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THEORETICAL AND EXPERIMENTAL STUDY OF A GENERALIZED PSEUDO-FORCES METHOD FOR SOURCE CHARACTERIZATION

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So-called pseudo- or substitution forces on the outside of a source body can be used to characterize the strength of compact structure-borne sound sources. Their use allows for multi-directional vibrations, whilst the measurement effort remains limited. However, the indirect nature of the method introduces some arbitrariness in substitution forces locations. This leads to the existence of equivalent but different sets of appropriate pseudo-forces. In the current work the theory and the practical validity are studied of a transformation of pseudo-forces into modal loads. This procedure would allow for an unambiguous definition of 'equivalent forces', increasing the practical exploitation possibilities of the method. At the same time the measurement effort needed for the characterization is kept limited. The validity and practicability of this new technique is tested by experiments on a compact source.

1. INTRODUCTION

Characterization of the strength of air-borne sound sources is well established for all kinds of machinery and it is commonly applied in industrial environments. Compared to this, the methods for characterizing the strength of structure-borne sound sources still appear to have a long way to go before arriving at a similar level of practical implementation.

The conventional linear theory for a structure-borne sound source is quite transparent and is a multi-dimensional mechanical equivalent of the Thévenin or Norton concepts. For a single, unidirectionally vibrating contact point, a source can be characterized with two parameters. With more degrees of freedom per mounting point and more mounting points the number of parameters needed increases rapidly. In practice, this large number is prohibitive for experimental characterization and therefore simplifications are inevitable. Due to the different perspectives of the various researchers, different simplifications are made. This has resulted in a wide variety of methods, see [1], all with advantages on some points and disadvantages on others.

The 'pseudo-forces method', see [2,3], was developed for compact sources with the perspective to reduce the measurement effort compared to the effort involved in the approaches which evaluate the source at its mounting points. The method allows for multi-dimensional vibrations and multiple-source mechanisms in the source. It also facilitates the measurements on a source running under load.

In this paper an attempt is made to broaden the scope of the method by transforming results of different pseudo-forces experiments with identical sources into similar load sets.

In this paper a 'source' means a mechanical component incorporating some sort of noise generating mechanism, e.g. a gearbox or a hydraulic pump. A 'receiver' means the structure onto which the source is mounted.

2. PRINCIPLE

In the pseudo-forces method the internal noise generating mechanism (NGM) is represented by a number of fictitious forces on the outer surface of the source.

It is assumed that the NGM is not affected by the mounting conditions. Note that this restriction does not only apply to the pseudo-forces method but to any method which describes the source as a 'black box' using general theory for linear systems.

If n forces are used for a source with n vibrational degrees of freedom (DOF), these forces can regenerate the vibrations of the source exactly. For example, in the case that a source acts as a rigid body, six forces appropriately distributed on the outer surface of the source can always reproduce any actual internal excitation *exactly*. Furthermore, this set will mimic the actual excitation in any built-in situation. This set of forces is an equivalent of the NGM and therefore a source property.

This principle is also valid in the general case of a non-rigid source, see [2,3]. An actual source, which is a continuous body, has an infinite number of DOFs, hence, strictly speaking, an infinite number of pseudo-forces would be needed. It is anticipated however, that in practice it may be sufficient to reproduce only the contributions of the most important DOFs. In that case an adequate result may be obtained using a limited number of forces, say five to ten. This assumption is the main simplification in the pseudo-forces approach. Experiments have shown very encouraging results on this point, see [3].

3. DETERMINATION OF PSEUDO-FORCES

The pseudo-forces can be determined based on measurements using an inverse technique. Since the pseudo-forces are a source property, they can be evaluated in almost arbitrary surroundings, using the following procedure:

- Select a number of n pseudo-force positions distributed at the outer surface of the source. The corresponding forces form a vector $\{F\}_{pseudo}$.
- Select a (preferably larger) number of m response positions located on the source and/or receiver structure. These responses form a vector $\{a\}$.
- Measure a $m \times n$ transfer matrix $[A]$ which relates forces and responses.
- Measure the responses with the source running under relevant operational conditions.
- Calculate the pseudo-forces using the (pseudo)inverse of $[A]$:

$$\{F\}_{pseudo}=[A]^{+}\{a\}_{running} \tag{1}$$

If the pseudo-forces are used in a diagnostic study of an existing machine, a direct link to the radiated sound of the receiver $\{p\}$ induced by the source is given by:

$$\{p\}_{radiated}=[H]\{F\}_{pseudo} \tag{2}$$

where $[H]$ is a (measured) transfer matrix which relates forces and sound pressures at particular positions.

4. PRO'S AND CON'S

Pro

- the source is evaluated at its outer surface, which is in general better accessible for transducers than the mounting points.
- the 'activity' can be determined from measurements in an (almost) arbitrary surrounding (e.g. in [3] results are presented for a source installed in a vehicle).
- all forces and responses can be chosen to be perpendicular to the surface (although other directions may be included at wish).

Con

Sets of pseudo-forces determined at different positions at the same source are not directly comparable, although they are fully equivalent.

5. EXPANSION OF THE METHOD

Two different sets of forces are related via a transformation matrix $[T]$:

$$\{F\}_{set\,1} = [T]\,\{F\}_{set\,2} \tag{3}$$

It can be shown that $[T]$ only depends on source body properties. A unique definition of substitution forces can be achieved by defining a transformation for each possible set of pseudo-forces, which transforms the pseudo-forces into a unique vector.

An ideal way of doing this would be to use the relation between the particular pseudo-forces and the actual NGM, $\{F\}_{NGM}$:

$$\{F\}_{NGM} = [T_{set\,1}]\,\{F\}_{set\,1} = [T_{set\,2}]\,\{F\}_{set\,2} \tag{4}$$

Of course, in general $[T_{set\,1,2}]$ will not be known and therefore (4) cannot be used. Another way to enforce uniqueness is defining a particular set of forces as the 'standard' set, but this is unattractive. A unique set of loads can however be created, using the dynamic properties of the *free source*. For a discretized case the responses $\{r\}$ of the free source are related as:

$$\{F\}_{pseudo} = (-\omega^2[M] + j\omega\,[B] + [K]\,)\,\{r\} \tag{5}$$

with $[M]$, $[B]$ and $[K]$ are mass, damping, and stiffness matrices of the source. This matrix equation has eigenvectors $\{\phi\}_1$ to $\{\phi\}_n$ stored in a matrix $[\Phi]$ and corresponding eigenvalues ω_1 to ω_n. The vector $\{r\}$ can be written as a summation of the eigenvectors (modeshapes), each one multiplied with its associated *participation coefficient* $\{\eta\}$:

$$\{r\} = [\Phi]\{\eta\} \tag{6}$$

Substituting (6) in (5) and multiplying with $[\Phi]^T$ then gives:

$$[\Phi]^T\{F\}_{pseudo} = \{Q\} = [\Phi]^T\,(-\omega^2[M] + j\omega\,[B] + [K]\,)\,[\Phi]\,\{\eta\} \tag{7}$$

where $\{Q\}$ are called *modal forces*. Since $[\Phi]$ are the eigenvectors, (7) is the completely diagonalized form of (5). Matrix $[\Phi]$ can be found experimentally using modal analysis of the dismounted source. For each set of pseudo-forces a transformation can be found by evaluating the eigenvectors at the pseudo-force positions. So the modal forces of the free source are found from:

$$\{Q\} = [\Phi_{at\,set\,1}]^T\{F\}_{set\,1} = [\Phi_{at\,set\,2}]^T\{F\}_{set\,2} = [\Phi_{at\,set\,i}]^T\{F\}_{set\,i}\;(= [\Phi_{at\,NGM}]^T\{F\}_{NGM}) \tag{8}$$

Eq. (8) shows that the modal forces $\{Q\}$ are independent of the choice of pseudo-force positions. Except for a scaling factor, $\{Q\}$ is uniquely defined. The scaling of $\{Q\}$ and $\{\eta\}$ depends on the scaling of $[\Phi]$, which is arbitrary. In the product of $\{Q\}$ and $\{\eta\}$ however this scaling factor must vanish. Thus for the i^{th} degree of freedom $Q_i\,\eta_i$ gives a completely unique measure of the 'activity'. Note that this product can have real and imaginary parts and is in units of energy. The matrix $[\Phi]$ can be scaled in such a way that the participation coefficients of all DOFs are unity:

$$\{r\} = [\Phi]\{\eta\} = [\Psi][1\ 1\ ....\ 1\ 1]^{T} \tag{9}$$

Using this type of scaling, the modal loads are as:

$$\{Q\}=[\Psi]^{T}\{F\}_{pseudo} \tag{10}$$

They have the same dimension as energy. In practice, a modal analysis of the free source will result in the identification of a (finite) number of q modes. This will generally not be equal to the number of n pseudo-forces used. The number of modes will at least be six (rigid body modes) plus some flexible modes, say in total about twelve. For practical reasons the number of used pseudo-forces will be rather limited. So, generally speaking, the number of modes q, in the frequency range of interest, will be higher than the number of pseudo-forces n. However, projection onto more modes than n doesn't give more independent information and projection onto less modes means loss of information. Therefore it appears to be most convenient to project the pseudo-forces onto n modes (so $[\Phi]$ is square). This means that a selection of n modes out of q must be made.

6. SELECTION OF MODES

This selection of a subset of modes is to some extent arbitrarily. Mathematically any subset is valid, therefore the selection must be made based on physical considerations. At a particular frequency the contribution of the various modes to the total response is not the same. It is rational to use those modes which have the most important contributions. Two criteria can be used to determine which mode is important. Firstly, a mode which's corresponding natural frequency is near to the particular frequency of interest is more likely than others. This can be expressed in a numerical criterion. Secondly, a mode of which the shape resembles the actual response is more likely than others. This can be expressed using the so called Modal Assurance Criterion, (see e.g. [6]). A mode which 'scores' high using both criteria is selected.

7. RECIPE

Summarising the previous sections, the recipe to determine the modal loads of a compact source now is:

1. Select n pseudo-force positions at the outer surface of the source. Locations are quite arbitrary. The number can be based on experience (say six to ten).
2. Measure m responses $\{a\}$ of the mounted, running source and/or framework. Mounting conditions are quite arbitrary.
3. Measure the $m\times n$ transfer matrix $[A]$ from the force to the response positions.
4. Pseudo-forces are found using $\{F\}=[A]^{+}\{a\}$.
5. Dismount the source.
6. Select p response positions (including the n force positions) distributed over the outer surface of the free source to create to vector $\{r\}$.
7. Measure a $p\times n$ transfer matrix $[S]$ from force to response positions.
8. Estimate the response of the free source $\{r\}=[S]^{+}\{F\}$.
9. Use the same matrix $[S]$ to identify q modes using modal analysis, which results in a $p\times q$ mode shape matrix $[\Phi]$.
10. Select n modes out of q resulting in $p\times n$ matrix $[\Phi_{subl}]$.
11. Determine n the participation coefficients $\{\eta\}=[\ \Phi_{subl}]^{+}\{r\}$.
12. Rescale $[\Phi]$ and $\{\eta\}$: $[\Phi_{subl}]\{\eta\} = [\Psi][1\ 1\ ....\ 1\ 1]^{T}$.

13. Evaluate the rescaled modeshapes at the n pseudo-forces positions, giving $n \times n$ $[\Psi_{sub2}]$.
14. Calculate modal loads $\{Q\}=[\Psi_{sub2}]^{T}\{F\}$.

8. RELATION TO 'MOUNTING POINT' METHODS

Some other 'activity' methods, see e.g. [4] and [5], characterize the source by its properties at the connection points of source and receiver. These methods are based on the same 'black box' model so there is no fundamental difference, but the simplifying assumptions are different. In fact, the pseudo-forces approach might be used as an indirect method to determine mounting point velocities of the free source, using:

$$\{v\}_{free} = [V]\{F\}_{pseudo}=[V][\Phi]^{-T}\{Q\} \tag{11}$$

where $[V]$ is a transfer matrix between pseudo-forces and $\{v\}_{free}$. $\{F\}_{pseudo}$ can be determined from measurements with the mounted source, and thus simplifying the application of a representative load case. $[V]$ can be measured using the dismounted source.

9. EXPERIMENTS AND RESULTS

Experiments were performed on a small electrical drive of a copier machine (AEG, 190 V, 50 Hz) by courtesy of OCE Nederland BV. The objective of the experiments was to test the hypothesis that different sets of pseudo-forces result in the same modal loads after transformation. A total of thirty different sets of pseudo-forces were used: different in terms of number, excitation positions and mounting conditions of the source. Figure 1 shows the three mounting conditions considered: free (fig. 1a), blocked (fig.1b) and mounted on a copier frame (fig. 1c).

First the sets of pseudo-forces were calculated using (1). For these fictitious excitations, responses on the source in different mounting conditions were recalculated to check the equivalence of different sets of forces. Figure 2, as an example, displays the average of all the measured responses and the average of all the calculated responses for two sets of pseudo-forces. These sets consisted of seven different forces acting on the free source. This figure shows that both sets of pseudo-forces fit the responses very well so that these sets can indeed be considered fully equivalent. Results for other sets were similar.

However, although the sets of pseudo-forces may be equivalent, mutual comparison of the pseudo-forces itself is not possible due to the fact that they act at different positions. Figure 3a and 3b show the pseudo-forces, used in the calculation of figure 2 for a particular frequency, in the complex plane. The arrows in the figure represent the pseudo-forces at the different locations, so that these arrows are not comparable in the figures 3a and 3b.

Modal analysis was performed on the free hanging drive, resulting in the modes labelled *a* up to *l*. After projection of the pseudo-forces onto these modes the resulting modal loads (labelled *A* up to *L*) are comparable. Figure 3c and 3d show these modal loads. Although, theoretically speaking, these modal loads should be equal, the results still show differences. This can be expressed by the standard deviation of the modal loads in terms of phase and amplitude. In figure 4, the levels of the modal loads and their standard deviation of a subset of the thirty experiments are represented for one frequency. The subset used consisted of five different sets of seven pseudo-forces, assessed for the free source. This figure shows a standard deviation of approximately 3 dB for each modal load.

In figure 5, the average levels of the modal loads for three subsets of experiments are displayed for one frequency. These subsets represent the experiments for seven pseudo-forces for each mounting condition. For some modes significant scatter between the results of different mounting conditions is seen.

Figure 6 shows the average standard deviation of all modal loads for each frequency used in the analysis. Results are shown for the same three subsets as used in figure 5 and for the totality of experiments in these three subsets. For the individual mounting conditions the frequency average of the standard deviation is about 3.7 dB. For the totality of the experiments it is about 5.5 dB.

10. DISCUSSION

The pseudo-forces method has several distinctive advantages compared to descriptors which need measurements at freely vibrating mounting points. However, in the previous studies on this new method the formulation had distinctive disadvantages as well. Sets of forces which meet the requirements for equivalence, in the sense that they provide a good reproduction of vibrations and sound, are still largely different. Causes are, for example, the differences in the number of forces included in a set and of their excitation positions and directions. In this paper a generalization of the method is sought by a modal projection of the pseudo-forces obtained from an arbitrary experiment. This leads to the definition of a set of modal loads as a source descriptor, which might be obtained in different environments and for different numbers and positions of pseudo-forces. The advantages of such a procedure are clear. Results obtained by different researchers and resulting from different choices of pseudo-forces in the experiments, do become equivalent. Moreover, a transformation is possible from one set of pseudo-forces into another set. This may be very practical for diagnostic experiments for low noise design purposes.

Looking at the results in this paper a significant scatter was found in modal loads derived from thirty different experiments. Averaged over all modal loads and over frequencies, a standard deviation of about 3.5 dB was found for experiments with different sets of pseudo-forces, but for the same mounting condition of the source. Another 2 dB increase of this standard deviation was found taking all the experiments for three very different mounting conditions. However, although these results require further study concerning the causes of the scatter, it would not be justified to be very pessimistic on the validity and the potential practical value of the proposed source descriptor. Fig. 2 and fig. 3 illustrate the fact that two rather different sets of modal loads, which were obtained from two different pseudo-forces experiments, do quite well in reproducing the source vibrations. Similar comparisons using results derived from other sets of pseudo-forces, both for the same and for different mounting conditions, show similar promising results. Therefore, the way this paper dealt with scatter in modal loads may be somewhat misleading and overemphasizing the differences.

Apart from several other methodological questions related to accuracy and robustness, further investigations are needed on the influence of scatter in the source vibrations themselves, which form part of the inputs for the inverse method. This situation is not different from what has to be done for other source descriptors.

In conclusion, the generalization of the pseudo-forces method seems practically feasible and worth of further investigation.

11. REFERENCES

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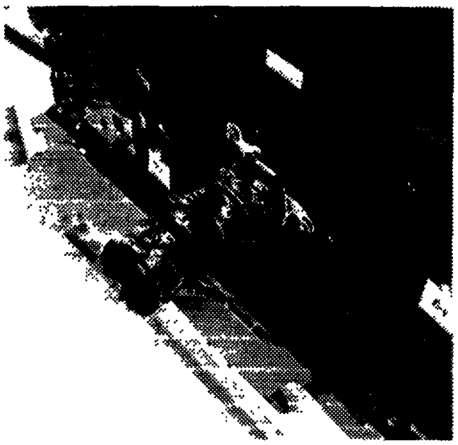
5. R.J. Pinnington, R.A. Fulford, M. Terry, *Proc. InterNoise '96*, Liverpool UK, p. 1587-1592. (1996)
6. D.J. Ewins. *Modal Testing: Theory and Practice.* Research Studies Press LTD, Letchworth, Hertfordshire, England, 1984.

12. FIGURES

a)

b)



c)

Figure 1. Mounting conditions of the source.

a) The free source.
b) The blocked source.
c) The source mounted on its framework.

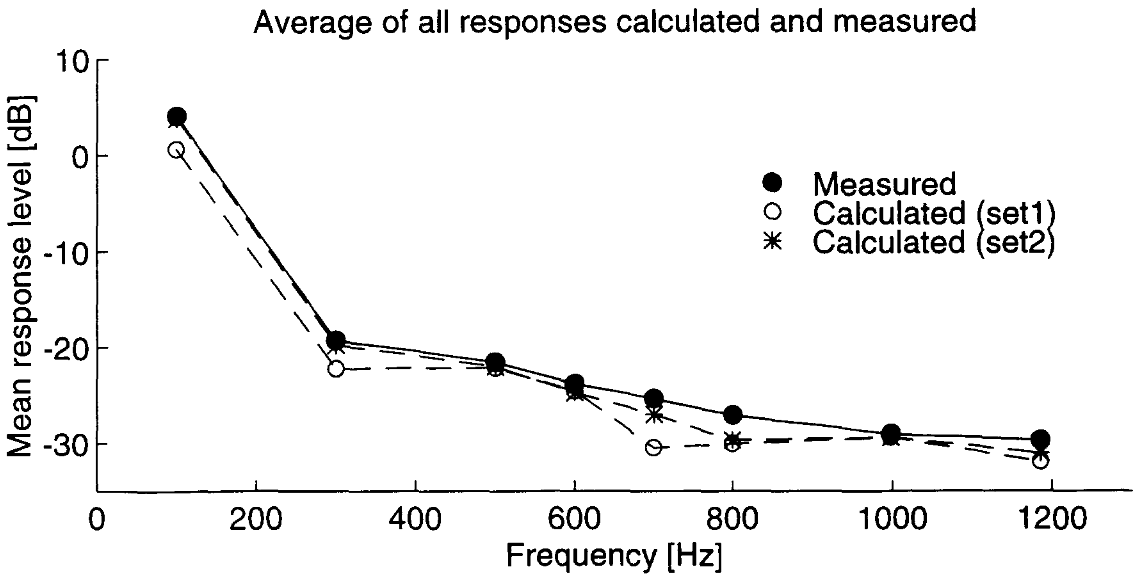


Figure 2. Average of all the responses on the free source, measured and calculated with two different sets of pseudo-forces. The pseudo-forces used had been derived from measurements on the free source

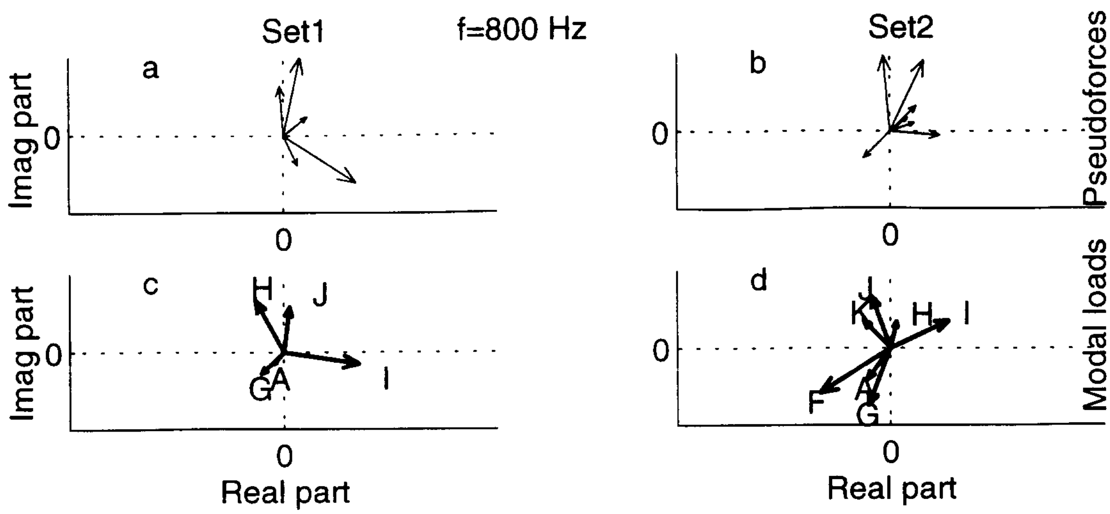


Figure 3. Pseudo-forces and modal forces of two sets. Modal projection of the pseudo-forces makes comparison possible.

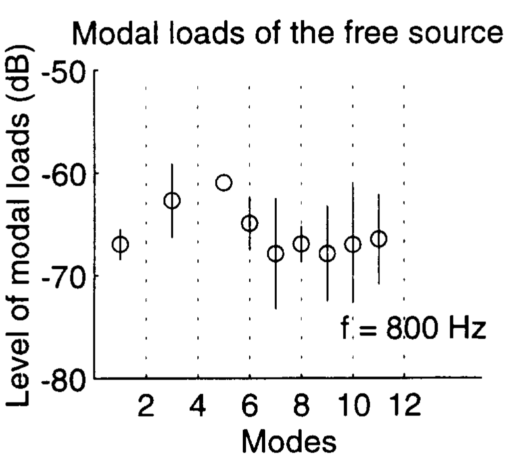


Figure 4. Average level of the modal loads $\{Q\}$ on the free source at 800 Hz.

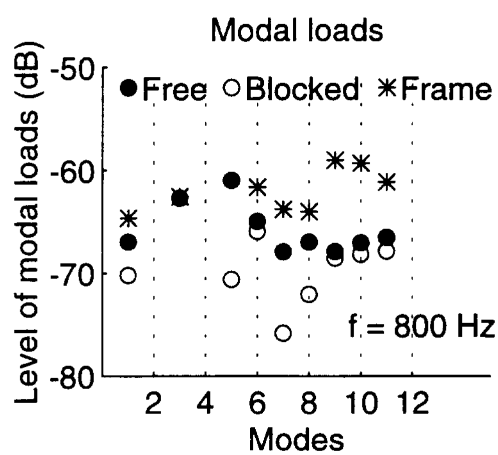


Figure 5. Average level of $\{Q\}$ in three mounting conditions.

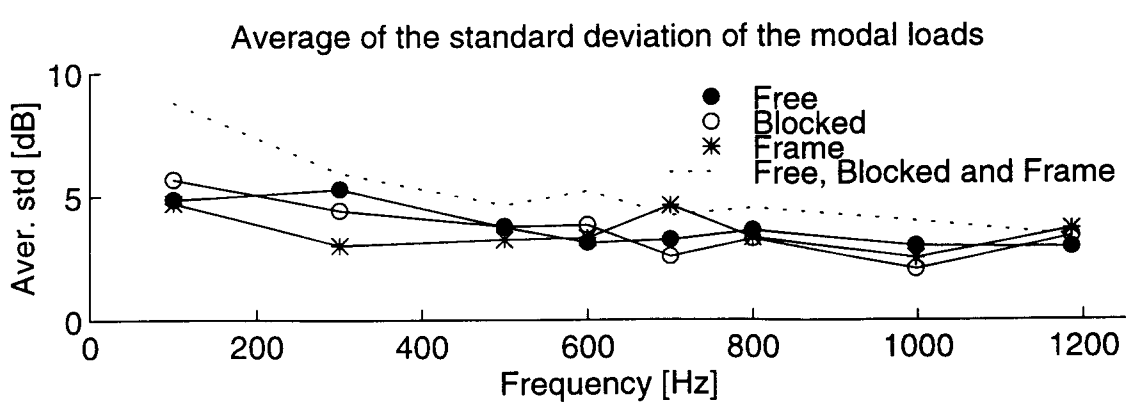


Figure 6. Average of the standard deviation over three subsets, according to the mounting conditions. The dotted line represents the total average of the standard deviation.