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### SMART SENSORS FOR MEASURING THE ACOUSTIC POWER MODE OF A PLANAR STRUCTURE

Yoshihiro Kikushima and Nobuo Tanaka Mechanical Engineering Laboratory, MITI 1-2 Namiki, Tshukuba Science City, Ibaraki 305 Japan

## Abstract

This paper considers the design of distributed parameter modal sensors called "smart sensors", with a particular emphasis on filtering the combination of appropriately weighted vibration modes providing a specific performance index in control strategy. First, by considering a practicability of the distributed parameter smart sensors using PVDF film sensors, one-dimensional smart sensor is presented. It is found that the approach done by the one-dimensional sensors holds only the necessary condition for sensing the transformed mode. This problem is overcome by introducing multiple one-dimensional smart sensors. Moreover, the design procedure for the multiple one-dimensional smart sensors for measuring the transformed mode is established. Then, an experiment is conducted, demonstrating the validity of the smart sensors. Finally, using the smart sensors, the minimization of the total acoustic power radiated from a vibrating plate is carried out.

# **1. INTRODUCTION**

Summarizing the active structural / acoustic control approaches reported so far, much research has been devoted to minimizing sound pressures at discrete sensing points located in the acoustic field. However, if the sound pressures at the sensory points are minimized (such as by employing an existing active control technique, e.g., filtered-x LMS algorithm), there is no guarantee that sound pressures at any other point in the acoustic field will be minimized or decreased. On the contrary, sound pressures at other points or areas may increase. In addition, there are a great number of cases in which such an approach turns out to be impractical. It is evident that the disadvantage found in the conventional active structural / acoustic control approach lies in the inheritance of a lumped parameter design methodology. At this point a necessity of introducing the root crux of the "intelligent structure" [1] arises, that is, the distributed parameter design methodology of control systems.

If it is possible to use distributed parameter sensors such as PVDF (polyvinylidene fluoride) films or optical fiber sensors, information on the vibration field can be obtained not from points but from lines or areas. As a result of enhancement of observability, global attenuation of the vibration level over the structure can theoretically be achieved, and the observability spillover [2] can be avoided. The application of PVDF film sensors can be traced back to Lee and Moon [3] for measuring modal amplitudes. Clark et al.,[4,], Collins et al., [5] and Lefebvre et al.,[6] also used the PVDF sensors as modal sensors. These efforts are, however, confined to the estimation of a particular single vibration mode of a structure. Other work (Snyder et al.,[7]; Snyder and Tanaka, [8] has involved the use of transformed modes consisting of a number of vibration modes on which weighting factors are imposed. Thus, in order to enable the estimation of not only a normal mode but also a general mode such as the transformed mode, a more general modal filtering methodology

is now needed.

This paper considers distributed parameter modal filtering, with particular emphasis on sensing the combination of appropriately weighted vibration modes to provide a specific performance index in control strategy. It is the purpose of this paper to establish a design procedure for "strip" distributed parameter modal sensors called "smart sensors", as well as to experimentally demonstrate the validity of the "smart sensors". First, by considering a practicability of the distributed parameter smart sensors using PVDF film sensors, one-dimensional smart sensor is presented. It is found that the approach done by the one-dimensional sensors holds only the necessary condition for sensing the transformed mode (acoustic power modes in this paper (Snyder and Tanaka,[7, 8])). This problem is overcome by introducing multiple one-dimensional smart sensors. Moreover, the design procedure for the multiple one-dimensional smart sensors for measuring the transformed mode is established. Then, an experiment is conducted, demonstrating the validity of the smart sensors. Finally, using the smart sensors, the minimization of the total acoustic power radiated from a vibrating plate is carried out.

### 2. Background

When considering the active control of structural vibration or structural / acoustic radiation, many of the global (error) criteria of interest can be expressed as quadratic functions of structural velocities or displacements. Included in this set are the criteria of structural kinetic energy, radiated acoustic power, and acoustic potential energy in a volume where the acoustic field is driven by the vibration of some part of the boundary. The general form of these error criteria is (Snyder et al., [9]; Snyder and Tanaka, [8])

$$J = \mathbf{w}^{\mathrm{H}} \mathbf{A} \mathbf{w},\tag{1}$$

where w is an  $(N_m \times 1)$  vector of complex modal displacements, and consideration has been limited to  $N_m$  modes; the superscript <sup>H</sup> refers to Hermitian (transpose conjugate) operation; and the  $(N_m \times N_m)$  weighting matrix, **A**, is usually frequency dependent for acoustic radiation criteria, such as acoustic power, and frequency independent for vibration criteria, such as structural kinetic energy. The error criterion of most interest in this series of papers is radiated acoustic power. If modal displacements are used in the vector w, the terms in the weighting matrix are defined by (Snyder et al.,[7]; Snyder and Tanaka, [7, 8, 9])

$$A_{ij} = \frac{\omega^3 \rho_0}{4\pi} \int_{S} \int_{S} \psi_i(\mathbf{r}_2) \frac{\sin kr}{r} \psi_j(\mathbf{r}_1) \, \mathrm{d}\mathbf{r}_2 \mathrm{d}\mathbf{r}_1, \qquad (2)$$

where  $\psi_i(\mathbf{r})$  is the value of the *i*th mode shape function at location  $\mathbf{r} = (\mathbf{x}, \mathbf{y})$  on the vibrating structure,  $\rho_0$  is the air density,  $\mathbf{r}$  is the distance between the points  $\mathbf{r}_1$  and  $\mathbf{r}_2$ ,  $\mathbf{k}$  is the acoustic wave number at frequency  $\omega$ , and the integrations are over the surface of the vibrating structure. The modal displacement vector  $\mathbf{w}$  is expressed as

$$\mathbf{w}^{T} = [\mathbf{w}_{1}(\mathbf{m}_{1}, \mathbf{n}_{1}), \mathbf{w}_{2}(\mathbf{m}_{2}, \mathbf{n}_{2}), \cdots \mathbf{w}_{N_{m}}(\mathbf{m}_{N_{m}}, \mathbf{n}_{N_{m}})]$$
 (3)  
where  $\mathbf{w}_{i}(\mathbf{m}_{i}, \mathbf{n}_{i})$  represents the *i*th modal amplitude; modes are ordered from the lowest resonance frequency to the highest; and  $\mathbf{m}_{i}$  and  $\mathbf{n}_{i}$  indicate the associated modal indices in the x and y directions, respectively. The weighting matrix **A** in Eq. (1) is typically real and symmetric, and hence can be diagonalized via an orthonormal transformation,

$$\mathbf{A} = \mathbf{Q} \Lambda \mathbf{Q}^{-1},\tag{4}$$

where  $\Lambda$  is the diagonal matrix of eigenvalues of  $\mathbf{A}$ , the subscript  $^{-1}$  denotes the inverse of the matrix, and the columns of the orthonormal matrix  $\mathbf{Q}$  are the corresponding eigenvectors of  $\mathbf{A}$ . Many of frequency dependent weighting matrices  $\mathbf{A}$  have eigenvectors which are approximately frequency independent over the range targeted for active control (Snyder et al.,[7]; Snyder and Tanaka, [8]), and so the frequency dependence can be attributed solely to the eigenvalues. Applying the transformation of Eq. (4) to Eq. (1), the global error criterion can be re-expressed as

$$J = \mathbf{w}^{\mathrm{H}} \mathbf{Q} \Lambda \mathbf{Q}^{-1} \mathbf{w} = \mathbf{u}^{\mathrm{H}} \Lambda \mathbf{u}, \tag{5}$$

where **u** is a transformed modal vector, defined by

$$\mathbf{u} = \mathbf{Q}^{-1}\mathbf{w} = \mathbf{Q}^{\mathrm{T}}\mathbf{w}, \tag{6}$$

where the superscript <sup>T</sup> denotes the transpose of the matrix, and the property  $\mathbf{Q}^{-1} = \mathbf{Q}^{T}$  has been used.

An important property of Eq. (5) is that the error criterion J is now expressed as a weighted summation of the squared amplitudes of the transformed modes,

$$J = \sum_{i=1}^{N_m} \lambda_i | \mathbf{u}_i |^2,$$
 (7)

where  $\lambda_i$  is the eigenvalue associated with the transformed mode  $u_i$ . The error criterion in Eq. (7) was first derived by Borgiotti [10] using the singular value decomposition method, then afterwards by Naghshineh and Koopman [11], Elliott and Johnson [12] and Snyder and Tanaka[8] using the principal transformation of the eigenvectors as shown above. Each transformed mode, which is actually some combination of structural modes (the proportions of which are defined by the associated eigenvector), is an orthogonal contributor to the error criterion of interest. If, for example, the error criterion is radiated acoustic power, the transformed modes each contribute independently to the overall sound power, and so may be viewed as "acoustic power modes" (Snyder et al., [9]; Snyder and Tanaka,[10]) or "radiation modes" (Elliott and Johnson,[12]).

### 3. One-dimensional distributed parameter smart sensors

#### **3.1 Fundamental characteristics**

Consider the one-dimensional PVDF film sensor as shown in Fig. 1, which is attached to a rectangular plate along the y direction with its width varying according to the shaping function  $\varphi(y)$ , and with constant thickness. Then, the sensor output amplitude is written as

$$\widetilde{y} = \Gamma_0 \int_{\gamma - \phi(y)}^{\gamma + \phi(y)} \int_0^{L_y} \left( e_{32} \frac{\partial^2 w(\mathbf{r})}{\partial x^2} + e_{31} \frac{\partial^2 w(\mathbf{r})}{\partial y^2} \right) d\mathbf{r} .$$
(8)

where  $e_{31}$  and  $e_{32}$  are the directional piezoelectric field constant, and  $\Gamma_0$  is the piezolaminar constant. Let the shaping function be described by

$$\varphi(\mathbf{y}) = \mathbf{q}_{n_j} \sin \frac{n_j \pi}{\mathbf{L}_{\mathbf{y}}} \mathbf{y}$$
(9)

nserting Eq. (9) into Eq. (8), the sensor output amplitude is found to be

$$\widetilde{y} = q_{n_j} \sum_{m_j=1}^{m_{n_j}} w_j(m_j, n_j) b_j(m_j, n_j) c_j(m_j)$$
 (10)

where the number  $m_{nj}$  shown in Eq.(10) is the total number of vibration modes with the modal index  $n_i$  in the y direction in the frequency range of interest;  $b_i(m_i, n_i)$  is defined as

$$\mathbf{b}_{j}(m_{j}, n_{j}) = \mathbf{L}_{y} \Gamma_{0} \left( \mathbf{e}_{32} (\frac{m_{j} \pi}{\mathbf{L}_{x}})^{2} + \mathbf{e}_{31} (\frac{n_{j} \pi}{\mathbf{L}_{y}})^{2} \right) , \qquad (11)$$

where j denotes the mode number while  $m_j$  and  $n_j$  are the corresponding modal indices; and  $c_j(m_j)$  is defined as

$$c_j(m_j) = \sin \frac{m_j \pi}{L_x} \gamma.$$
(12)

where  $\gamma$  is the location of the sensor in the x direction. In the derivation process of Eq. (10), the

following approximation is used:

$$\varphi(\mathbf{y}) = \sin(\frac{m_j \pi}{L_x} \mathbf{q}_{n_j} \sin \frac{n_j \pi}{L_y} \mathbf{y}) \cong \frac{m_j \pi}{L_x} \mathbf{q}_{n_j} \sin \frac{n_j \pi}{L_y} \mathbf{y} \quad \text{if } \frac{m_j \pi}{L_x} \mathbf{q}_{n_j} <<1.$$
(13)

The important point to note in Eq.(10) is that the characteristics of modal orthogonality can not be applied in both the x and y directions as was done for the two-dimensional PVDF film sensor case, but merely in one direction (the y direction in this case). Thus, the one-dimensional PVDF film sensor shaped by the function Eq. (9) has the potential to measure the amplitude of all modes with a given modal index in one direction. Therefore, further work is required to enable the sensing system to isolate the measurement of individual two-dimensional modes.

#### 3.2 Smart sensing system methodology

It is worthwhile summarizing the fundamental characteristics of one-dimensional PVDF film sensors as follows:

**Point 1;** Attention should be paid to the form of the PVDF film sensor output shown in Eq. (10), expressed as the superposition of the terms in which each modal amplitude is being multiplied by an appropriate coefficient. This means the output of the PVDF film sensor already contains a series of modal amplitudes in itself. Therefore, the estimation of the modal amplitudes is not necessary; instead, adjusting the coefficients of the shaping function is needed to make the output of the PVDF film sensor equal to the transformed modal amplitude.

**Point 2;** Utilization of one-dimensional PVDF film sensors for the purpose of measuring the transformed mode turns out to be a necessary condition, and not a necessary and sufficient condition. To cope with this problem, multiple one-dimensional PVDF film sensors must be used.

**Point 3;** One may ask how many one-dimensional PVDF film sensors are needed to obtain the transformed mode uniquely. To answer this question, we need to get back to Eq. (10) describing

the PVDF film sensor output. The number  $m_{nj}$  denotes the number of the vibration modes filtered out by a single shaping function (or a basis function) described in Eq. (9). Supposing that the transformed mode in the frequency band of interest consists of *m* basis functions in the y direction, for instance, and each function filters

out  $m_{nj}$  vibration modes, then  $\sum_{j=1}^{m} m_{nj}$  is the total

number of vibration modes which a single one-

dimensional PVDF film sensor shaped with a

series of the basis functions will scoop out.

Here, an important point to note arises; a series of basis functions can be superimposed onto a single one-dimensional PVDF film sensor, and so the necessary number of one-dimensional PVDF film sensors to determine the transformed mode is

 $m_{max} = \max(m_{nj} \mid j = 1, 2, 3, \cdots m)$  (14)

which is the maximum number of modes included

in any of the basis functions, and is not the total



Fig. 1 Shaping of one-dimensional smart sensor

number of measured vibration modes,  $\sum_{j=1}^{m} m_{nj}$ , which makes it possible to reduce significantly the

required number of the one-dimensional PVDF film sensors. As seen from Eq. (10), the introduction

of  $m_{max}$  one-dimensional PVDF film sensors attached at different locations means the introduction of  $m_{max}$  equations. Therefore, the coefficient of the basis function which has the maximum number of modes,  $m_{max}$ , can be solved with the same number,  $m_{max}$ , of equations. So can other coefficients of the basis functions be done as there are a sufficient number of the equations.

**Point 4;** Shaping a PVDF film with appropriate fundamental functions is equivalent to using point sensors together with a large number of read, multiply, and addition operations in the associated digital signal processing. In other words, the output of the smart sensors, *per se*, is the desired error signal for the controller; thus, the direct-sensing of acoustic power modes becomes possible, with a corresponding large reduction in the digital signal processing requirement.

**Point 5**; By using the symmetry of the vibration modes for placement of the PVDF film sensors, the burden of calculating the shaping function will be reduced.

#### 3.3 Design procedure for one-dimensional modal sensors

Taking into consideration the points described above, the design procedure for the one-dimensional PVDF smart sensors is outlined in Fig. 2, which consists of eight steps, beginning with the determination of the frequency range of interest to finally determining the coefficients of the shaping function. At Step 7, a count of the necessary number of one-dimensional smart sensors is indicated. These steps will be demonstrated here for the problem of resolving "acoustic power modes" on a rectangular plate in two frequency ranges. However, the same steps could be used for other structures of regular geometry, such as circular plates.

Illustrated in the top in Fig. 3 are the shaping functions of the smart sensors for measuring the first acoustic power mode (j = 1). In exactly the same way, the design procedures for the odd/ even, even/odd and even/even modes can be carried out, and these shaping functions for the second (odd/even modes), the third (even/odd modes) and the fifth (even/even modes) acoustic power modes are also depicted in Fig. 3. Note that the fourth acoustic power mode falls into odd/odd modes, so that the shaping function of the fourth acoustic power mode is skipped in Fig. 3.

It should be noted that a combination of the seven odd/odd modes with an appropriate coefficient on each mode can be sensed with only three one-dimensional PVDF film sensors; the signal obtained by simply combining these three sensor outputs electrically resolves the desired odd/odd transformed modal amplitude, thereby eliminating numerical processing.



Fig. 2 Design procedure of the onedimensional smart sensors to measure the acoustic power mode

### 4. Experiment

The specification of PVDF film sensor used in experiment is as follows: the thickness of the PVDF film is 25  $\mu$ m; the aluminum electrode of thickness of 60 nm is attached to both sides of the PVDF film; and the piezoelectric field intensity constants  $e_{31}$  and  $e_{32}$  are 0.0105 C/m<sup>2</sup> and 3.5 x 10<sup>-4</sup> C/m<sup>2</sup>, respectively. A steel test panel measuring 88 cm x 1.8 m x 9.0 mm was supported on knife edges fixed to the perimeter of an enclosure with ferroconcrete walls of 10 cm thickness. Inside the enclosure covered with absorbent material, an electro-dynamic shaker located at x =0.645 m, y = 1.451 m was installed to excite the plate, which was mounted on the bottom of the enclosure with anti-vibration rubber pads

Figure 4 shows a schematic diagram of adaptive feedforward control system that has been installed in the experimental rig. The outputs of the smart sensors go into the adaptive feedforward controller through weighting filters, providing to the controller as an error signal.



Fig. 3 Shaping functions of the smart sensors for odd/odd mode, odd/even mode, even/odd mode and even/even mode designe in the frequency range up to 500Hz

The driving point inertance, shown as the top curve in Fig.5, was obtained by the disturbance shaker driven by a broadband white noise between 10 and 200 Hz. As seen from the figure, there are seven vibration modes from the (1.1)to the (2,3) mode. One-dimensional smart sensors for measuring the first, second and third acoustic power modes were bonded to the plate. The smart sensor for the first acoustic power mode was attached along the central line of the plate in the y direction, while another single onedimensional smart sensor for the second acoustic power mode was attached along the central line of the plate in the y direction, and a pair of onedimensional smart sensors for the third acoustic power mode, were attached at  $x = (1/3)L_x$  and  $(2/3)L_x$ . To implement the system, PVDF film strips with a maximum width of 7mm were used. The negative value of the shaping function was realized in practice by reversing the polarity of the sensor, by mounting it upside down.



Schematic diagram of adaptive Fig. 4 feedforward control system for suppressing radiated acoustic power from a vibrating plate

Illustrated in the second through fourth of the curves from the top in Fig.5 are the spectrums of the smart sensor outputs for the first through third acoustic power modes. For comparison, the numerical results of the power mode frequency characteristics are also given in Fig. 6. For the first acoustic power mode sensor, the (1,1) and (1,3) modal resonances are dominant, which is the desired response of the sensor. Due to shaping errors of the shaping function on the PVDF film strips, effects of the (1,2) and (1,4) modes which are undesired are also present in the experimental results; however, these levels are negligible; approximately 30dB below the amplitude of the dominant modes. With the second acoustic power mode sensor, the desired (1,2) and (1,4) mode appears; at a level 40dB below the amplitude of the desired (1,2) mode. As to the third acoustic power mode sensor, the (2,1) and (2,3) modes are clearly dominant, which is the desired feature. As a whole, a good agreement can be found in the experimental and analytical results.

Three-inputs one-output adaptive feedforward control system was formulated in the experimental devices. Figure 7 shows the control effect on the suppression of the acoustic power in the frequency range up to 200Hz, showing that the attenuation obtained in each vibration mode is 16 dB for (1,1) mode, 11 dB for (1,2) mode, 13 dB for (1,3) mode, 5 dB for (2,1) mode and 5dB for (2,3) mode. Thus the practicability of the smart sensors is experimentally verified.



Fig. 5 Spectrum of smart sensor outputs (Experimental results)

### 5. Conclusions

By considering the impracticability of two-dimensional sensors, an alternative modal filtering approach based upon onedimensional smart sensors has been presented. It was found that this approach satisfies the necessary condition for sensing the required transformed acoustic power mode. This was done by introducing multiple one-dimensional smart sensors attached at appropriate locations to the plate



Fig. 6 Spectrum of smart sensor outputs (Numerical results)

structure. A design procedure was established, which gives the required number as well as appropriate locations for the one-dimensional smart sensors for measuring the transformed acoustic power modes. Finally, an experiment was conducted, demonstrating the validity of the one-dimensional smart sensors in sensing the required transformed modes.

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