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MODE STUDIES IN MUSICAL INSTRUMENTS

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ABSTRACT

All pitched musical instruments rely on resonances in some form for frequency selection and amplification. In most percussion instruments, string instruments and string keyboard instruments a membrane, the instrument body or a sound board is largely responsible for radiation from the instrument. The normal modes of these structures thus have a significant influence on the sound heard by an audience. Among the techniques used to study such normal modes are optical and electronic holographic interferometry, and computer animated modal analysis. These techniques will be discussed and illustrated with examples on Handbells, Guitars, and Caribbean Steel Pans.

INTRODUCTION

Musical Acoustics is primarily concerned with the production, transmission and perception of musical sounds. This involves music generated by acoustic and electronic instruments, as well as synthesized musical sounds. In addition, computer control of such musical elements as pitch, dynamics and rhythm add a dimension to a performance which is just beginning to be explored. While these concerns are primary, they are by no means exhaustive or exclusive. Interest in recording and reproduction technology is certainly important to musical acousticians, as is architectural acoustics, particularly in the context of concert hall acoustics. Furthermore, the diversity of measurement technologies utilized to study musical instruments leads to vital interests in apparently only marginally related field such as structural acoustics which lends modal analysis as one of the important tools to study the normal modes of a number of instrument bodies,The implication by these introductory comments that current work in musical acoustics is principally experimental is somewhat misleading since much of the work in the field is directed at theoretical modeling of instruments. The initial work of both experimentalists and theoreticians is directed at gaining a better insight into the nature of each instrument and the interactions between their individual elements. In some instances such insight has enabled instrument makers to modify and improve an instrument, or at least understand what principles lead to instrumental consistency.

The excellent text by Fletcher and Rossing[1] on the Physics of Musical Instruments explores at some depth the theoretical basis needed for an understanding of sound production and radiation of a number of members of the major musical instrument families along with the experimental references which provide details necessary to appreciate individual instrumental subtleties. The sophistication and the comprehensive scope of that work recommend it highly to any serious worker in acoustics with an interest in musical instruments. With that comment, and the direction for a more inclusive study, this paper will concern itself with the tools employed to study those vibrational characteristics of some musical instruments which are primarily relevant in coupling that sound to the outside world and radiating the sound to the audience.

All musical instruments rely on a power supply to excite and drive an oscillator. Enhancement of certain vibrational frequencies coupled from that oscillator to some other structural element of the instrument is related to the imposition of boundary conditions. These resonances are then either radiated directly or coupled to some other instrumental element, where additional boundary conditions further modify the spectral components of the radiated sound. A very direct example is the violin, where the bow functions as the power supply, the string serves as the oscillator, the terminations at the nut and the bridge provide the boundary conditions responsible for the selection of the harmonic resonances, with the intonation also influenced by the elastic properties of the string, the tension, and the linear mass density. The bridge in turn couples the string vibration to the violin body, where the shape of the top plate, the rib connections to the back plate, and many other instrumental details with their inherent boundary conditions further modify the normal mode structure of the vibrational pattern of the entire instrument, and thus determine the nature of the audible spectrum perceived by the audience.

While all these different elements of the many musical instruments play an important role in the ultimate tone quality of the sounds produced, this paper will be concerned primarily with those instrumental elements which couple the sound directly to the radiation field. A number of experimental techniques will be discussed in their application to several musical instruments. The primary experimental approaches to studying normal mode vibrations in musical instruments discussed here include holographic interferometry, electronic holography and impact excited modal analysis with computer animation. These techniques will be illustrated with examples of mode studies[2] on hand bells[3,4,5], guitars[6,7], and steel pans[8,9].

EXPERIMENTAL TECHNIQUES

All three methods of studying normal mode vibrations under consideration here require the choice of an excitation point or an observation point on the structure. The location of the drive point is critical, since the choice of a point on a nodal line for a particular resonance, excludes that mode from observation because a drive point at a non-moving location could not transfer energy to that mode, alternatively an observation point at such a location would not monitor mode motion. In a sense this says that if the normal mode pattern is known, it is easy to study the modes. Clearly this begs the question. In a real setting it is at least important to have a rough idea where the nodal lines are expected to be located so that the nodal lines of those modes can be avoided which have the most significant impact on the tone color of the instrument. The experimental procedures for almost all mode studies will therefor include a number of preliminary steps. Initially it is important to identify the frequencies of the strongly radiating modes, consequently the first measurements usually include a simple spectral mapping of the entire instrument. Preliminary mapping of mode shapes is then done relatively easily by driving the instrument at a resonance frequency, at some reasonably asymmetric point to avoid obvious nodal line locations, and monitoring the near sound field with a microphone very close to the vibrating plate. Nodal lines are identified by the phase reversal in a Lissajou figure on an oscilloscope when the observing microphone passes over a nodal line. Armed with this preliminary resonance frequency table and the crude map of the mode shapes of the most significant normal modes of the instrument, the careful, more detailed mode study of the instrument can begin.

Holographic Interferometry

Holographic imaging relies on phase shifts introduced by reflection from an object into one of two self-coherent optical beams. The subsequent recording of the interference pattern between these two beams preserves this phase shift information as the hologram. Photographic processing stabilizes that information, and interaction of the original un-shifted reference beam with the holographic recording restores all the optical information of the original object as the holographic reconstruction. The holographic plate serves, as it were, as a window on the world of the holographic reconstruction. Perspective changes of the observer result in perspective changes of the observed object analogous to position changes which cause observation of different parts of a real three dimensional object. In this process the object beam serves as a modulation on the reference beam, so the relative beam intensities need to be such that the object beam intensity must not exceed the intensity of the reference beam, otherwise information is lost. In practice intensity ratios from 1:10 to 1:3 are optimal. More critical than relative beam intensities is relative beam stability since relative beam phase shifts of the order of fractions of a wavelength introduced by vibrations during exposure negate holographic phase relation requirements. This suggests that vibration isolation is an essential requirement for holographic mode studies.

Mode information is recorded as an interference pattern between two holograms. The two holograms are obtained from the same object during vibration in a single normal mode vibrational pattern. During resonance vibration the instrument finds itself periodically in the position of extreme displacement. At that point the instrument is instantaneously stationary, furthermore, the motion in the neighborhood of that turn-around point is minimal. During the time-average recording of that instrument moving in a regular, periodic fashion, two holograms are recorded corresponding to the two positions of the instrument near the two turn-around points. The difference in displacement is recorded as an interference pattern, with path differences of odd multiples of half wavelengths appearing as dark and even multiples as bright fringes. Fringe counts can be related to vibration amplitudes. Figure 1 shows the optical beam arrangement for making holograms.

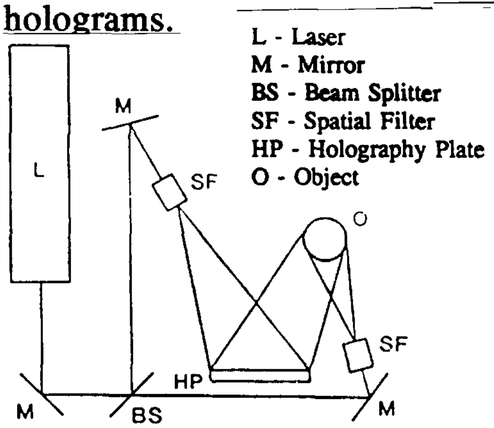


Fig. 1 Holography Instrumentation

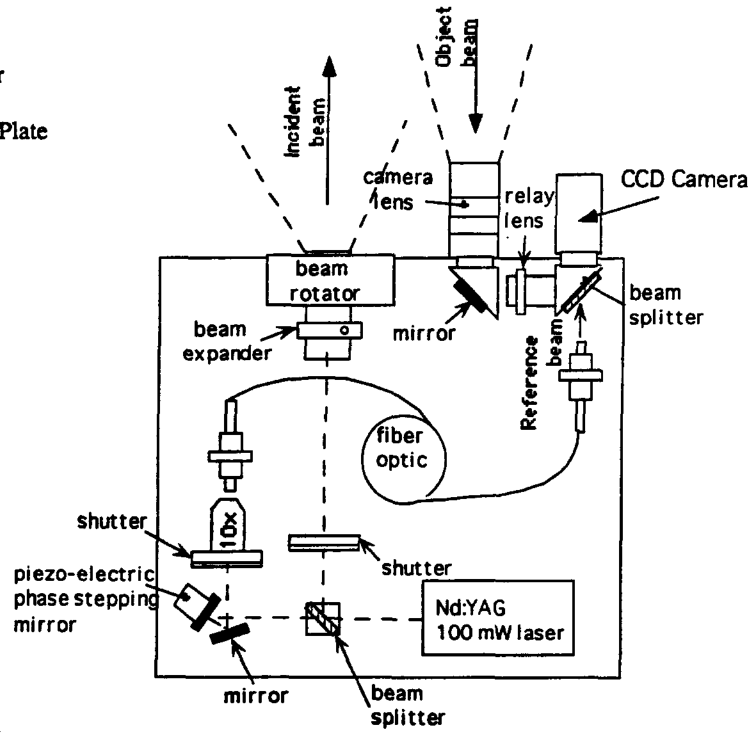


Fig. 2 Electronic Holography Instrumentation

Computer aided electronic holography makes it possible to observe the image forming process nearly in real time. Furthermore, the process introduces fibre optic techniques which enable optical phase shift introduction for interference fringe contrast optimization. Figure 2 illustrates the instrumental arrangement for electronic holography.

Modal Analysis

Modal analysis is an alternative coherent signal processing technique which enables normal mode representation in computer animation. In this technique a dual channel FFT analyzer prepares a transfer function from an excitation signal and a response signal on the same musical instrument. From the peaks on the transfer function resonance frequencies can be extracted . In obtaining a large number of such transfer functions, generally the response point is fixed and the excitation point is moved over the predetermined points on the instrument. The computer algorithm then fits these transfer functions to a model of coupled harmonic oscillators and subsequently animates the motion at the resonance frequencies. Excitation and response symmetry permits the use of a single drive point and shifting the response point over a predetermined grid of points. This technique is used in extending the method to the modeling of radiated sound fields while maintaining phase coherence with the source of the sound. Fifure 3 shows a block diagram of the experimental arrangement for modal analysis.

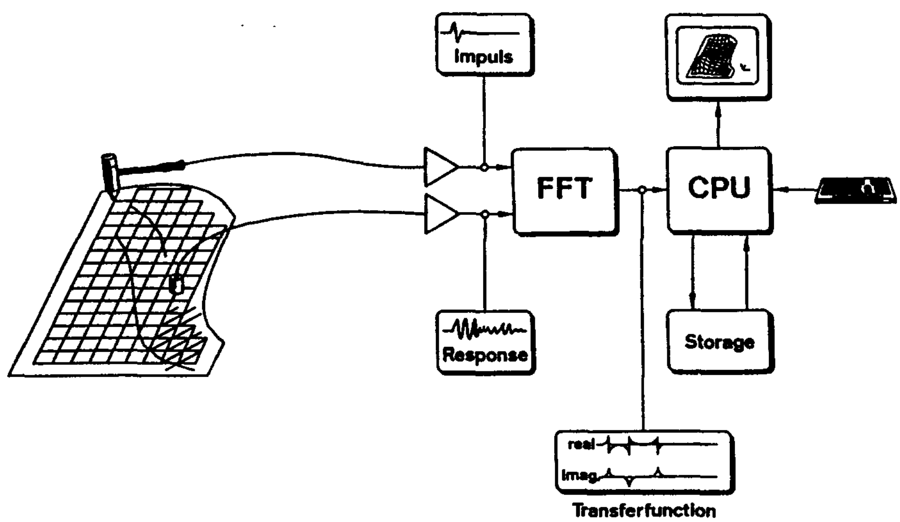


Fig. 3 Experimental arrangement for modal analysis of a piano sound board

Hand Bells

The high degree of material homogeneity combined with the cylindrical symmetry in a hand bell results in a number of very well defined normal modes which are related in such a way that a systematic pattern almost reminiscent of the periodic chart of the elements can be prepared.

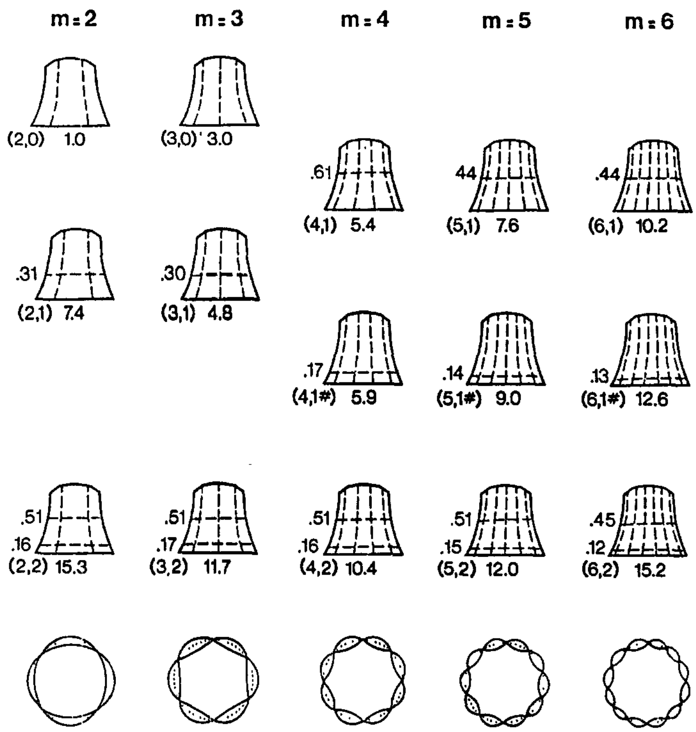
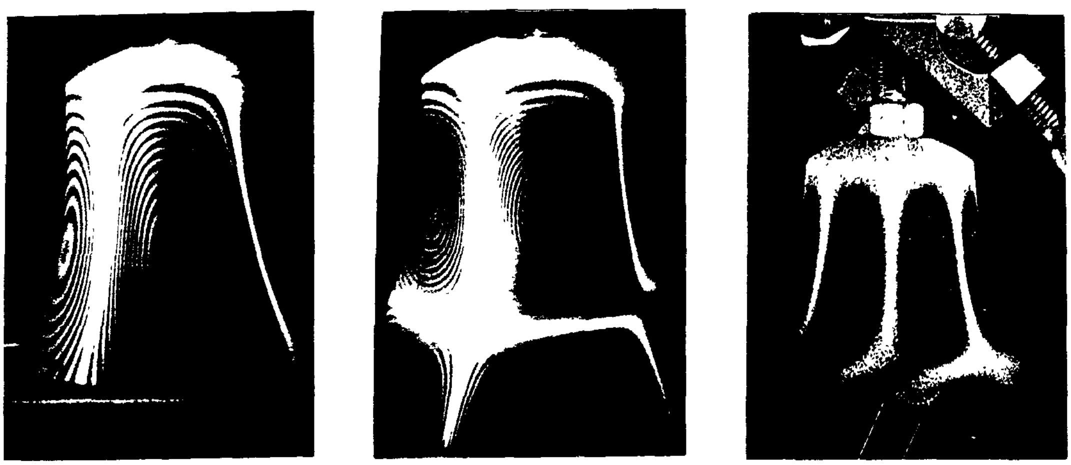


Fig. 4 Systematic pattern of Hand Bell modes

The modes are generally labeled by two indices with the number of nodal lines crossing the crown indicated by the first index and the number of circumferential nodal lines given by the second. The portion of the mode area enclosed by nodal lines on the side, the crown of the bell at the top and the mouth of the bell at the bottom can be viewed as a curved plate of varying thickness with the following boundary conditions: The crown effectively clamps the top of the section, the nodal lines on the sides essentially provide a hinged boundary, and the mouth end is basically free. That picture provides a suggested conceptual explanation for the mode pattern observed.



(3,0) mode (3,1) mode (5,1)# mode

Fig. 5 Holographic interferograms of Hand Bell Modes

The (3,0) mode shows a truncated "bulls eye". The (3,1) mode shows a circumferential nodal line about half way up the bell wall. No (4,0) mode is observed, instead a (4,1)* mode appears in its place, where the * identification is used for all modes where the circumferential nodal line occurs very close to the mouth of the bell. A conceptual explanation can be given in terms of a one dimensional analogue. A vibrating string has a node at both ends for fixed boundaries. If, as in the case of the bell mouth, the boundary condition at one end of the string is relaxed, two options present themselves. The string can be terminated by two springs perpendicular to the string, in which case the force is proportional to the displacement and thus the position will be in phase with the spring motion. This causes the resulting loop to be truncated and the actual nodal point lies beyond the loop. This corresponds to the situation observed with the (2,0) and the (3,0) modes. On the other hand, the string could be terminated by a mass, free to move in a direction perpendicular to the string. Now the force on the mass is proportional to the acceleration of the mass, which for harmonic motion is 180o out of phase with the displacement. For this setting the length of the string exceeds the resonant loop length by a small amount. This corresponds to the * modes where the mouth motion is "mass-like". This analogy is related to the width to height ratio of a single "bull's eye" section of a modal pattern. For the lowest modes such as the (2,0) and (3,0) modes the height is not excessively greater than the width of the central portion of the mode. In that situation the free mouth section is able to move in phase with the entire mode and it thus behaves "spring-like". The observed holographic interferogram for that situation is a truncated series of concentric ellipses. For the higher modes, (4,0) or (5,0), the stiffness of the plate with the hinged boundary condition on the side precludes a vibrational pattern with an excessively large height in comparison to the width and the lowest mode reduces the effective height of the section by shifting the nodal line from the exterior to the interior of the mode. This results in "mass-like" behavior of the free mouth for that mode with the associated circumferential nodal line close to the mouth for these modes.

Fig. 6 shows an example of a single frame taken from modal analysis animation of the (5,1)* hand bell mode.
For western bells with cylindrical symmetry the azimuthal orientation of the modes is determined by the impact point of the clapper. For Chinese bells with almond-shaped cross- section two modes are possible with lateral symmetry. The impact point determines which mode is excited,

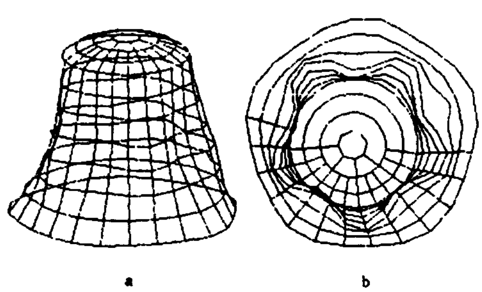
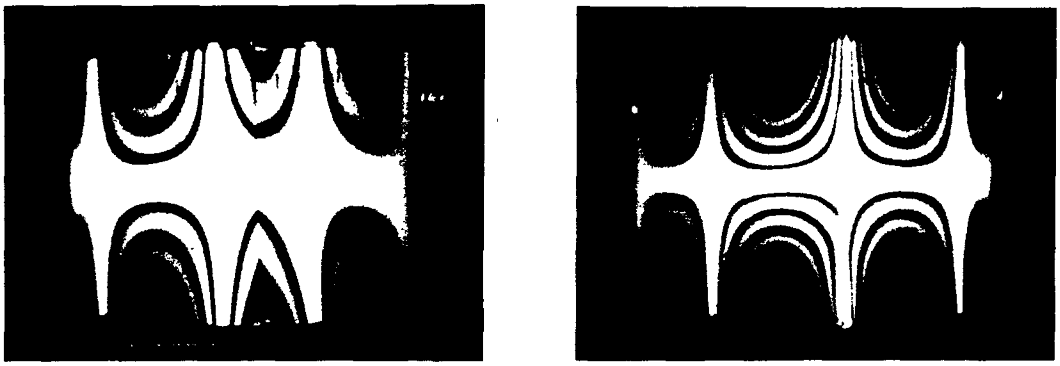


Fig. 6. The (5,1#) mode in a G_4 handbell at 3500 Hz. a. side view b. viewed from below

of the mode orientation is fixed. Holograms of these modes illustrate quite clearly why the frequencies of modes with nodal lines along the spine of the bell have a higher frequency than their corresponding counterpart. Bending wave reflections at the edge introduce a phase shift only in those modes which do not have a nodal line along the spine. The phase shift cuts a section from the standing wave, leaving the effective total wavelength slightly longer resulting in a somewhat reduced frequency. The discontinuity in the slope of the bending wave in the left picture of figure 7 shows the location of the spine in the almond shaped shell. The mode shown in the picture on the right side has a nodal line at the spine location.



(5,1)a 1671 Hz (5,1)b 1875 Hz

Fig. 7. Holographic interferograms showing vibrational modes of an almond shaped shell viewed from the spine.

Guitars

In comparison to hand bells, guitars are far less homogeneous and therefore guitar mode patterns are significantly less systematic. A further complexity is introduced by the fact that individual elements of the instrument have their own resonances, which in turn are modified by their interactions with each other. Figure 8 show examples of holographic interferograms of a front and back plate normal mode, and fig 9 shows a selected frame from a guitar modal analysis animation.

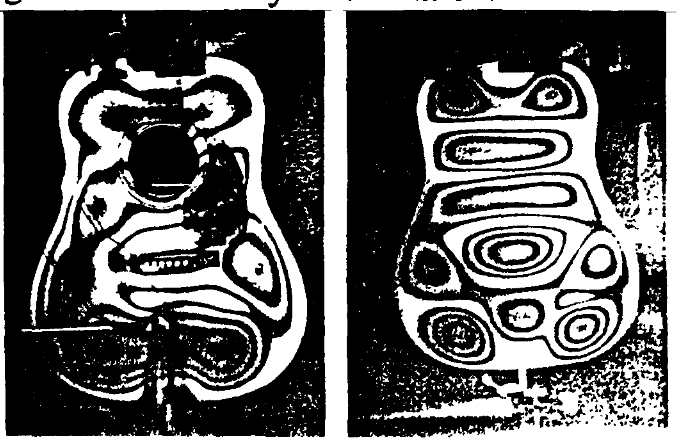


Fig. 8. Modes of a Martin D28 Guitar
Top plate 785 Hz Back plate 783 Hz

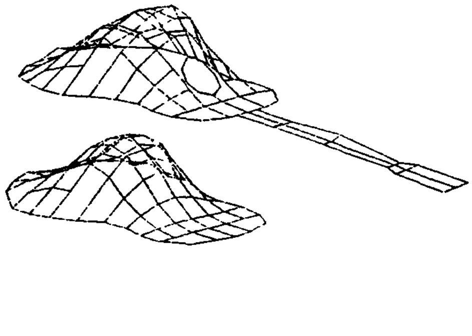
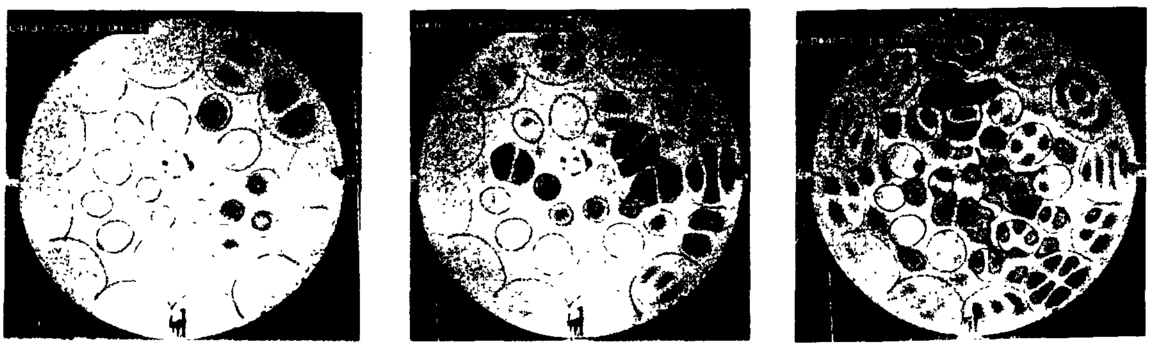


Fig. 9. Baritone guitar mode
135.9 Hz

Steel Pans

Steel pans will be discussed in detail in another paper, however, since their nonlinear behavior introduces an additional element of complexity it is appropriate to make some comments at this point. Imbedded in the bowl of the pan head are numerous tonal sections. Since they are all located in the same steel matrix a number of coupling mechanisms are responsible for exciting several sections when an impulse is given to only one. Studying those many interacting responses suggests that a real-time approach to imaging the many modes would be very desirable. The images in figure 10 were made with commercial electronic holography instrumentation. The examples shown give a clear indication of note coupling as well as non-linear behavior evidenced in amplitude dependent mode shapes. In the left hologram the three note areas in the upper left corner are tuned in harmonic relationships, so that the small note section vibrates at its fundamental frequency, the section above that has its second harmonic at that frequancy, ande section just below that and to the right has its fundamental an octave and a fifth below that so it vibrates with its third harmonic frequency. In the middle picture the three note sections to the right are also related, but this time the second, third and fourth harmonics coincide, so that the harmonic relationship is shifte up. The picture on the right shows evidence of nonlinear coupling as well, since not all of the excited note sections are harmonically related.



775.9 Hz 1357.5 Hz 2138.9 Hz

Fig. 10. Electronic Holographic Interferograms of Double Tenor pan

Conclusion

It is clear from the examples given that a number of tools are available to study the resonant behavior of musical instruments. When several independent techniques present a consistent mode picture one can discuss the nature of the instrument with some degree of confidence. While the experimental determination of a mode pattern enhances our understanding of that particular instrument, and it may even be of some value to the instrument maker, there are many more additional features which need to be understood and controlled before 20th century "Strads" can be produced. A recurring dream of the following "three-ring-circus" comes to mind:

"My friend Carleen Hutchins builds a violin, our mutual friend Oliver Rogers determines the elastic constants, the geometric and mass parameters of the various parts of the instrument and then uses finite element analysis to model that violin, Carleen gives me the instrument, and I do the modal analysis. Subsequently Ollie and I compare notes and we adjust the measured parameters within possible errors until the finite element animation matches the modal analysis picture. Then we go to Carleen and ask her: " Is that really where you wanted the resonance frequencies to be?" And she says: "No, it would be nice if the second mode were a little higher and the fourth mode a little lower, furthermore it would be better if the nodal line were a little closer to the middle." So Ollie adjusts the parameters until he has the modes to Carleen's liking, and he tells Carleen to shave off a little to the left of the soundpost. She does that and I run another modal analysis test to confirm that the predicted changes actually accomplish what was desired".

It is conceivable that some day such a cooperation between artisan, theoretician and experimentalist could result in a consistently improved instrument. Until then we will continue to attempt to gain insight into the nature of those instruments whose beautiful sounds have enriched all our lives.

REFERENCES

1. Neville H. Fletcher, and Thomas D. Rossing, *The Physics of Musical Instruments,* Springer-Verlag, New York (1991)

2. Uwe J. Hansen, and Thomas D. Rossing, *Zur Darstellung schwingender Strukturen durch Holographie oder Modalanalyse,* Qualitätsaspekte bei Musikinstrumenten, ed. Jürgen Meyer, Moeck Verlag Celle, 1988

3. Rossing, T. D., and Perrin, R., *Vibration of bells,* Applied Acoustics **20**, (41-70), 1987

4. Rossing, T. D., and Sathoff, H. J., *Modes of vibration and sound radiation from tuned hand bells.* J. Acoust. Soc. Am. **68**, (1600-1607) 1980

5. Rossing, T. D., Hampton, D. S., Richardson, B. E., and Sathoff. H. J., *Vibrational modes of a Chinese two-tone bell.* J. Acoust. Sic. Am. **83** (369-373) 1988

6. Richardson, B. E., and Roberts, G. E., *The adjustment of mode frequencies in guitars: A study by means of holographic interferometry and finite element analysis.* Proc. SMAC 83, Royal Swedish Academy of Music, Stockholm

7. Rossing, T. D., Popp, J., and Polstein, D., *Acoustical reponse of guitars.* Proc. SMAC 83. Royal Swedish Academy of Music, Stockholm

8. Hansen, U. J., Rossing, T. D., Hampton, D. S., and Bork, I., *Modal Analysis Applied to Musical Instruments: The Caribbean Steel Pan.* Proc. 4Th Intntl. Congr. Sound & Vibr., St. Petersburg, Russia, 1530 - 1520 (1996)

9. Hampton. D. S., Alexis, C., and Rossing, T. D., *Note coupling in Caribbean steel drums.* Paper GG3, Acoust. Soc. Am., Miami, (1987)