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# The Relevance of the Biot Theory for prediction of Sound Transmission through Partitions incorporating Porous Layers.

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A car floor is normally comprised of a decorated soft trim on a heavy layer with a net weight of 4 - 8 kg/m$^2$ and 2 - 4 mm thickness, which is backed by a porous or fibrous material. This is either simply fitted to the steel floor pan or glued to it. Such a system essentially constitutes a double leaf partition with a high damping filler. In order to facilitate design of carpeting systems to provide optimal sound insulation properties, a good model of sound transmission through such a structure is required. Several models of sound propagation through porous materials are available. These models have been compared and in particular the Biot model has been studied. This model predicts that three distinct waves will propagate through a porous material. Under what circumstances must each of these waves be included in the model and when can we neglect one or more of them? Do porous plastics behave in a similar way to fibrous materials? As is often the case, evaluation of a theoretical model is hampered by lack of experimental values of parameters. We have therefore sought to relate measurable quantities to the theoretical parameters.

## 1. Introduction

In multiple layer partitions, the fibrous material is often less than 1 cm in thickness. Moreover in floor pans it is often compressed by the trim and heavy layer. How does the porous layer function under these circumstances and how can its sound and vibration transmission characteristics be modelled?

The porous layer will have several functions. It will act as a sound attenuating layer for sound transmission from one side of the structure to the other. It will act as a compressible spacer, which will produce a resonance at the 'double wall' resonant frequency. Further, depending on how it is attached to the outer layers, it may also act as a vibration damper. If it is glued to one or more of the outer layers it will add extra impedance to the layer(s), generally reducing the amplitude of vibration except possibly at resonant frequencies. It will normally add some resistive damping to the structure at least if it is integrally bonded to an outer, elastic layer.

In order to be able to predict the effectiveness of a given thickness of a given porous material, we need to model in some detail sound transmission through the multiple layer structure. Let us assume air-borne sound excitation of the steel plate. The steel constitutes an elastic plate and can be modelled using well established theories. The porous material needs to be modelled both in its vibration damping capacity and in its sound attenuation role.

Unfortunately a rigorous model is found to include a large number of parameters, some of which are difficult to measure. It may well prove easier and more effective to use sound transmission through the porous layer in order to determine the parameter values! Nevertheless, a theoretical model will be useful in improving our understanding of the sound and vibration transmission mechanisms and allow us to make informed judgements about choosing or even designing a porous material for a given application.

## 2. Porous materials

The material in the porous layers is two phase, ie. air plus a solid skeleton. The porosity is high and typically greater than 95%. If the material is compressed by say 10% the porosity will be reduced by approximately the same amount.

The porous materials used belong either to the class of porous, fibrous materials like rockwool or to the class of partially reticulated foams. In car floor pans, plastic foams seem to be more commonly used. There are continuous air channels through the foam, but also a fraction of the air-volume is occupied by closed pores. Are these materials isotropic or do the open channels have a preferred direction?

Fibrous materials generally consist of separable layers of randomly orientated fibres, which are matted together to a high degree. Thus it may be supposed that in the plane of the layers, there is good contact between the fibres and that stresses and therefore wave motion can be propagated through the layers. However the degree of cohesion between the parallel layers is much lower than between the fibres in each layer, so it may not be appropriate to assume that waves can be propagated through the solid material in the direction perpendicular to the layers.

## 3. Models

The theoretical model developed by Biot predicts that three different wave types can propagate through a two-phase medium consisting of a solid phase, which can support shear stresses, and a fluid phase, which in itself cannot. The existence of these three wave types has been demonstrated by Plona 1982 and Johnson 1985 among others.

Interestingly, three waves are also found to propagate through regular crystal structures, see Lau and McCurdy, 1997. This theory has its foundation in the Christoffel equations for wave propagation in an elastic continuum, which yields equations in terms of the elastic constants. The theory developed by Biot is built on the classical theory of elasticity. The number of elastic constants needed to describe wave propagation depends on the degree of symmetry in the material.

Consider the stresses and strains on a 'particle' of the material, which is cubic in its equilibrium state. Hooke's law states that there is a linear relationship between stress and strain of a particle, ie. that each stress component has a linear relationship with each strain component, which it causes :

$$\{\sigma_{ij}\} = \begin{pmatrix} c_{11} & c_{12} & c_{13} & c_{14} & c_{15} & c_{16} \\ c_{21} & c_{22} & c_{23} & c_{24} & c_{25} & c_{26} \\ c_{31} & c_{32} & c_{33} & c_{34} & c_{35} & c_{36} \\ c_{41} & c_{42} & c_{43} & c_{44} & c_{45} & c_{46} \\ c_{51} & c_{52} & c_{53} & c_{54} & c_{55} & c_{56} \\ c_{61} & c_{62} & c_{63} & c_{64} & c_{65} & c_{66} \end{pmatrix} \{e_{ij}\}, \qquad (1)$$

where $\{\sigma_{ij}\} = (\sigma_{11} \quad \sigma_{22} \quad \sigma_{33} \quad \sigma_{23} \quad \sigma_{31} \quad \sigma_{12})$, the order of subscripts for each component being switchable, and with six corresponding components in the strain vector.

This equation applies to any elastic material, but is also applicable to viscoelastic materials or those showing hysteresis as long as the strains are sufficiently small. Although the material may be heavily loaded at equilibrium, the extra strains produced by wave motion will usually be small enough for this assumption to be valid.

The matrix in equation (1) contains 36 elastic coefficients. However sine the matrix can be shown to be symmetric, the number of independent coefficients is reduced to 21. Mirror or rotational symmetry of the material causes some of the coefficients to became zero and for simple relationships to exist between some non-zero coefficients, further reducing the number of independent coefficients.

One type of symmetry, which may be appropriate for describing porous materials is so-called transverse isotropy. A material with this symmetry, in addition to having three orthogonal planes of mirror symmetry, also possesses one axis of symmetry such that the material properties along lines at right angles to this axis are equivalent. For fibrous materials this axis would be perpendicular to the plane of the fibre matrices. For plastic foams the axis would be perpendicular to the direction of mass loading and may be parallel to the direction through which gas percolates the plastic during production, producing a degree of non-isotropy. Equation (1) is then reduced to :

$$\{\sigma_{ij}\} = \begin{pmatrix} c_{11} & c_{11}-2c_{66} & c_{13} & 0 & 0 & 0 \\ c_{11}-2c_{66} & c_{11} & c_{13} & 0 & 0 & 0 \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 \\ 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} \end{pmatrix} \{e_{ij}\}. \qquad (2)$$

Biot initially considers the stresses and strains in the solid frame and the interstitial fluid separately. Since the fluid cannot support shear (except in boundary layers where they are taken into account by coupling factors), there can only be axial components of stress $\sigma_{ij}$ in the fluid. Equation (2) taking in to account, stresses and strains in the fluid, becomes :

$$\left\{\sigma_{ij}, s\right\} = \begin{pmatrix} c_{11} & c_{11}-2c_{66} & c_{13} & 0 & 0 & 0 & c_{17} \\ c_{11}-2c_{66} & c_{11} & c_{13} & 0 & 0 & 0 & c_{17} \\ c_{13} & c_{13} & c_{33} & 0 & 0 & 0 & c_{37} \\ 0 & 0 & 0 & c_{44} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & c_{44} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & c_{66} & 0 \\ c_{17} & c_{17} & c_{37} & 0 & 0 & 0 & c_{77} \end{pmatrix} \left\{e_{ij}, \varepsilon\right\} \tag{3}$$

where s and $\varepsilon$ are the stresses and strains respectively in the fluid
and zero coupling has been assumed between stresses in the solid and shear strains in the fluid, which cannot sustain shear. This leaves us with 8 non-zero, independent coefficients.

These equations have been written with somewhat different notation by Biot and Willis 1957 as follows :

$$\left.\begin{aligned} \sigma_{xx} &= Pe_{xx} + Ae_{yy} + Fe_{zz} + M\varepsilon \\ \sigma_{yy} &= Ae_{xx} + Pe_{yy} + Fe_{zz} + M\varepsilon \\ \sigma_{zz} &= Fe_{xx} + Fe_{yy} + Ce_{zz} + Q\varepsilon \\ \sigma_{yz} &= Le_{yz} \\ \sigma_{zx} &= Le_{zx} \\ \sigma_{xy} &= Ne_{xy} \\ s &= Me_{xx} + Me_{yy} + Qe_{zz} + R\varepsilon \end{aligned}\right\} \tag{4}$$

in which $P = A + 2N$.

In this set of equations, 1/N and 1/L clearly represent the shear moduli of the frame in the direction parallel to the axis of rotation and perpendicular to it, respectively. Now if we write equations (i), (ii), (iii) and (vi) of the set (16) in matrix form, the inverse of the matrix

$$\begin{Bmatrix} P & A & F & M \\ A & P & F & M \\ F & F & C & Q \\ M & M & Q & R \end{Bmatrix} \text{ can be written as } \begin{Bmatrix} a & d & g & h \\ d & a & g & h \\ g & g & b & m \\ h & h & m & n \end{Bmatrix}.$$

The inverse stress-strain relations become :

$$\left.\begin{aligned} e_{xx} &= a\sigma_{xx} + d\sigma_{yy} + g\sigma_{zz} + hs \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ &\cdots\cdots\cdots\cdots\cdots\cdots \\ \varepsilon &= h\sigma_{xx} + h\sigma_{yy} + m\sigma_{zz} + ns \end{aligned}\right\}\text{etc.} \tag{5}$$

Hence two Young's moduli can be identified as :

$$\frac{1}{E_1} = \frac{\partial e_{xx}}{\partial \sigma_{xx}} = \frac{\partial e_{yy}}{\partial \sigma_{yy}} = a, \text{ and } \frac{1}{E_2} = \frac{\partial e_{zz}}{\partial \sigma_{zz}} = b \tag{6}$$

and similarly three Poisson's rations can be defined, eg. as :

$$\nu_1 = -\frac{\left(\frac{\partial e_{yy}}{\partial \sigma_{xx}}\right)}{\left(\frac{\partial e_{xx}}{\partial \sigma_{xx}}\right)} = \frac{d}{a} = E_1 d \text{ etc.} \tag{7}$$

Thus measurements of 2 shear moduli, 2 Young's moduli and 3 Poisson's ratios enable the coefficients *a, b, d, g, p* and *q* to be determined with a redundancy of one modulus. As always, there may be inconsistency in the static values measured and the dynamic values of the elastic moduli most relevant to wave propagation.

By applying the Euler equation to the stresses given by the tensors above the following wave equations are derived :

$$\left.\begin{aligned} &\nabla^2((P-A)e+Q\varepsilon)=\frac{\partial^2}{\partial t^2}(\rho_{11}e+\rho_{12}\varepsilon) \\ &\nabla^2(Qe+R\varepsilon)=\frac{\partial^2}{\partial t^2}(\rho_{12}e+\rho_{22}\varepsilon) \\ &N\nabla^2\omega=\frac{\partial^2}{\partial t^2}\left(\rho_{11}\vec{w}+\rho_{12}\vec{\Omega}\right) \\ &0=\frac{\partial^2}{\partial t^2}\left(\rho_{12}\vec{w}+\rho_{22}\vec{\Omega}\right) \end{aligned}\right\} \tag{8}$$

where $w$ and $\Omega$ are the curl of the particle displacement vectors in the solid and fluid respectively and the effective coupling density $\rho_{12}$, corresponds to the "additional apparent mass loading" $\rho_a$ of each phase due to the presence of the other. Thus :

$$\left.\begin{aligned} \rho_{11} &= \rho_s + \rho_a \\ \rho_{22} &= \rho_f + \rho_a \\ \rho_{12} &= -\rho_a \end{aligned}\right\} \tag{9}$$

where $\rho_{11}$ is the net effective density of the solid phase and $\rho_s$ is the actual density of the solid and likewise for the fluid.

The first two of these equations represent coupled compressional wave equations in $e$ and $\varepsilon$ respectively, each one having a different wavenumber or wave speed. The wave described by the first of these equations is primarily propagated in the solid frame, the second is primarily propagated in the fluid. The last two equations describe a shear wave, propagated through the solid frame with fluid loading, with a single valued wavespeed.

Thus far we have four wave equations and six elastic moduli, but as yet there has been no attempt to include damping mechanisms in to the equations! Note the elastic moduli describing the elasticity of the frame need to be measured in vacuo.

In the first 1956 paper Biot introduces damping in to the wave equations by introducing a damping term, $b\left(\vec{u}-\vec{U}\right)$ proportional to the fluid velocity relative to the frame velocity but independent of frequency. The factor $b$ is identified as $b = \eta\phi^2/k$, where $\eta$ is the fluid viscosity, $\phi$ is the porosity and $k$ is here the 'permeability' of the porous material. In the second part of the 1956 paper, the higher frequency range is considered for which Poiseuille flow can no longer be assumed. The factor $b$ for Poiseuille flow is replaced by a factor $bF\left(a\sqrt{\frac{\omega}{\nu}}\right)$, where a is a characteristic radius of the pores, $\omega$ is the radial frequency and $\nu$ is the dynamic viscosity of the fluid and F is a complex function, which was determined by Biot for cylindrical pores. Dispersion curves are shown in the paper for the different wave types as are also attenuation coefficients as functions of frequency for certain combinations of parameter values.

Acoustic energy losses in porous media are primarily due to viscosity and thermal conduction. Such losses can be described mathematically, see eg. Makarov and Ochmann 1997. However in order to incorporate damping analytically into the stress/strain relationships, it is probably necessary to assume that the air channels through the material are cylindrical. This would require extra data relating to thermal conductivities of the two phases and viscosity of the fluid but also about the effective mean diameter of the pores and their effective length per metre length of path in a given direction. This data could only realistically be obtained from direct measurements.

An alternative approach is to simply to model sound attenuation by assigning an imaginary part to the wavenumber of each wave. Again the imaginary component of the wavenumbers can only be found experimentally.

Bolton et al. 1996, demonstrate clearly the importance of the boundary conditions on wave propagation and net transmission loss measured for a double wall structure. The lowest transmission loss across the whole frequency range is shown when the porous layer is bonded to the massive layers on either side. This indicates that at least for the structure studied loading of the vibrating panels is a less important function of the porous layer than attenuation of sound propagated from one side to the other.

Interestingly Bolton et al. 1996 found that for the structure studied the transmission loss was rather insensitive to the values of the parameters of the foam except at higher frequencies (above about 1000Hz) for a 27 mm thick layer of polyurethane foam. Possibly approximate values only of the parameters may be sufficient for prediction purposes. They also found that the relative magnitudes of the different wave types depended on the boundary conditions.

Measurement procedures for sound transmission through porous materials have been investigated by Bolton et al 1996 and Bolton and Green 1993. Plona 1980 has demonstrated the reality of the three different wave types using a pulse technique. Perhaps applying the pulse technique to a range of thicknesses of a porous material, the relative amplitude of the waves and the attenuation rate could be measured.

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