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Invited Paper

FINITE ELEMENT ANALYSIS OF AN INDUSTRIAL REACTIVE SILENCER

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Abstract

Classical analytical models used for prediction of the performance of reactive silencers are limited to conditions where the dimensions of the duct and resonators are small compared to the wavelength of the sound. Finite Element Analysis does not suffer from such limitations and has therefore been used to analyse the design of a reactive silencer for the exhaust stack of a 980MW power station. To assist in the design process, resonators of various dimensions were analysed using FEA which has led to the derivation of expressions for the resonance frequencies of slot-type rhomboid shaped resonators as a function of the geometry. An important design issue is the influence that adjacent resonators have on the overall performance of the system. It was found that when resonators of similar resonance frequency are in close proximity, they can interact and lead to a decrease in the overall performance compared to that of a single resonator.

1 INTRODUCTION

The work presented here is a Finite Element based numerical and experimental investigation of a reactive silencer designed to reduce the noise emitted from the exhaust stack of a 980 MW coal fired power station. Conventional analytical techniques, such as transmission line theory, that are typically used to evaluate the performance of silencers were not suited to this application for two reasons; first, the wavelength of the sound was similar in size to the resonator dimensions and second, non-plane wave conditions existed in the duct.

Rhomboid shaped resonators (see Figure 1) were used in the design because they have several distinct advantages over rectangular shaped resonators: they have less tendency to become filled with debris (Vér, Biker & Patel 1978); they permit longer quarter wave tubes, hence lower frequencies may be achieved than with conventional right angled tubes; and finally resonators with orifices that face away from the flow (ie inhibit inflow) tend to have lower self noise (Rockwell & Naudascher 1978, Panton 1990).

A literature survey of classical analytical models for resonators revealed that most research deals with either circular or rectangular shaped cavities and principally with resonator dimensions which are small compared to the wavelength of sound. In addition most papers treat the throat/cavity junction as a flange (ie the throat does not intrude into the cavity) which meets at right angles to the cavity wall, unlike the current design where the throat enters into the cavity at an angle of 45°.

The limitation of current analytical models led to a FEA investigation of the resonance frequencies of rhomboidal shaped resonators which accounted for the irregular geometry and higher order effects associated with large dimensions. This involved calculating the Transmission Loss (TL) of resonators of various dimensions and performing a regression analysis on the collected data.

From this data set, individual resonators or cells were selected (based on their resonance frequencies) and added to the silencer section to meet the desired silencer performance.

The Finite Element Analysis (FEA) of multiple cells revealed that interactions occur between cells of similar resonance frequency which are in close proximity to each other. This interaction may decrease the TL compared to that achieved when the cells are placed well away from each other ($> \lambda/4$, where λ is the wavelength of sound).

2 THE PROBLEM AT HAND

Two large Induced Draft (ID) fans are used to disperse the exhaust gases from the power station's boiler through an exhaust stack. The fans generate a combination of broadband noise and strong tones at the fan's blade passage frequency (BPF) and associated harmonics.

While dissipative silencers have been used successfully in clean air systems, attempts to use dissipative silencers on the flue gas from coal fired power stations have repeatedly failed because of accumulation of fly ash and other particulates in the absorptive material and perforated facing plate. It has been shown that this can result in a rapid reduction in transmission loss of up to 50% in the space of 2 years (Fryer 1989, Bjork 1994).

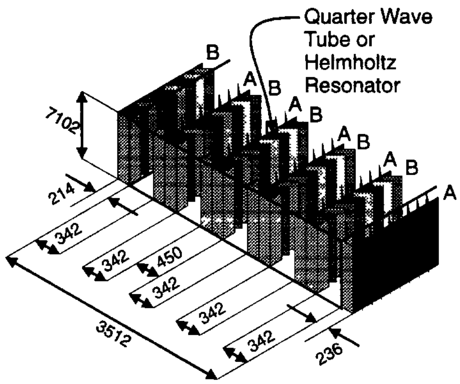

Figure 1: Schematic of the silencer.

Reactive silencers provide a feasible alternative to the more conventional dissipative systems and do not suffer from a reduction in performance from build up of particulates. In the case of broadband silencers, the performance may even increase with contamination as the particulates act to increase the acoustic damping. In addition, reactive silencers offer better low frequency performance than dissipative silencers of similar dimensions.

The fans have 12 blades and rotate at either 735 RPM or 984 RPM, resulting in fundamental BPF's of 150 Hz and 200 Hz respectively. The dimensions of the exhaust duct were 7m high $\times$ 3.5m wide, the gas temperature was 140°C and the volumetric flow rate was 300 kg/m^3. From these parameters it was determined that the speed of sound and fluid density were approximately c = 407 m/s and ρ = 0.867 kg/m^3 respectively, resulting in a Mach number of $M = 0.03$ in the main duct and $M = 0.06$ in the airway between the resonators.

The silencer consisted of 5 parallel rows of resonator cells , referred to as baffles. Figure 1 illustrates the arrangement.

3 THE FINITE ELEMENT METHOD

Two dimensional finite element models using the software package Ansys (©Ansys Inc.) were used to predict the TL of resonators of various dimensions. The models used FLUID29 acoustic elements (Ansys 1995*b*) with only pressure degrees of freedom. A unit dynamic pressure was applied to the duct, upstream from the silencer. Anechoic end conditions were applied to the ends of the ducts by setting the acoustic impedance equal to the characteristic impedance of the fluid ρc. An inherent constraint of 2D modeling is that only plane-wave conditions exist in the vertical direction which may lead to errors above the cut-off frequency of the first cross mode.

The TL across the silencer was calculated via three methods: first, difference in maximum squared pressure levels upstream and down stream of the silencer; second, using the two-microphone technique (Chung & Blaser 1980) ; and third, by integrating the acoustic intensity of the right traveling pressure wave downstream of the silencer, to determine the acoustic power loss across the silencer (Munjal 1987). Although, for a single resonator in a duct the first method significantly overestimates the TL as would be expected from the pressure doubling on reflection (increase of 6 dB), this is not the case when the silencer geometry is complicated. In the interests of simplicity, it was found that the first technique adequately calculated the TL across the silencer, particularly when one considers the errors involved in numerical prediction.

4 MULTI-DIMENSIONAL EFFECTS

4.1 Prediction of resonance frequency : Effect of resonator and duct geometry on the resonance frequency

For many silencer systems the assumption that plane wave conditions hold is valid because the critical dimensions of the system are significantly less than $\lambda/2$. However, when the frequency is increased so that cross-modes occur, then the performance of the resonators will be degraded and can no longer be evaluated using plane wave theory (Ihde 1975). In large industrial silencers, these conditions generally occur above frequencies of the order of 500Hz.

Because of the rhomboidal shape of the quarter wave tubes and the throat of the Helmholtz resonators, it was difficult to define the equivalent length and width of the throat. For long-narrow throats, the length and width were approximately equal to the line parallel and normal to the throat walls respectively. However, for short-wide throats, the relationship is less clear. For this reason when deriving an empirical expression for the resonance frequencies of the resonators it was necessary to have several higher order terms involving both the throat width and length to account for the uncertainty in the geometry of the throat.

Panton & Miller (1975) derived an expression for the resonance frequency of a Helmholtz resonator, which takes account of the impedance of the cavity, for a resonator with a long narrow cavity,

$$f_r = \frac{c}{2\pi}\sqrt{\frac{S}{V\,l_{eff} + \frac{1}{3}L^2 S}} \tag{1}$$

which is applicable to resonators which satisfy $\kappa L < \pi/2$; $L < \lambda/4$, and where κ is the wavenumber, l_{eff} is the effective throat length, V is the volume of the cavity, S is the throat

	Helmholtz Resonator		Quarter Wave Tube	
	Minimum	Maximum	Minimum	Maximum
g	0.050	0.800	0.050	0.800
l	0.005	0.070	0.100	0.500
d	0.050	0.150	0.050	0.100
L	0.080	0.236	N/A	N/A
D	0.100	0.350	N/A	N/A

Table 1: Dimensional limits of the resonators in the data set (with all dimensions in metres).

area and L is the depth as shown in Figure 2. The preceding expression formed the basis for a multiple regression analysis on the FEA results for the 2D Helmholtz resonators. The analysis of the 130 Helmholtz resonators with the geometric dimensions shown in Table 1 formed the data set. It was found that the effective length for a Helmholtz resonator with a narrow slot was given by the following empirical relationship (with a correlation coefficient of 0.9998)

$$\begin{aligned} l_{\mathit{eff}} &= 0.0835 + 0.1358D + 0.4334L - 0.0502\ln(L) + 0.0140\ln(g) + 0.0493\ln(d) \\ &\quad + l\,[3.8871 - 9.6155d + 0.7403\ln(D) + 0.3997\ln(L) - 9.6110LD] \end{aligned} \tag{2}$$

where all dimensions are in metres and $c = 407\text{m/s}$. Figure 2 illustrates the dimensions

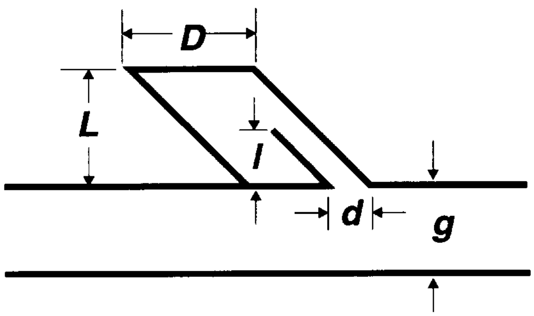

Figure 2: Dimensions of a Helmholtz resonator.

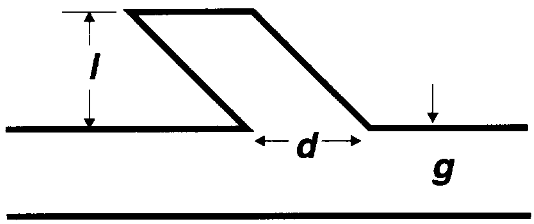

Figure 3: Dimensions of a quarter wave tube resonator.

of the Helmholtz resonator and duct. The expression is only strictly correct for the case considered of a single resonator mounted in the wall of the duct. Thus little effort was made to non-dimensionalise the expression. As a side note Chanaud (1994) showed that by placing the orifice at the end of the resonator face (see figure 2) rather than in the centre of the face, the resonance frequency can be lowered. The placement of the orifices at the end of the cell in the current design assisted in achieving the necessary low frequency performance and helped to negate the increase in the resonance frequency that occurs when a slot orifice is used instead of a square orifice.

The expression for the fundamental resonance frequency f_r of a quarter wave tube is given by

$$f_r = \frac{c}{4l_{\mathit{eff}}} \tag{3}$$

The above expression formed the basis for a multiple regression analysis on the FEA of the quarter wave tubes. Analysis was conducted on 45 quarter wave tubes, bounded by the geometric dimensions listed in Table 1. The dimensions of the quarter wave tube are shown in Figure 3. The effective length for a quarter wave tube which satisfies equation (3) empirically is given by

$$l_{eff} = -0.0569 + 1.5845l - 0.1432l^2 + 0.2028d + \frac{0.0205g}{l} - \frac{0.0040l}{g} \tag{4}$$

where $c = 407$m/s. It should be noted that equations (2) and (4) for the effective lengths are by no means definitive. The calculation of the resonance frequencies (and effective lengths) used a rigid walled duct with no other elements attached to the duct wall. When additional cells are added to the system, subsequent modification of the duct geometry (hence impedance) as well as modal coupling will alter the resonance frequency of the cells. Hence, equations (2) and (4) will provide only an estimate of the effective lengths of installed resonators. It was found that adjacent resonators may alter the resonance frequency of a single resonator in a duct by up to 15%, depending on the proximity of the adjacent resonator and the difference in resonance frequencies of the two resonators.

4.2 Influence of adjacent resonators on Transmission Loss

When designing a series of resonators to be inserted into a silencer, it is essential that resonators with similar resonance frequencies are not placed too closely together, otherwise the benefit of additional resonators will not be realised.

This effect can be seen with classical transmission line theory. When two identical resonators are placed adjacent to each other, the effective throat area is doubled, which approximately doubles the TL near resonance, ie an increase in 6dB is observed. However, when the resonator centres are placed at a spacing of $\lambda/4$ the TL in dB is raised by a factor of 2.

In addition, the coupling of modes when resonators are closely spaced produces a shift in the resonance frequencies, producing two distinct frequencies; one lower and one higher than the original frequency. This "de-tuning" of the resonators may have a detrimental effect on the overall performance of the silencer by creating "holes" in the TL spectrum and is particularly apparent when the resonators have similar resonance frequencies.

Figure 4 shows the sound pressure level of two rhomboidal quarter wave tubes separated by 80mm. Dimensions of the tubes are $d = 100$mm, $l = 214$mm , $g = 430$mm. As can be seen from the figure both cells are resonating and are almost 180° out of phase (determined through phase plots of pressure). When the two cells are excited 180° out of phase and have a spacing of less than a quarter of a wavelength, the TL is significantly reduced because the cells tend to cancel one another's effectiveness by a dipole effect. Figure 5 shows the TL of the system in figure 4 as a function of frequency. The figure clearly shows that the TL afforded by two adjacent resonators is significantly less than that provided by even a single cell.

The spacing shown here is the worst case situation and spacings of greater than $\lambda/4$ provide a much better attenuation. However, the analysis does highlight the need for careful consideration of where cells are placed in relation to the others.

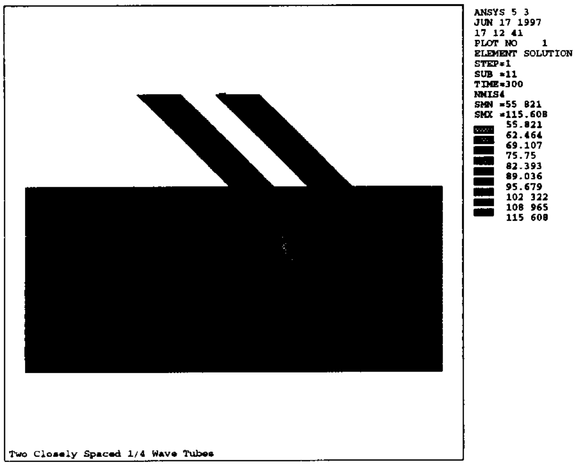

Figure 4: Sound Pressure Level contour plot of two closely spaced rhomboid quarter wave tubes.

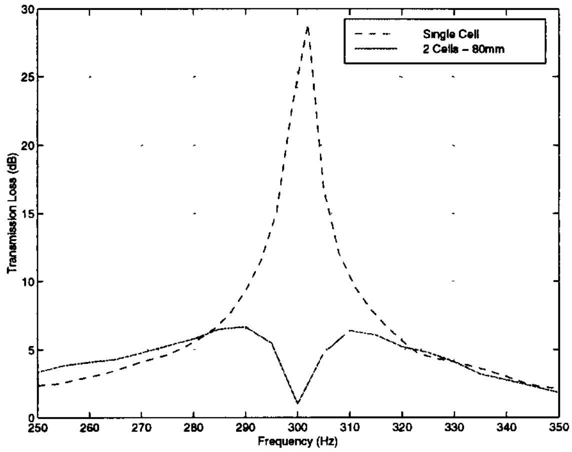

Figure 5: Transmission loss for two closely spaced rhomboid quarter wave tubes.

5 DESIGN CHARTS

The silencer for the power station was designed to attenuate broad band noise through the use of Helmholtz resonators and tonal noise through the use of quarter wave tubes.

To "tune" a quarter wave tube, it is essential that the gas temperature and hence the speed of sound, remains constant. Small variations in the gas temperature will change the wavelength of the sound, which will mean that the quarter wave tubes will be significantly less effective at the desired frequency, which can degrade the overall performance. In general, if gas temperature variations can be expected, then it is wiser to use Helmholtz resonators to meet the acoustic specifications than quarter wave tubes as the former have broader TL characteristics.

The design process involved creating series of design charts derived from equations (2) and (4), with resonance frequency as a function of the dimensions of the resonators. From these curves individual resonators or cells were selected (based on their resonance frequencies) and added to the silencer section to meet the desired silencer performance.

Because the silencer was constructed from individual resonators, deficiencies in the TL were identified. Additional cells were added to increase the TL in regions of poor performance. This procedure was repeated until the silencer met the required performance.

6 FINAL DESIGN

Once a general design was found, ten variations of this design were investigated to optimise the overall performance, each design taking approximately 12 hours to evaluate on a Silicon Graphics R8000. Figure 6 shows a plot of the resonators used in the finite element model. Approximately 16,000 nodes and elements were used.

The Ansys examples manual (Ansys 1995*a*) recommends a minimum of 15 elements per wavelength to obtain accurate results. The model described here had approximately 60 elements per metre which indicates that the results might have significant errors at frequencies above 1500 Hz.

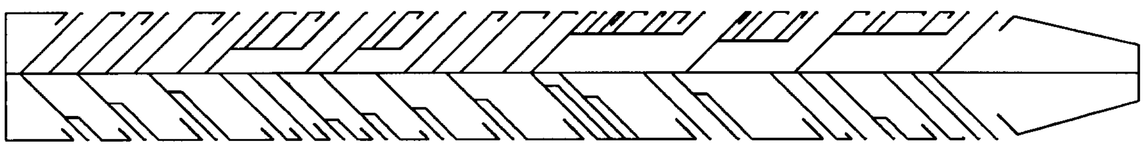

Figure 6: The final design of the reactive silencer.

6.1 Fabrication & Structural Compliance

In the previous sections it was assumed that the walls of the baffles were rigid. In reality, walls of finite thickness such as those fabricated from sheet metal are compliant. The impedance of the wall is generally not an issue for most reactive silencers such as automobile mufflers which have a highly curved walls with a large thickness to length ratio. However, if the walls of the baffles are thin compared with the length of the panels then the structure may not be sufficiently rigid to prevent coupling between the structure and the cavity, especially when the resonance frequencies of the acoustic modes are close to the resonance frequencies of the low order panel modes. This structural-acoustic coupling may severely limit the performance that can be obtained in reactive devices. Three mechanisms act to reduce the TL (although it appears from modelling that the former mechanism dominates): firstly, a compliant structure will limit the impedance change that can be developed across a reactive element, thereby limiting the TL across the element; secondly, the modal coupling that occurs between the structure and cavity tends to "de-tune" the resonators; and finally, once the energy enters the structure it may be re-radiated downstream from the reactive element, effectively short circuiting the resonator.

Figure 7 shows the effect that varying the wall thickness has on the TL of the silencer. The results are presented from the 50Hz to 400 Hz third octave band. The structural FEM had insufficient nodes to adequately model above 400Hz due to node and element limits. It should be noted that the TL calculated using a fully coupled model is highly dependent on the structural boundary conditions and the results shown in Figure 8 are simply to demonstrate the influence wall compliance has on the TL of the system.

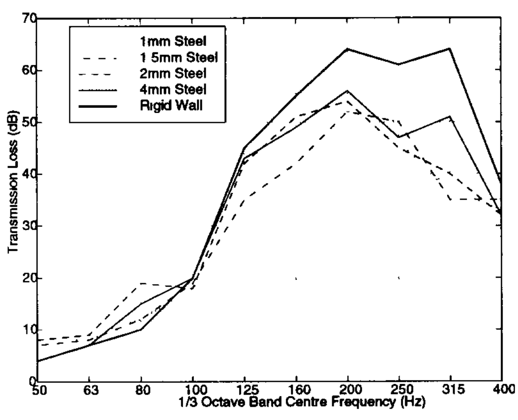

Figure 7: The effect of wall thickness on silencer performance.

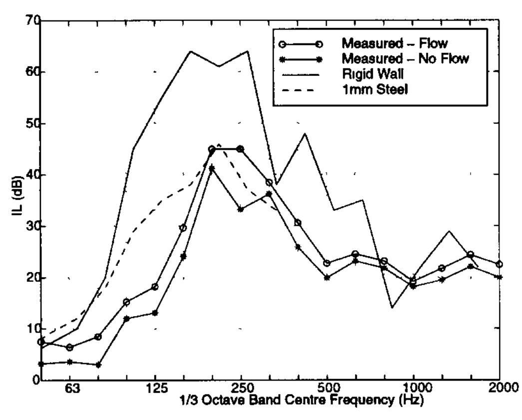

Figure 8: Predicted versus measured transmission loss for the final silencer design.

Figure 9: Test section made from sheet metal.

6.2 Measured Performance

A full-scale section of the silencer baffle was tested at the NATA registered acoustics chambers at the CSIRO division of Building, Construction and Engineering, Melbourne. The test section measured 900mm high, 1190mm wide and 4000mm in length and was formed by placing adjacent sides of the designed baffle (see Figure 9) along the test section of duct and measuring the insertion loss into a reverberation chamber. Figure 8 shows the predicted and measured insertion loss for the final design. During testing of the silencer with air flow some of the third octave band measurements suffered from a poor signal to noise ratio so the measured results shown have been adjusted to remove the effects of the background noise levels. The measured and predicted results show good correlation, although the the measured IL is slightly less than predicted for a 1mm fabrication, particularly in the 125Hz octave band. The lower measured values were not unexpected as the structural model had a low element density which artificially stiffened the structural model, thereby increasing the predicted IL results.

7 CONCLUSIONS

Finite element methods were used successfully to design a reactive silencer for the exhaust stack of a coal fired power station. Classical analytical models were not suitable for the design process. This led to the development of a technique utilising FEA to predict the transmission loss which occurs for rhomboid shaped resonators attached to a main airway. Separate regression analyses were performed for Helmholtz and quarter wave tube resonators to provide an expression for the effective length as a function of the system geometry.

When these resonators were placed in series, interactions between resonators of similar resonance frequency reduced the transmission loss compared to when the same resonators were separated by more than $\lambda/4$. A compliant fabrication may severely degrade the performance of an undamped reactive silencer.

Two-dimensional Finite Element Analysis provides a technique suitable for the design and evaluation of large industrial reactive silencers and should suitable computing resources be available then the design may be further optimised using automated linear optimisation routines.

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