustic pressure, $p(\vec{\mathbf{r}})$, at any location, $\vec{\mathbf{r}}$ within a cavity can be expressed as an infinite summation of the product of rigid-wall acoustic mode shape functions, ϕ_i, and the modal amplitudes of the cavity, p_i.

$$p(\vec{\mathbf{r}}) = \sum_{i=1}^{\infty} p_i \phi_i(\vec{\mathbf{r}}) \tag{1}$$

The acoustic potential energy within an enclosure provides a suitable global error criterion for controlling the sound transmission from a coupled structure. This is given by the following

$$E_p = \frac{1}{4\rho_0 c_0^2} \int_V |p(\vec{\mathbf{r}})|^2 d\vec{\mathbf{r}} \tag{2}$$

where ρ_0 is the density of the acoustic fluid (air), c_0 is the speed of sound in the fluid and V is the volume over which the integral is evaluated. This volume may be the entire cavity or it may be a smaller volume within the cavity. By limiting the volume over which the integral is evaluated, controller effort is not expended in controlling non-essential regions. Subsequently, the attenuation within the zone is greater than when the total acoustic potential energy of the cavity is used as the cost function. Such an approach may be suitable when controlling the sound around the heads of passengers in automobiles or aircraft.

Substitution of the modal expansion (1) evaluated over n_a acoustic modes into (2) leads to the expression

$$E_p = \mathbf{p}^{\mathrm{H}}\Lambda\mathbf{p} \tag{3}$$

where $\mathbf{p}$ is the (n_a x 1) vector of acoustic modal amplitudes and Λ is a (n_a x n_a) diagonal weighting matrix, the diagonal terms of which are $\Lambda_{ii} = \frac{1}{4\rho_0 c_0^2}\int_V \phi_i^2(\vec{\mathbf{r}})dV(\vec{\mathbf{r}})$.

The matrix expression for the pressure in the cavity arising from the vibration of the structure is given by (Snyder and Hansen, 1994)

$$\mathbf{p} = \mathbf{Z}_a\mathbf{v} \tag{4}$$

where $\mathbf{Z}_a$ is the (n_a x n_s) modal structural-acoustic radiation transfer function matrix and $\mathbf{v}$ is the (n_s x 1) structural modal velocity vector. Substituting equation (4) into (3), the potential energy arising from the vibration of the structure is given by

$$E_p = \mathbf{v}^{\mathrm{H}}\Pi\mathbf{v} \tag{5}$$

where the frequency dependent (n_s x n_s) error weighting matrix Π is given by $\Pi = \mathbf{Z}_a^{\mathrm{H}}\Lambda\mathbf{Z}_a$. The weighting matrix Π is real symmetric and may be diagonalised via an orthonormal transformation. This leads to an expression for the potential energy as a function of an orthogonal set of structural modes, commonly referred to as radiation modes,

$$E_p = \mathbf{v}^{\mathrm{H}}\Pi\mathbf{v} = \mathbf{v}^{\mathrm{H}}\mathbf{U}\mathbf{S}\mathbf{U}^{\mathrm{T}}\mathbf{v} = \mathbf{w}^{\mathrm{H}}\mathbf{S}\mathbf{w} \tag{6}$$

where the unitary matrix $\mathbf{U}$ is the (real) orthonormal transformation matrix representing the eigenvector matrix of Π, the (real) diagonal matrix $\mathbf{S}$ contains the eigenvalues (singular values) of Π and $\mathbf{w}$ is the (n_s x 1) radiation mode modal amplitude vector. By writing the potential energy in the form of equation (6), we have a set of structural velocity patterns (radiation mode shapes) which are orthogonal contributors to the error criterion (in this case, the potential energy in the volume). Therefore, a reduction in the amplitude of any of the radiation modes will directly result in a reduction in the potential energy.

The physical significance of the eigenvectors and eigenvalues is interesting. The eigenvalue can be considered a radiation efficiency (or coupling strength) and the associated eigenvector gives the level of participation of each normal structural mode to the radiation mode; thus it indicates the modal transmission path.

To evaluate (6) it is necessary to know the magnitude of the modal amplitudes of the radiation modes. In a physical system, these amplitudes may be estimated by decomposing the vibration at several discrete locations on the structure, ie

$$\mathbf{w} = \mathbf{Z}_t \mathbf{v}_e \tag{7}$$

where, $\mathbf{v}_e$ is the (n_e x 1) vector of velocity levels for the discrete structural sensors and $\mathbf{Z}_t$ is the (n_{rm} x n_e) radiation mode structural transfer function matrix (or modal filter matrix) which relates the vibration velocity levels at the discrete error sensor locations to the modal amplitudes of the radiation modes. Using the principle of modal orthogonality it can be shown that (Cazzolato & Hansen, 1997)

$$\mathbf{Z}_t = \Psi_e \mathbf{U} = \Theta_e \tag{8}$$

where Ψ_e is the (n_e x n_s) mode shape matrix at the sensor locations and Θ_e is the mode shape matrix of the radiation modes evaluated at the error sensor locations.

It will be shown in the following section that only a very small number of the radiation modes contribute to the sound transmission into the cavity over a narrow frequency range. This means that using radiation mode sensors rather than microphones, the number of inputs into the control system can theoretically be reduced.

It has been shown by Cazzolato and Hansen (1997) that the eigenvalues and eigenvectors can be highly frequency dependent. Subsequently the radiation mode shapes are also frequency dependent and therefore equation (6) as it stands does not easily lend itself to practical implementation. To overcome the frequency dependence of the mode shapes it is possible to select a frequency, f, at which to fix or "normalise" the eigenvector matrix. The acoustic potential energy given by (6) can now be approximated as

$$E_p \approx \mathbf{v}^{\mathrm{H}} \mathbf{U}_f \hat{\mathbf{S}}_f \mathbf{U}_f^{\mathrm{T}} \mathbf{v} = \mathbf{w}_f^{\mathrm{H}} \hat{\mathbf{S}}_f \mathbf{w}_f \tag{9}$$

where $\mathbf{U}_f$ is the eigenvector matrix corresponding to the chosen frequency, f, $\mathbf{S}_f$ is the diagonalised frequency normalised eigenvalue matrix and $\mathbf{w}_f$ is the modal amplitude vector of the frequency independent radiation modes. The mode shapes and the modal filter may still be calculated using equation (8).

This results in a non-orthogonal set of equations at all frequencies apart from the "normalisation frequency". However, at frequencies close to the normalisation frequency the resulting eigenvalue matrix is highly diagonal and can be approximated by only using the diagonal elements with little loss of mass from the matrix and accuracy in the error criterion.

With the mode shapes of the radiation modes now independent of frequency, the modal decomposition can be performed using shaped sensors, doing away with the need for a modal filter (see Figure 1). The use of shaped error sensors and means for relating the charge output to the flexural motion of the surface to which they are fixed, is discussed in detail by Lee and Moon (1990).

3 NUMERICAL SIMULATION OF SOUND TRANSMISSION THROUGH A CURVED PANEL INTO A COUPLED CAVITY

The coupled curved panel/cavity system shown in Figure 1 has been numerically investigated to determine the effectiveness of using radiation modal sensing to actively minimise the sound transmission into a cavity. The curved panel was made of 1 mm thick aluminium with simply supported end conditions. The other 5 walls of the cavity, measuring 0.985m x 0.420m x 0.250m, were rigid. The panel and the cavity were modelled individually using the FEA package ANSYS, then coupled using modal coupling theory (Snyder and Hansen, 1994) within MATLAB. The natural frequencies of the coupled system are shown in Table 1.

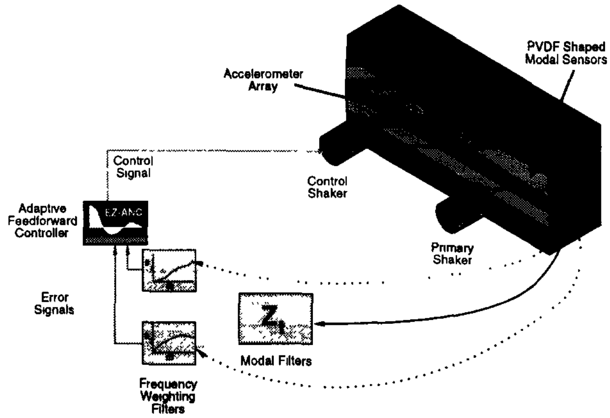

Figure 1 : Schematic showing the setup of the numerical experiment

Table 1 : Natural frequencies of the coupled vibro-acoustic system

Mode	Frequency (Hz)
$[1,1]_s$	73
$[2,1]_s$	94
$[2,2]_s$	117
$[1,2]_s$	135
$[2,3]_s$	158
$[0,1,0]_a$	185
$[3,1]_s$	205
$[2,4]_s$	207
$[1,3]_s$	212
$[3,2]_s$	219
$[3,3]_s$	241
$[2,5]_s$	256
$[1,4]_s$	259
$[3,4]_s$	276
$[1,5]_s$	290

s = structural, a = acoustic

A single primary shaker was located slightly off centre approximately a quarter of the way along the panel and a single control shaker was located symmetrically at the opposite end as shown in Figure 1.

Singular Value Decomposition was used to calculate the eigenvectors and eigenvalues of the radiation matrix. It was decided to optimise the system to control the first longitudinal acoustic mode. Using the technique given in Equation (9), the radiation mode shapes have been fixed to the shape that they have at the natural frequency of the first longitudinal mode, viz 185 Hz. The magnitude of the frequency normalised eigenvalues (radiation efficiencies) are shown in Figure 2. The mode shapes of the first 2 corresponding radiation modes are shown in Figures 3 and 4.

It is clear from Figure 2 that below 120 Hz the zeroth radiation mode dominates the sound transmission into the cavity. Between 120 Hz and 270 Hz the first mode becomes the most efficient. At the "cross over" frequencies (viz, 120 and 270 Hz) two modes contribute to the sound transmission and subsequently control will be ineffective with only a single control force.

It becomes clear when looking at Figures 3 and 4 that the mode shapes of the radiation modes looks very much like the acoustic mode shape on the surface of the panel. For systems with low acoustic modal density, this always occurs when the equations are normalised so that the mode shapes are fixed to the shape they have at a frequency corresponding to the natural frequency of the acoustic system. In fact, when this is the case, the acoustic mode shape at the panel surface can be used instead of the mode shape of the radiation mode to decompose the modal amplitudes with little loss in accuracy.

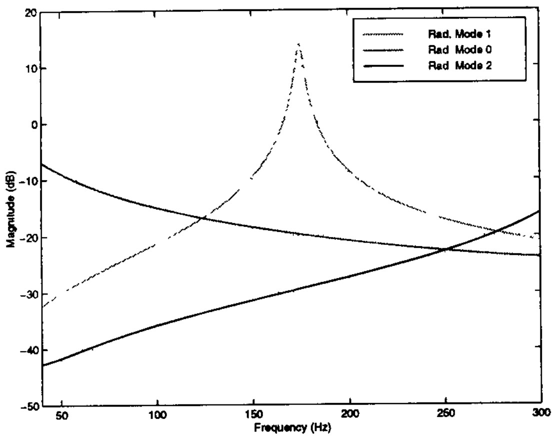

Figure 2 : Radiation efficiencies of the first 3 radiation modes fixed to 185 Hz

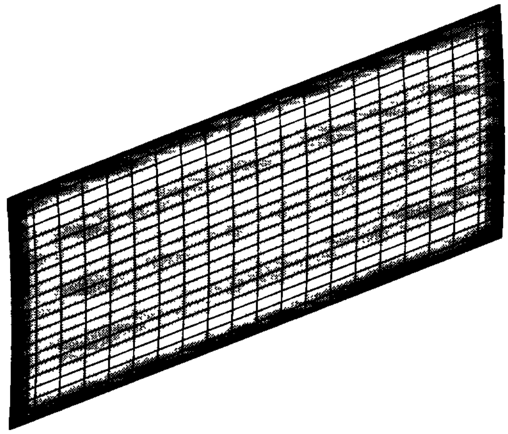

Figure 3 : Mode shape of the zeroth radiation mode at 185 Hz.

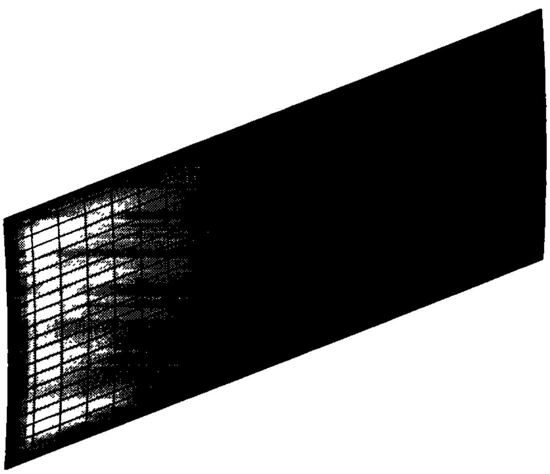

Figure 4 : Mode shape of the first radiation mode fixed to 185 Hz.

In Figure 5, the reduction in the total acoustic potential energies obtained when using conventional error criteria; namely, the acoustic potential energy (providing an upper limit of control performance), structural kinetic energy and the pressure at a single microphone are compared. As expected, minimising the structural kinetic energy does not necessarily lead to a reduction in the acoustic potential energy.

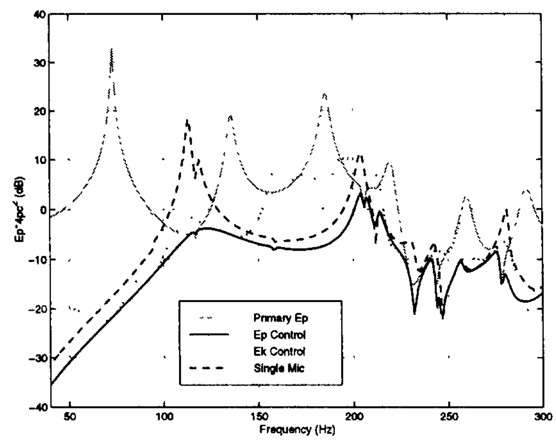

Figure 5 : Reduction in acoustic potential energy using various cost functions

In Figures 6 and 7, a comparison is made between using 11 discrete sensors and a continuous surface sensor to control the zeroth and first radiation modes respectively. The control achieved by sensing the first 2 radiation modes using continuous sensors is almost as effective as directly sensing the potential energy. As has been observed previously (Lee and Moon, 1990), the use of discrete sensors suffers from spatial sampling problems which leads to significant "leak through" of undesired modes. This contamination of the error signal results in sub-optimal performance.

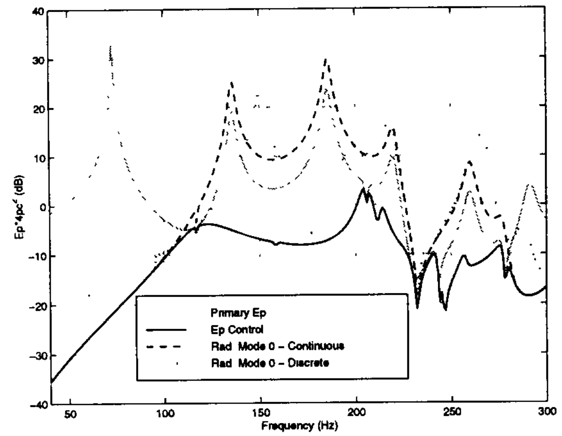

Figure 6 : Acoustic potential energy using zeroth radiation mode sensors.

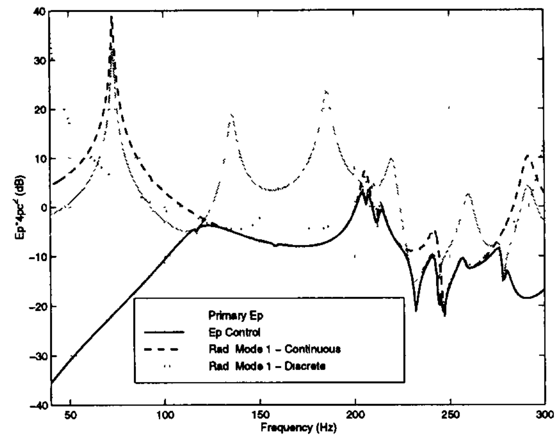

Figure 7 : Acoustic potential energy using first radiation mode sensors.

4 EXPERIMENTAL SETUP

A preliminary experimental investigation of the curved panel-cavity system was undertaken in order to confirm the theory derived by Cazzolato and Hansen (1997). The experimental configuration is shown in Figures 8 and 9.

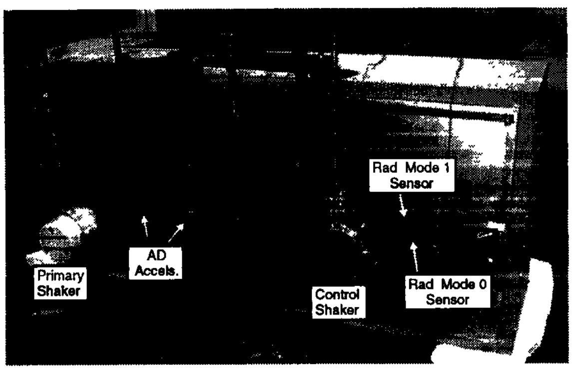

Figure 8 : Experimental setup for active control of sound transmission

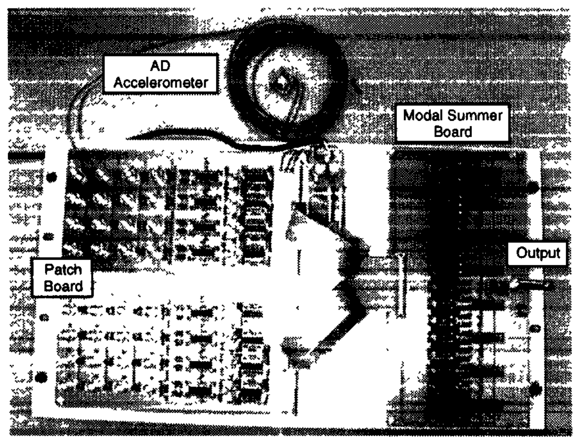

Figure 9 : Patch board and modal summing board

A Ling V203 shaker was used to provide the primary driving force and a B&K Type 8001 impedance head was used to measure the input force. A B&K Type 4810 mini-shaker was used as the control source.

Because many discrete error sensors are required for the modal decomposition in a practical system it was necessary to use a very low cost accelerometer. The Analog Devices ADXL05 accelerometer was selected for use as it is very inexpensive at $30 each and has a high sensitivity, contains internal amplification and has the necessary bandwidth for active noise and vibration applications. The accelerometers were attached to a patch board (see Figure 9) which provided the 5V dc supply voltage for the ADXL05 internal op amps and allowed calibration of the individual accelerometers. The buffered output from the patch board was fed into the modal summing board where the accelerometer signals were weighted according to the desired mode shape using trim pots and the fed into then 32 channel summer.

Only a single modal summing board was used which was found to be adequate for the current study. However, in most applications it would be necessary to monitor several radiation modes which would require additional summing boards. With the present layout these can be simply daisy chained to the single patch board.

The continuous sensors were made from shaped strips of 28 micron thick Cu-Ni electrode Polyvinylidene Fluoride (PVDF) film. The PVDF is sensitive to strain (rather than displacement as are the accelerometers) and consequently needed a profile equal to the second spatial derivative of the desired modes (Lee and Moon, 1990). These were cut to shape using a sharp blade and then attached to the curved panel using double sided tape. Copper tape backed with a conductive adhesive was used to provide an adequate connection from the film to a charge amplifier. It was necessary to buffer to output from the film via a high impedance charge amp to ensure that the cut-off frequency of the high-pass filter circuit formed by the PVDF was well below the frequency range of interest.

The potential energy of the cavity was estimated using 5 microphones randomly located throughout the interior. Numerical simulations and experience showed that this was an adequate number of microphones to estimate the potential energy of a small cavity using a single structural control source. The single microphone used during the tests was optimally located in one of the rear corners of the backing box.

5 EXPERIMENTAL RESULTS

The transfer functions between the driving force and the microphones, the zeroth radiation mode and the first radiation mode sensors are shown in Figures 10, 11 and 12 respectively. It has been shown that for simply supported structural systems that the even-order radiation modes are formed from the odd-order structural modes, and vice-versa. Therefore the zeroth radiation mode sensor, designed to sense the bulk compression acoustic mode, should only respond to the odd-order structural modes shown in Table 2. Likewise, the first radiation mode sensor, shaped to sense the primary longitudinal acoustic mode, should only respond to the even-order structural modes.

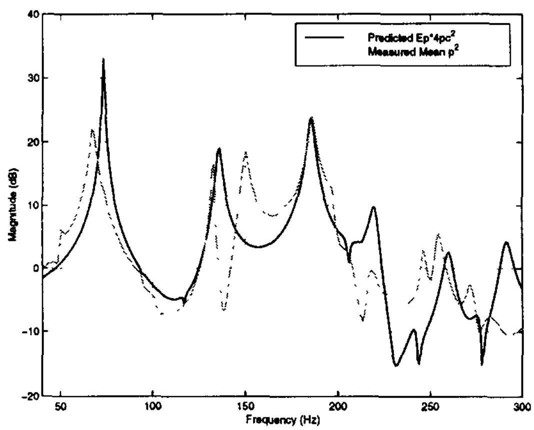

Figure 10 : Measured and predicted transfer function between the 5 microphones and driving force

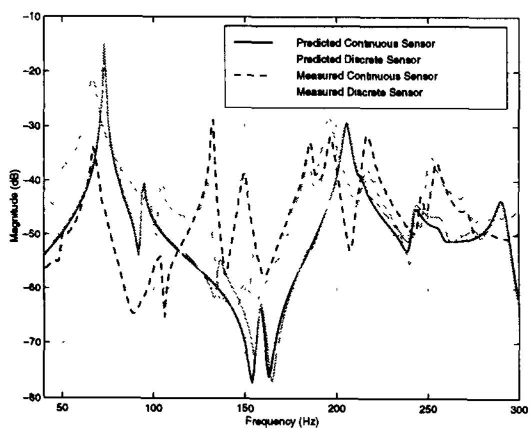

Figure 11 : Measured and predicted transfer function between the zeroth radiation mode sensor and driving force

As can be seen in Figures 11 and 12 both the PVDF and discrete radiation mode sensors respond strongly to the desired structural modes; however, it can also be seen that there is some "leak-through" of undesired structural modes due to imperfections in the film pattern and errors associated with discrete spatial sampling. It is worth noting that the (straight) zeroth radiation mode sensors are more sensitive to the even-order structural modes than the (half sine) first radiation mode sensors are to the odd-order structural modes. This phenomenon was also experienced by Lee and Moon (1990) when applying shaped sensors to measure normal structural modes on a beam, although no explanation was given as to the cause.

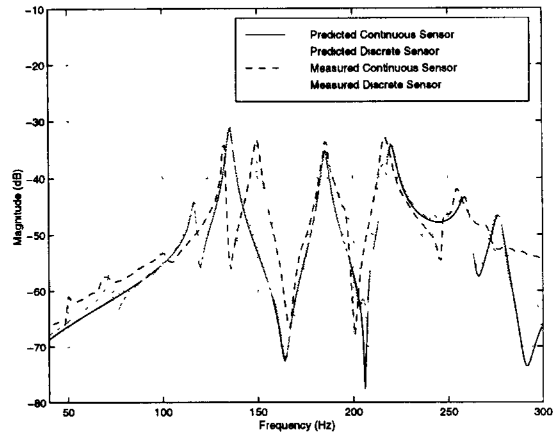

Figure 12 : Measured and predicted transfer function between the first radiation mode sensors and driving force

Table 2 : Natural frequencies and modal participation factors of the structural modes

Structural Mode	Resonance Frequency (Hz)	Structural Mode Participation Factor	
		Radiation Mode 0	Radiation Mode 1
[1,1]	77	-0.859	0
[2,1]	96	0	0
[2,2]	129	0	0
[1,2]	144	0	0.818
[2,3]	180	0	0
[3,1]	199	-0.284	0
[2,4]	204	0	0
[1,3]	204	0.272	0
[3,2]	220	0	0.273
[3,3]	234	-0.092	0

As to be expected, the discrimination quality of the real sensors is not as effective as the numerical simulation predicted. However, even with only seven accelerometers and a single strip, the frequency response curve shows that radiation modal sensing will still work in systems with a low modal density.

Active Structural-Acoustic Control

The mean square pressure level of the five microphones with and without active sound transmission control is shown as a function of frequency in Figures 13, 14 and 15. Control using the 5 microphones as error sensors provides the maximum attenuation threshold that can possibly be achieved. Figure 13 shows that using a single microphone provides good control at most frequencies. This is only because the acoustic modal density is very low in the frequency range of interest, particularly below 250 Hz.

Because only a single radiation mode was used for the error signal at any one time, the frequency weighting filter shown in Figure 1 was not required. In practice, for single channel control a band pass filter for each radiation mode signal is adequate. For multiple channel control it is often possible to neglect the filter altogether (Cazzolato and Hansen, 1997).

Control using the continuous zeroth radiation mode sensor within the design bandwidth (0-120 Hz) is somewhat disappointing. This is believed to be due to the "leak-through" of undesired modes noted earlier. Contrary to the numerical simulation, the controlled pressure level when using the discrete zeroth radiation mode sensor as an error sensor was lower than that obtained when using the equivalent continuous sensor. This is believed to be due to inaccuracies in the shape of the PVDF film. As expected, at frequencies above 120 Hz control leads to an increase in the mean square sound pressure level within the cavity.

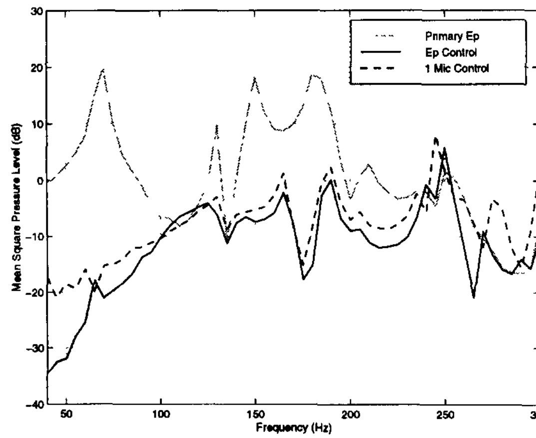

Figure 13 : Mean square pressure level using all five mics. and a single mic. as error sensors.

Control using the continuous first radiation mode sensor within its operational bandwidth (120-270 Hz) is high, with almost optimum control achieved between 120 Hz and 200 Hz. However, control above 200 Hz is poor. This is because the second longitudinal mode begins to contribute to the acoustic potential energy. This was not experienced in the numerical simulation because the shakers were placed close to the nodes of the structural modes that excite the second acoustic mode. As before, control outside the design frequency band leads to an increase in the mean square sound pressure within the cavity. The discrete first radiation mode sensor behaves as expected and provides less control than the equivalent continuous sensor.

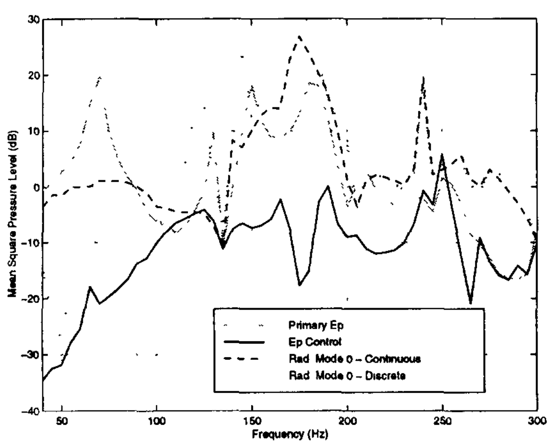

Figure 14 : Mean square pressure level using the zeroth radiation mode sensors.

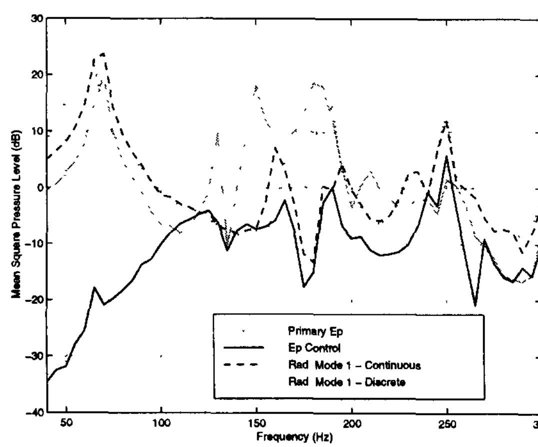

Figure 15 : Mean square pressure level using the first radiation mode sensors.

Although the level of control achieved when using the modal sensors rather than using the discrete microphones as error sensors was less, the expected benefits of modal sensing, namely rapid controller convergence and high stability were definitely observed.

6 CONCLUSIONS

It has been shown both numerically and experimentally that it is possible to use structural sensors to actively minimise the transmission of sound from a structure into a coupled enclosure. The preliminary results indicate that the although technique is feasible, the sensors appear to be very sensitive to "leak through" of undesired modes. The continuous sensors are more accurate modal sensors than discrete sensors due to spatial sampling errors for all but the zeroth (bulk compression) radiation mode. The modal sensors were shown to reduce the convergence time of the controller and increase the controller stability when compared to that using discrete microphones. Although not presented here, it is possible to constrain the cost function in such a way as to only control specific regions in the cavity. This constrained global approach has the advantage that controller effort is not expended in controlling non-essential regions. This "virtual microphone" is the subject of a future paper.

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