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**PRACTICAL REALISATION ISSUES IN ADAPTIVE ACTIVE  
VIBRATION CONTROL**

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**ABSTRACT**

This paper presents an investigation into practical real-time realisation issues of active vibration control (AVC) systems. An adaptive active control system is considered. The control strategy is realised for a cantilever beam system in transverse vibration. The unwanted vibrations in the structure are assumed to be due to a single point disturbance of broadband nature. An AVC system is designed on the basis of optimum cancellation of broadband vibration at an observation point along the beam. Practical issues related to the design and implementation of the system are high-lighted and discussed. An adaptive algorithm consisting of a simulation, control and identification processes is developed. This is implemented on a number of computing domains involving high-performance reduced instruction set computer (RISC) processors and digital signal processing (DSP) devices. A comparison of the results of the implementations, on the basis of real-time computation performance, is made to establish merits of development of fast processing methods in real-time active control applications.

**Keywords:** Active vibration control, adaptive control, digital signal processing, real-time control.

**1. INTRODUCTION**

Active vibration control (AVC) consists of artificially generating cancelling source(s) to destructively interfere with the unwanted source and thus result in a reduction in the level of vibration (disturbance) at desired location(s). This is realised by detecting and processing the vibration by a suitable electronic controller so that, when superimposed on the disturbances, cancellation occurs.

The interference of the cancelling wave with the disturbance in the propagation medium principally results in a pattern of cancellation and reinforcement; the disturbance is reduced in some regions and reinforced in other regions. The physical extent of cancellation in the medium is affected by the geometrical arrangement of system components. This is, further, closely related to the observability, controllability and stability requirements of the system. Thus, for a robust design and performance of the system, it is required to consider the geometrical arrangement of the system at the design stage. Such a consideration in the context of design of active noise control (ANC) systems has previously been reported (Tokhi, 1995; Tokhi and Leitch, 1991a, 1992a). However, little is known in the area of AVC.

Due to the broadband nature of the disturbance the control mechanism in an AVC system is required to realise suitable continuous-frequency characteristics so as to achieve cancellation over a broad range of frequencies of the disturbance. This, in turn, requires a frequency-domain characterisation of system components and development of design relations accordingly. Such a strategy can easily be achieved by adopting a systems approach (Tokhi and Leitch, 1992b).

In practice, the spectral contents of the disturbance vary due to operating and/or environmental conditions. Moreover, the characteristics of system components are subject to variation. This implies that the control mechanism in an AVC system is further required to have an adaptive capability to track these variations so as to achieve and maintain the required level of performance (Elliott *et al.*, 1987; Eriksson *et al.*, 1987; Fuller *et al.*, 1892; Snyder and Hansen, 1992; Tokhi and Hossain, 1994; Tokhi and Leitch, 1991b).

At the implementation level, the control mechanism can either be realised in analogue or digital or hybrid (combined analogue and digital) form. Due to the numerous advantages offered by digital techniques, it is commonly preferred to implement controllers in digital form. However, in active control applications care must be taken to ensure that the delay due to sampling, and the effects of non-integer plant delays are taken into account.

A further consideration in implementing controllers in digital form is that of real-time computing requirements. The performance demands of modern control systems require the employment of complex and computationally intensive algorithms. These, in turn, place hard constraints on computing capabilities of the digital processor utilised in implementing the control process. Such requirements are found difficult to meet with conventional computing methods as the computational requirements of the control process do not match the computing capabilities of the digital processor. Alternative high-performance computing techniques utilising digital signal processing (DSP) and parallel processing (PP) techniques could provide suitable solutions to such hard real-time constraints (Tokhi *et al.*, 1992).

This paper presents an investigation into practical realisation issues of active control systems with specific focus on AVC systems. The paper is organised as follows

Section 2 presents the AVC system. A cantilever beam system in transverse vibration is considered as a test and verification platform in this work. The AVC system is designed on the basis of optimum cancellation of vibration along the beam. Section 3 describes practical issues related to the design and digital implementation of the system. The algorithms involved in the AVC system are implemented on a number of uni-processor and multi-processor digital platforms. The processors utilised include the Texas Instruments TMS320C40 (C40) parallel DSP devices, Intel 80i860 (i860) RISC processor and the Inmos T805 (T8) transputers. Results of these implementations are presented and discussed. The paper is finally concluded in Section 4.

## 2. ACTIVE VIBRATION CONTROL SYSTEM

### 2.1 The flexible beam structure

Consider a cantilever beam system with a force  $U(x,t)$  applied at a distance  $x$  from its fixed (clamped) end at time  $t$ . This will result in a deflection  $y(x,t)$  of the beam from its stationary position at the point where the force has been applied. In this manner, the governing dynamic equation of the beam is given by (Virk and Kourmoulis, 1988)

$$\mu^2 \frac{\partial^4 y(x,t)}{\partial x^4} + \frac{\partial^2 y(x,t)}{\partial t^2} = \frac{1}{m} U(x,t) \quad (1)$$

where,  $\mu$  is a beam constant and  $m$  is the mass of the beam. Discretising the beam in time and length using central finite difference (FD) methods, a discrete approximation to equation (1) can be obtained as (Virk and Kourmoulis, 1988)

$$Y_{k+1} = -Y_{k-1} - \lambda^2 S Y_k + \frac{(\Delta t)^2}{m} U(x,t) \quad (2)$$

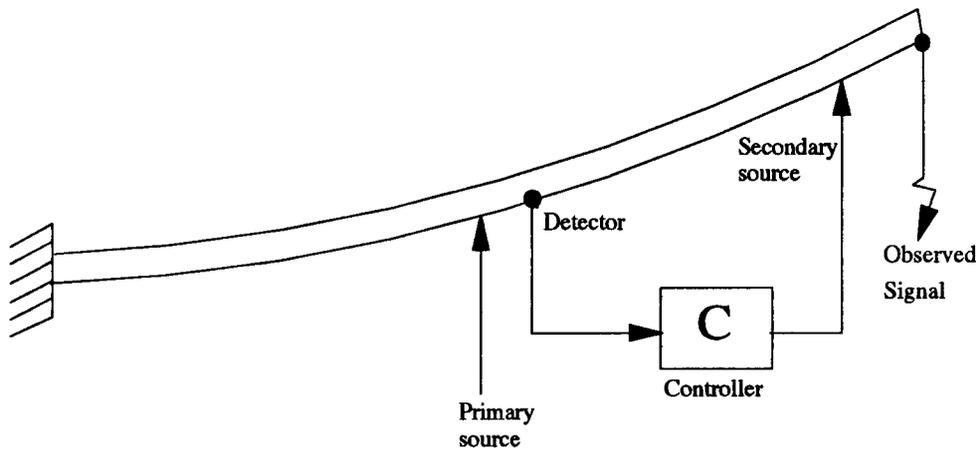
where,  $\lambda^2 = [(\Delta t)^2 / (\Delta x)^4] \mu^2$  with  $\Delta t$  and  $\Delta x$  representing the step sizes in time and along the beam respectively,  $S$  is a penta-diagonal matrix (the so called stiffness matrix of the beam),  $Y_i$  ( $i = k+1, k, k-1$ ) is an  $(n-1) \times 1$  matrix representing the deflection of end of sections 1 to  $n$  of the beam at time step  $i$  (beam divided into  $n-1$  sections). Equation (2) is the required relation for dynamic simulation of the beam which can be implemented on a digital processor easily.

### 2.2 Active control structure

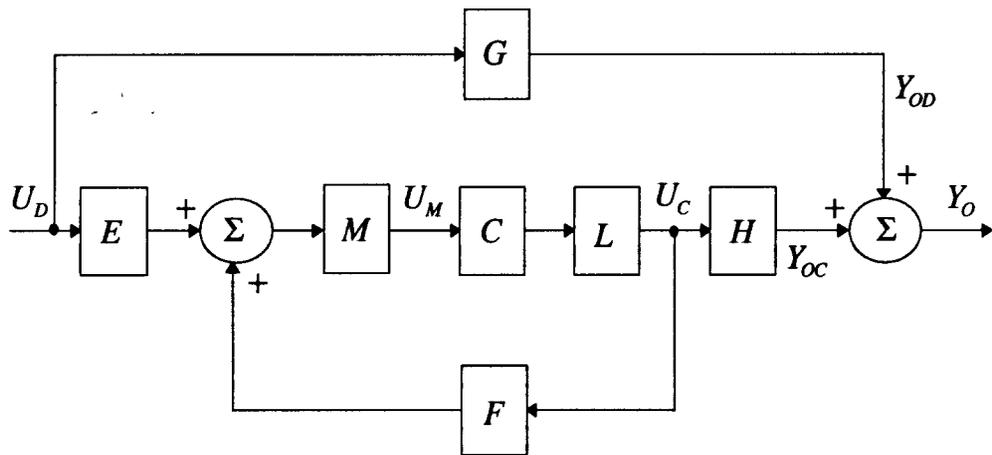
A schematic diagram of an AVC structure is shown in Figure 1(a). The unwanted (primary) disturbance is detected by a detection sensor, processed by a controller to generate a cancelling (secondary, control) signal so that to achieve cancellation at the observation point. Figure 1(b) shows the corresponding block diagram of the AVC structure, where  $U_D$  and  $Y_{OD}$  represent the disturbance signals at the source and observation points respectively,  $U_C$  and  $Y_{OC}$  are the secondary (control) signals at the source and observation points respectively,  $U_M$  and  $Y_O$  are the detected and the observed signals respectively.  $E$ ,  $F$ ,  $G$  and  $H$  represent the characteristics of the paths, along the beam, between the primary source and the detector, the secondary source and the detector, the primary source and the observer, and the secondary source and the observer respectively.  $C$ ,  $L$  and  $M$  are the characteristics of the controller, the secondary source and the detector respectively.

The objective in Figure 1 is to achieve total (optimum) vibration suppression at the observation point. Synthesising the controller on the basis of this objective yields (Tokhi and Leitch, 1991b)

$$C = \frac{G}{ML\Delta} ; \quad \Delta = GF - EH \quad (3)$$



(a) Schematic diagram.



(b) Block diagram.

Figure 1: Active vibration control structure.

Equation (3) represents the required controller design relation for optimum cancellation of the disturbance at the observation point.

### 2.3 Adaptive active control

The controller characteristics in equation (3) can be realised either as a fixed digital or analogue or hybrid (combined analogue and digital) controller. In practice, the characteristics of sources of disturbance vary with operating conditions, for instance, leading to time-varying spectra. Moreover, the characteristics of transducers, sensors and other electronic equipment used in the AVC system are subject to variation resulting from environmental effects, ageing, etc. Under such circumstances an AVC system is required to be capable of updating the controller characteristics according to these changes so that the required level of performance is achieved and maintained. This can be achieved by adopting a self-tuning control strategy which allows on-line design and implementation of the controller. Thus, manipulating the controller design relation in equation (3) yields (Tokhi and Leitch, 1991b)

$$C = [1 - Q_1/Q_0]^{-1} \quad (4)$$

where,  $Q_0$  and  $Q_1$  represent the equivalent transfer functions of the system (with input at the detector and output at the observer) when the secondary source is off and on respectively. Equation (4) is the required controller design rule which can easily be implemented on-line on a digital processor. This leads to a self-tuning AVC algorithm comprising the processes of identification and control. The process of identification involves obtaining  $Q_0$  and  $Q_1$  using a suitable system identification algorithm. A recursive least squares (RLS) parameter estimation algorithm is used here to estimate  $Q_0$  and  $Q_1$  in the discrete-time domain in parametric form. The process of control, on the other hand, involves designing the controller according to equation (4) and implementing this in real-time.

### 3. PRACTICAL REALISATION ISSUES

#### 3.1 Design related

In designing an AVC system, a careful consideration of the acoustic feedback loop, due to secondary source radiation reaching the detector, that can cause the system to become unstable is required. Moreover, for given source, sensor and necessary electronics, a study of the dependence of the controller characteristics on the transfer characteristics of the propagation paths along the beam and, hence, system geometry, giving an insight into the complexity and practical realisation aspects of the controller, is important.

The interference of the primary and secondary signals in an active control system, effectively, leads to a pattern of zones of cancellation and reinforcement in the propagation medium; the disturbance is cancelled in some regions and reinforced in others. The physical extent of cancellation primarily depends on the maximum frequency of the disturbance and geometrical arrangement of system components. Moreover, to satisfy the observability and controllability requirements of the system, the detector should be placed such that to provide good measurement of the disturbance. Similarly, the secondary source and the observer should be placed so as to allow the controller be practically realisable.

Equations (3) and (4) give the required controller characteristics for optimum suppression of broadband vibration at the observation point. The controller thus designed can be guaranteed to be causal by making the number of zeros either be equal to or less than the number of poles. This can be achieved by selecting suitable system model structures at the estimation/measurement process.

Note in equation (3) that, for given secondary source and detection sensor, the characteristics of the required controller are determined by the geometric arrangement of system components. Among these, it is possible with some arrangements that the function  $|\Delta|$  will be zero or close to zero, requiring the controller to have impractically large gains. Thus, if such a situation is not avoidable, as is the case with feedback active control structures, then a compromise between practical realisation of the controller and system performance has to be reached by the designer.

As noted in Figure 1, the secondary signal reaching the detection point form a feedback loop that can cause the system to become unstable. Therefore, an analysis of the system from a stability point of view is important at the design stage. For practical systems a measure of absolute stability is not useful; a system that has an extremely long and oscillatory transient

response is unlikely to be accepted. In this respect, a measure of relative stability can provide a more acceptable design criterion. This can be achieved, through the utilisation of the Nyquist's stability criterion, in terms of gain and phase margins (Tokhi, 1995; Tokhi and Leitch, 1991a). Using the block diagram in Figure 1 and the controller design relation in equation (3) yields the characteristic equation of the closed-loop system between  $U_D$  and  $U_C$  as

$$\text{Ch. Eq} = 1 + (EH/FG - 1)^{-1} \quad (5)$$

This is the required relation for analysing the system from a stability viewpoint. Note that this is expressed in terms of the transfer characteristics of the propagation paths along the beam in Figure 1. Therefore, the stability of the system is affected by the geometrical arrangement of system components.

### 3.2 Implementation related

In implementing the self-tuning active control algorithm, described earlier, several issues of practical importance need to be given careful consideration. These include properties of the disturbance signal, robustness of the estimation and control, controller stability and processor related issues such as wordlength, speed and computational power.

In practice, the disturbance may also excite those dynamics of the system which are not of interest. In such situations, care must be taken to condition the input signal properly before sampling. Robustness of the control algorithm is related to the accuracy of the estimated plant model. This, in turn, depends on the properties of the input signal, proper initialisation of the parameter estimation algorithm, model order and accuracy of computation (Tokhi and Leitch, 1991b). The computational accuracy is related to the processor's dynamic range of computation, determined by the processor wordlength and type of arithmetic. With a processor supporting fixed-point arithmetic, for example, it is important to take necessary precautions against problems due to overflow and inaccuracies due to truncation/rounding of variables (Tokhi and Leitch, 1991b).

In employing the controller design criterion for optimal cancellation of the disturbance, a problem commonly encountered is that of instability of system, especially when non-minimum-phase models are involved. Note in the controller design relation in equation (4) that such a situation will result in a non-minimum-phase and unstable controller, with the unstable poles approximately cancelling the corresponding zeros that are outside the stability region. Thus, to avoid this problem, either the estimated models can be made minimum-phase by reflecting their non-invertible zeros into the stability region and using the resulting models to design the controller or, once the controller is designed, the poles and zeros that are outside the stability region can be reflected into the stability region. In this manner, a factor  $(1 - pz^{-1})$  corresponding to a pole/zero at  $z = p$  in the complex  $z$ -plane, that is outside the stability region, can be reflected into the stability region by replacing the factor with  $(p - z^{-1})$ .

It is important at the identification level that simultaneous sampling of the input and output signals takes place. The input/output samples obtained are subject to delay through A/D conversion process. Provided such delays at both input and output ends are equal, a correct estimate of the plant model can be obtained. However, any difference between the

two delays will reflect itself as a phase lag in the estimated model with respect to the plant. Such a delay will result in if the A/D converters at the input and output ends are not identical, and in practice, even with identical A/Ds such a delay is possible.

In active control applications, where the accuracy requirements of the phase characteristics of the digital controller are crucial, the phase delay introduced due the sample and hold mechanism in an implementation process can have a significant effect on the performance of the system. This requires the realisation of a delay compensating function in cascade with the controller. The introduction of a delay compensation, however, will increase high-frequency controller gain and can cause instability in the system. Although, the instability problem at high-frequencies can be avoided by band-limiting the controller output, the problem may not be crucial in an active control system where the intention is to reduce low-frequency disturbance.

### **3.3 Real-time implementation**

To investigate the nature and real-time processing requirements of the self-tuning AVC algorithm, it was divided into the processes of control and identification. Thus, the control algorithm which consists of the implementation of the controller, includes the simulation algorithm as an integral component. The identification algorithm consists of parameter estimation of the models  $Q_0$  and  $Q_1$  and calculation of the required controller parameters according to equation (4). The beam simulation algorithm is also considered as a distinct algorithm. Both the beam simulation and control algorithms are predominately matrix based.

The algorithms were implemented on a number of digital processing domains, including uni-processor domains comprising single C40, i860 and T8 processors, two homogeneous parallel architectures comprising two C40s (C40+C40) and two T8s (T8+T8) and two heterogeneous parallel architectures comprising an integrated C40 and T8 (C40+T8) and an integrated i860 and T8 (i860+T8) system. The execution times achieved in implementing the simulation, control and identification algorithms over 20000, 20000, and 1000 iterations respectively are shown in Table 1. This clearly shows that, in practice, a mismatch between the computing requirements of the algorithm and computing capabilities of the computing domain results in inefficiency in the real-time implementation of the algorithm. While the powerful vector processing resources of the i860 are exploited in case of the simulation and control algorithms, its performance in case of the identification algorithm is not as impressive as the single C40 and multiple C40 and T8 architectures.

## **4. CONCLUSION**

The practical issues of design and implementation of adaptive AVC systems have been investigated and discussed. It has been shown that the geometrical arrangement of system components plays an important role on the performance of the system as far as the physical extent of cancellation, observability, controllability and stability requirements of the system and practical realisation of the controller are concerned. At the implementation level, properties of the disturbance signal, processor related hardware and software issues, delays due to A/D conversion process at the model identification level and controller implementation level as well as effects of non-minimum-phase models have been high-lighted and discussed. It has been demonstrated that, for an efficient and cost-effective implementation of an algorithm to be achieved in real-time a close match between the

computing requirements of the algorithm and computing capabilities of the processor is required. This can be achieved by adopting a strategy incorporating DSP and PP methods.

Algorithms	Architectures						
	i860	C40	T8	C40+C40	T8+T8	i860+T8	C40+T8
Simulation	0.38	2.30	3.86	1.62	3.00	0.99	1.77
Control	0.41	2.68	3.69	1.92	3.00	1.04	1.96
Identification	0.35	0.17	0.67	0.13	0.30	0.31	0.31

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