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THE LIMITATION IN THE SEA PREDICTION OF POWER TRANSMISSION AND ENERGY DISTRIBUTION

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Abstract This paper presents a numerical study of the SEA prediction accuracy in a coupled plate system. It was shown that the parameters controlling the prediction accuracy are the geometric mean of modal overlap factor and number of coupled modes in the frequency band of analysis. In the low frequency bands where few coupled modes are present, both the prediction error and standard deviation (in terms of driving force locations) of the numerical results are large. The "travelling wave" model and SEA may not be appropriate. In the medium frequency bands where the modal number is neither large nor small, the "travelling wave" model and SEA are applicable but their prediction errors are not negligible. The prediction error and standard deviation generally decrease as the geometric mean of modal overlap factor increases. However, the geometric mean of modal overlap factors cannot be used as a sole parameter for judging the prediction accuracy. For the same geometric mean value of modal overlap factors but different dissipation loss factors, the prediction errors could be different. The increase of modal number can reduce the fluctuation and standard deviation, but cannot reduce the prediction error. The increase of dissipation loss factors can reduce not only the fluctuation and standard deviation but also the prediction error. In the high frequency bands where both the modal number and modal overlap factor are large, the standard deviation becomes small and the prediction error is negligible.

INTRODUCTION

Statistical energy analysis (SEA) provides a useful tool to study the vibrational behaviour of complex structures. In SEA the system being studied is discretised into subsystems and the vibrational behaviour is described by the mean values of subsystem energies and power flows. The prediction can be made of the vibrational behaviour on the basis of known values of the power inputs, dissipation and coupling loss factors of a chosen SEA model. The prediction accuracy is assessed by the prediction error and standard deviation. The prediction error results from the fact that the chosen SEA model does not fully represent the actual dynamical system. The standard deviation characterises the uncertainty of the actual vibrational behaviour.

The dynamical responses and power flows in coupled beams or plates have been studied using SEA by several authors[1-4]. It was shown that the geometric mean of modal overlap factor has a strong effect on the accuracy of SEA prediction. When the geometric mean of modal overlap factor is considerably high, the deviation of actual value from SEA prediction is usually small and negligible, the "travelling wave" estimation of coupling loss factor agrees well with the exact calculation. In many practical situations, however, the geometric mean of modal overlap factors in the system may not be large, the deviation of actual value from SEA prediction could be high.

This paper presents a computational study of the accuracy in the SEA prediction of dynamic response and power flow in a plate system. This system consists of two steel plates and a steel beam, as shown in Figure 1. The surface dimensions of plates 1 and 2 are assumed to be 1.9x1.6m and 2.6x1.6m respectively. The thickness of plate 2 is 2mm. The neutral planes of the plates are in the same level and also coincide with the neutral plane of the beam. All the external boundaries of the system are assumed to be simply supported. Bending vibration is generated in one plate by a point force and in another plate by the power transmitted via the beam. The displacement response of this system can be described by the classical Bernoulli-Euler bending theory. Numerical results of the power transmission and energy distribution can be obtained from the exact solution of the Bernoulli-Euler bending equations and the boundary conditions.

COUPLING LOSS FACTOR

Coupling loss factor in SEA characterises the power flow between subsystems. Usually it is experimentally determined in practice. But for simple cases, it can be predicted on the basis of the "travelling wave" model in terms of wave transmission coefficient τ_{12} [5]

$$\eta_{12} = \frac{2c_B L_y \tau_{12}}{\pi\omega S_1} \tag{1}$$

where $\omega=2\pi f$ is the radian frequency; c_B and S_1 are the bending wavespeed and surface area of plate 1; L_y is the coupling length.

The plate system can be modelled into two SEA subsystems with each plate being defined as one SEA subsystem. If each of two subsystems is driven by an external force in turn, from the power balance equations the coupling loss factor η_{12} from subsystem 1 to 2 is given by [6]

$$\eta_{12} = \frac{\eta_{2T}}{R_{12}^{(1)} - R_{12}^{(2)}} \tag{2}$$

where $\eta_{2T}=P_2/\omega E_2^{(2)}$ is the total dissipation loss factor of subsystem 2; P_2 is the power input into subsystem 2 and $R_{12}=E_1/E_2$ is the energy ratio; superscript *(1)* or *(2)* means that only subsystem 1 or 2 is driven directly by an external force.

Figure 2 shows a comparison of the predicted (using equation (1)) and exactly calculated (using equation (2)) (spatial and frequency mean) coupling loss factors from plate 1 to 2 and the standard deviations. The dissipation loss factor of plate 1 is assumed to be 0.02. Three exactly calculated results correspond to different dissipation loss factors

(η_1=0.02 and 0.2) of plate 2 and different thicknesses (h_1=2mm and 5mm) of plate 1 which corresponds to different modal densities (n_1=0.484 mode/Hz and 0.194 mode/Hz). Both the predicted and exactly calculated coupling loss factors depend on modal density and frequency. However, the predicted values are independent of the dissipation loss factors of the coupled plates while the exactly calculated results depend very much on the dissipation loss factors. This dependence decreases with increasing frequency. This means that the coupling loss factor between finite plates is a function of the dissipation loss factors, modal densities and frequency, of which the combined effect can be described by the geometric mean of modal overlap factor $M = \sqrt{M_1 M_2}$ where $M_1=n_1(f)\eta_1 f$ and $M_2=n_2(f)\eta_2 f$ represent the modal overlap factors of plates 1 and 2 respectively. Generally, both the prediction error and standard deviation decrease with increasing the geometric mean of modal overlap factor. However, M cannot be used as a sole parameter for judging the prediction accuracy. For the same value of M but different dissipation loss factors, the prediction errors are different. The coupling loss factor is also the function of modal number. In the low frequency bands where the wavelengths are larger than or comparable with the surface dimensions of the plates and few modes are present, the fluctuation of exactly calculated results is large. Both the standard deviation and prediction error are great despite M is not small. The "travelling wave" model may not applicable. For example, for the coupling loss factor where η_2=0.2 in the frequency band of 16Hz where no coupled mode is present, the prediction error is more than 5dB although M=0.5. In the medium frequency bands where the modal number is neither large nor small, the exactly calculated coupling loss factor curve becomes smoother and the standard deviation becomes smaller. The "travelling wave" model can be used but the prediction error is visible (1~3dB) depending on the geometric mean of the modal overlap factors. In the high frequency bands where their centre frequencies are larger than 400Hz, both the modal overlap factor and modal number are very large, the coupling loss factor can be predicted accurately on the assumption that all waves are incident on the coupling boundary in the normal direction.

POWER TRANSMISSION

The exact estimation of the power flow between the plates can be made from the solution of the Bernoulli-Euler bending equations and the boundary conditions. Figure 3 shows the spatial mean power flows from plate 1 to 2 with different modal densities and dissipation loss factors (n_1=0.484 mode/Hz and η_1=0.02 or 0.2; n_1=0.194 mode/Hz and η_1=0.02). Each spatial mean value is the average over 12 exactly calculated results. Similar to the coupling loss factor, the spatial mean power flow depends on dissipation loss factors, modal densities and frequency, that is, it is a function of the geometric mean of modal overlap factor. The power flow generally increases with increasing modal densities and dissipation loss factors. For the curves with the same modal density but different dissipation loss factors, the response maximum and minimum locations coincide at low frequencies where the wavelengths are longer than or comparable with the surface dimensions of the plates. The increase of damping results only in a reduction (or increase) of the response maximum (or minimum) values. If the spatial average is not taken, the power flows for different source positions are different and fluctuate around its mean value. Although the fluctuation is large at low frequencies, the response maximum locations of the curves coincide, which indicates that the variation (or the effects of uncertainty) is small.

However, the variation increases as frequency increases. At high frequencies where the geometric mean of modal overlap factor is large, the variation in frequency response is also large. It is more likely that a maximum location of one response curve may coincide with a minimum locations of another.

If both spatial and frequency averages are taken, the fluctuation of exactly calculated power flow can be reduced to a great extent, as shown in Figure 4, which shows the one-third octave frequency band values of the same data as those shown in Figure 3 and also their corresponding SEA predicted values and standard deviations. Both the prediction error (the level difference between the prediction and the mean value of exactly calculated results) and standard deviation are the function of modal overlap factors. A small geometric mean of modal overlap factor usually corresponds to large values of prediction error and standard deviation. In the low frequency bands where the wavelengths are larger than or comparable with the surface dimensions of the plates, both the standard deviation and fluctuation of exactly calculated results are large. This indicates that the prediction cannot be correctly made using SEA because few coupled modes are present in the frequency band of analysis. As frequency increases, the bandwidth of analysis is broadened and the modal number increases. When the modal number is neither large nor small, SEA could be used but the prediction error is not negligible. In some frequency bands, the prediction error is larger than the standard deviation, that is, the prediction falls outside of the 95% confidential interval. This is because a few modes are present and the responses of some modes are very small in the frequency band. For example, for the curve (h_l=2mm and η_l=0.02) the predicted modal number is 7 in the one-third octave frequency band with centre frequency of 63Hz but only 4 modes actually exist and two modal responses are very small. In the high frequency bands where both the modal number and modal overlap factor become very large, the predicted and exactly calculated results agree well and the standard deviation is within 1dB.

DYNAMIC RESPONSE

The estimation of energy distribution in the plate system, which is made on the basis of the Bernoulli-Euler bending equations and boundary conditions, will not result in any approximation error. For the analysis made in a wide frequency band, the mean values of the energies in a coupled system can be predicted using SEA from the power balance equations. Figure 5 compares the predicted and averaged (exactly calculated) energy levels of the source and receiving plates in the system. Figure 6 shows the corresponding standard deviations. It is shown in Figure 5 that the energy level of a plate is affected by the dissipation loss factor of the coupled plate. In the low frequency bands (for example, the centre frequencies are smaller than 20Hz in Figures 5a and 5b or 50Hz in Figure 5c), the fluctuation in the exactly calculated energy levels of both source and receiving plates is large. The reason is that the wavelengths are comparable with the surface dimensions of the plates and the number of coupled modes is very small in the frequency band of analysis. As frequency increases, the modal number increases and the fluctuation decreases. In the medium frequency bands, the prediction error and standard deviation generally decrease with increasing modal overlap factor. For the same modal overlap factor, however, the prediction error and standard deviation in the source plate are usually smaller than those in the receiving plate. The energy of the receiving plate is over-predicted in most frequency bands. This is because the transmitted power is over-predicted (see Figure 4). In the higher frequency bands where the centre frequencies are above 400Hz, the spatial and frequency mean values of exactly calculated energy levels agree well with the predicted values, the prediction error is negligible.

CONCLUSIONS

This paper presents a computational study of the accuracy in the SEA prediction of power transmission and dynamic response in a coupled plate system. It is shown that two principal parameters which control the prediction accuracy are the modal overlap factor and the number of modes in the frequency band of analysis. In the low frequency bands where the wavelengths are larger than or comparable with the surface dimensions of the plates, few modes are present, the fluctuation of the numerical results is large. The "travelling wave" model and the SEA prediction may not be applicable. In the medium frequency bands, the modal number is neither large nor small, SEA can be used but its prediction error is not negligible. The SEA prediction error and the standard deviation of the numerical results generally decrease with increasing the geometric mean of modal overlap factors. However, the geometric mean of modal overlap factors cannot be used as a sole parameter for judging the prediction accuracy. For the same geometric mean value of modal overlap factor but different dissipation loss factors, the prediction errors could be much different. The increase of modal number can reduce the fluctuation and standard deviation, but cannot reduce the prediction error. The increase of dissipation loss factors can reduce not only the fluctuation and standard deviation but also the prediction error. In the high frequency bands where the wavelengths are much smaller than the surface dimensions of the plates, both the geometric mean of modal overlap factor and the number of coupled modes becomes large. The standard deviation of the numerical results becomes small and the SEA prediction error can be negligible.

The dynamical response and power transmission in a finite plate system are the function of source location and modal overlap factor. In the narrow frequency bands, the variation in the shapes and resonance peak locations of the individual curves of each variable for different source locations is small at low frequencies where the wavelength is larger than or comparable with the surface dimensions of the plates, but it increases with increasing frequency. The spatial mean of each variable fluctuates around its mean value and the level difference between them decreases with increasing modal overlap factor.

ACKNOWLEDGMENTS

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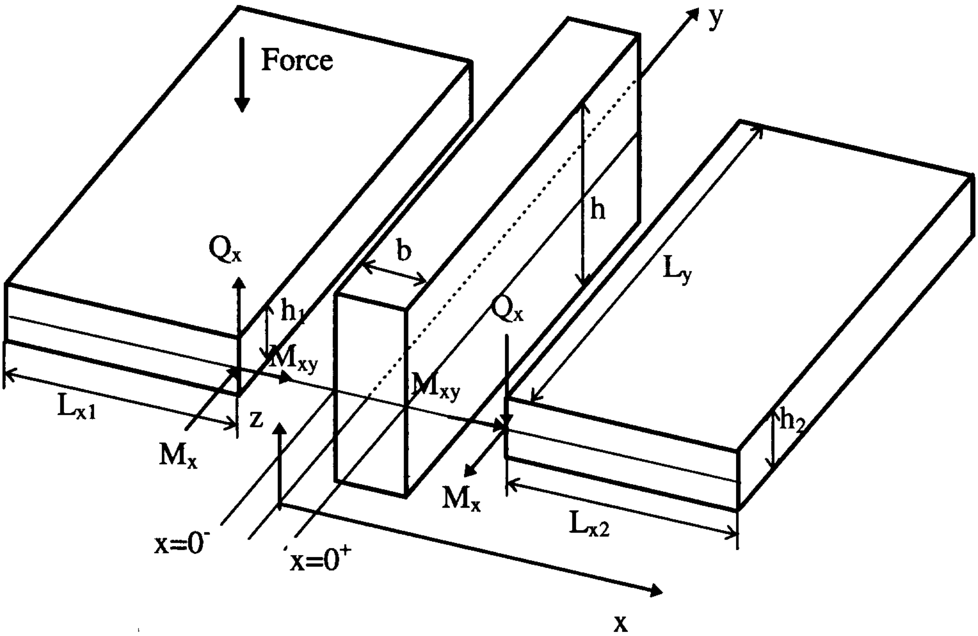


Figure 1. Coordinates and load conventions of a two-coupled plate system.

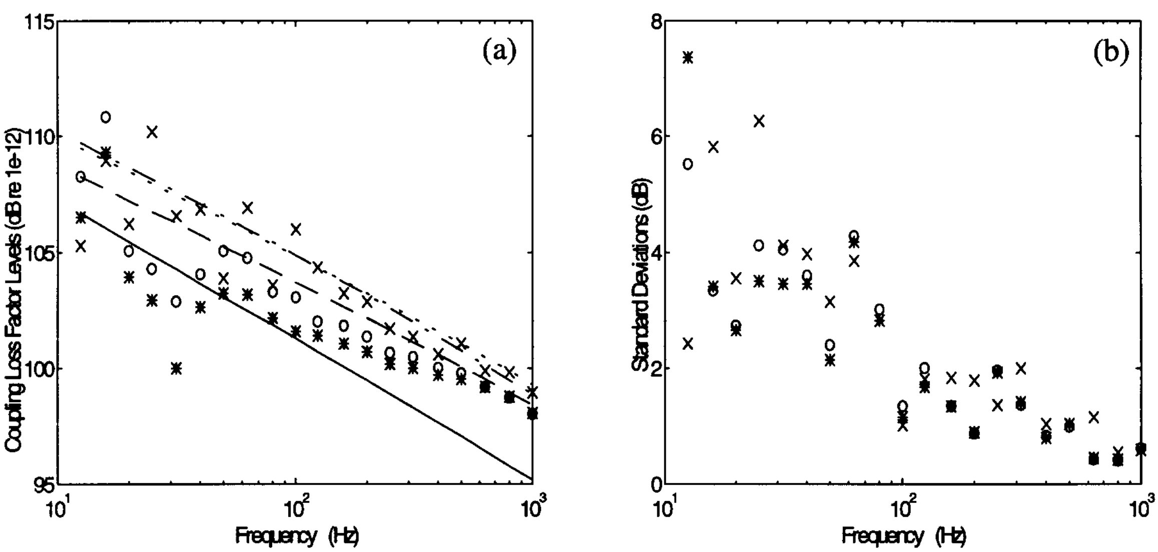


Figure 2. (a) Coupling loss factors predicted with incident angle of 90^0 (h_1=2mm: — — — — — ; h_1=5mm: - - - - - - - - ;) or averaged from 0^0 to 90^0 (h_1=2mm: ———— ; h_1=5mm: — - — - — - -)) and their exact solutions (h_1=2mm and η_2=0.2 (O); h_1=2mm and η_2=0.02 (∗); h_1= 5mm and η_2=0.02 (X)).

(b) Standard deviations.

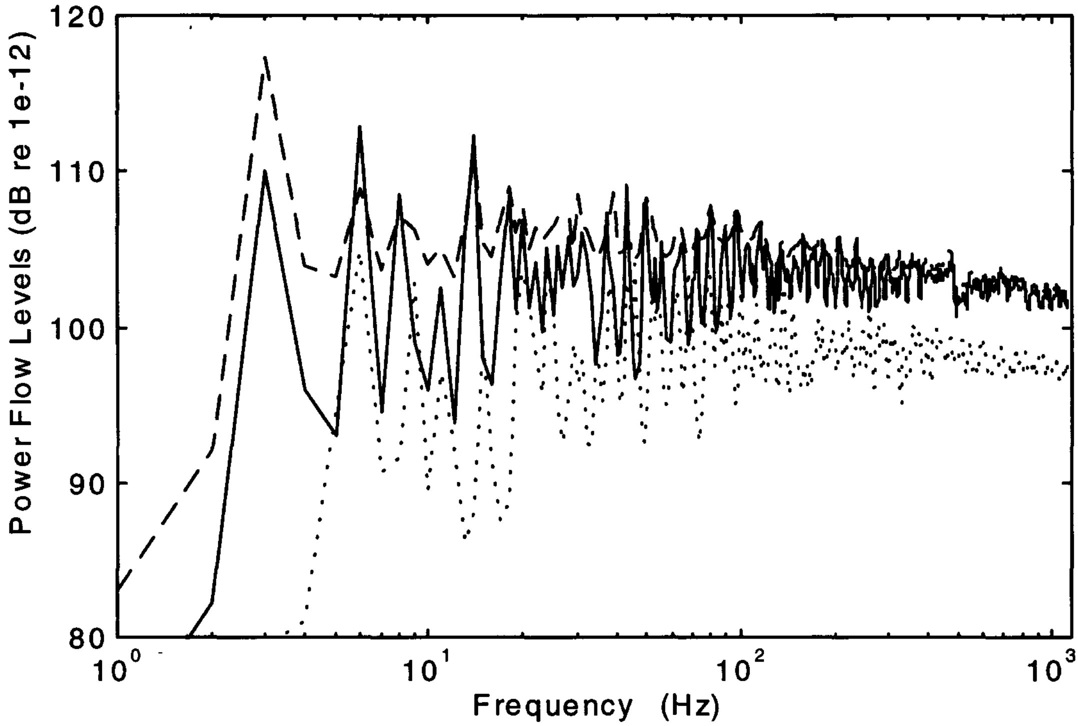


Figure 3. Exact power flows from plate 1(1.9x1.6m) to plate 2(2.6x1.6x0.002m). n_1=0.484 mode/Hz and η_1=0.02 (————); n_1=0.484 mode/Hz and η_1=0.2 (— — — — ·); n_1=0.194 mode/Hz and η_1=0.02 (· · · · · · · · ·).

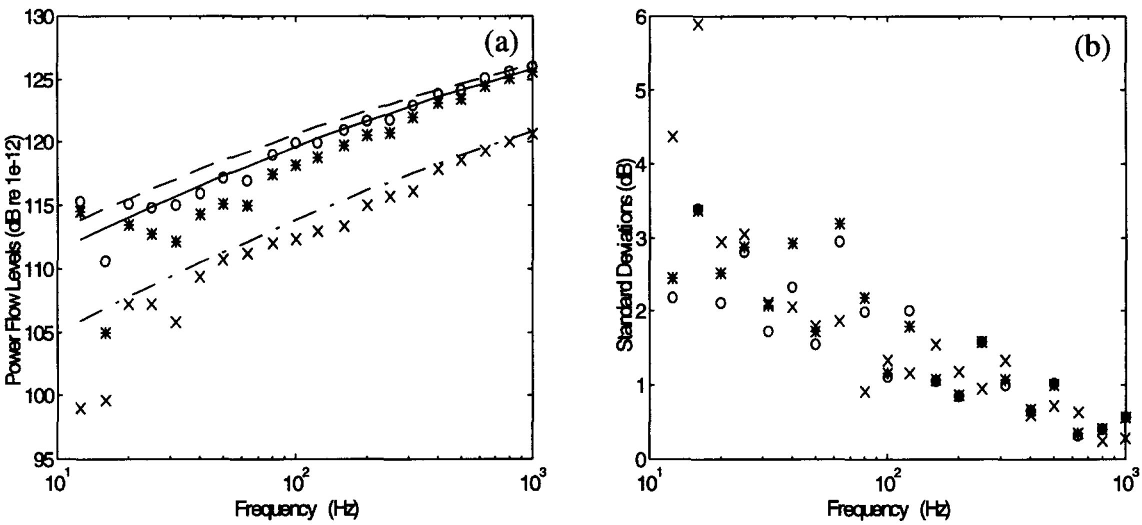
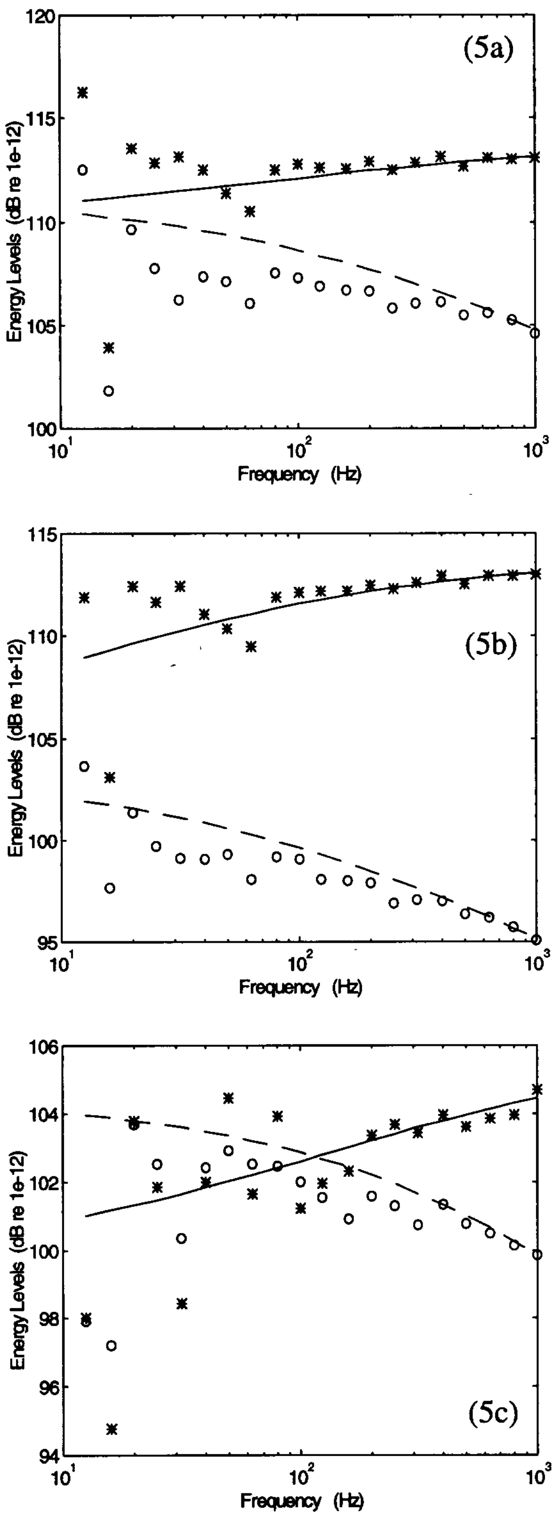


Figure 4.(a) Power flows from plate 1 (1.9x1.6m) to plate 2(2.6x1.6 x0.002m) predicted (h_1=2mm and η_1=0.2(— — — — —); h_1=2mm and η_1=0.02 (————); h_1=5mm and η_1=0.2 (— · — · — ·) and their exact solutions (h_1=2mm and η_1=0.2 (O); h_1=2mm and η_1=0.02 (✳); h_1=5mm and η_1=0.2 (X)).

(b) Standard deviations.



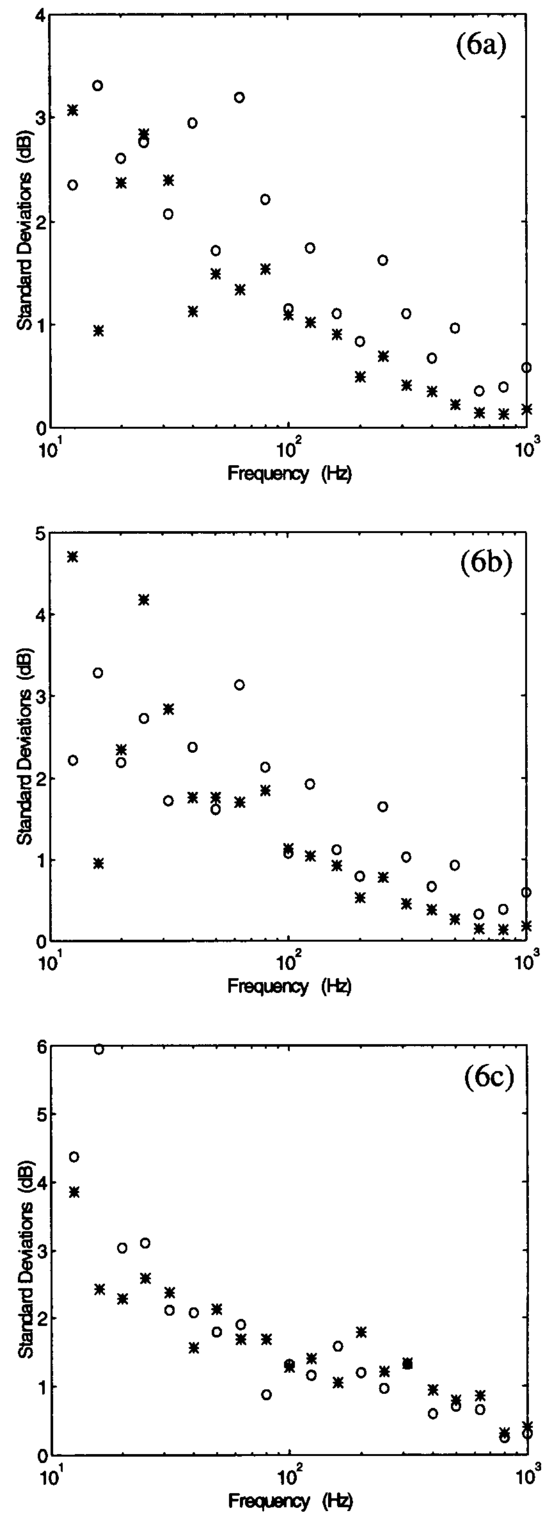


Figure 5. Predicted and exact energy levels in the source (1.9x1.6m; ———— , ✱) and receiving (2.6x1.6x0.002m, – – – – –, O) plates.

Figure 6. Standard deviations of the energy levels in source (✱) and receiving (O) plates.

(a) h_1=2mm and η_2=0.02; (b) h_1=2mm and η_2=0.2; (c) h_1=5mm and η_2=0.02;