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Invited Paper

MODAL FILETERING AND CONTROL OF A PLATE USING ONE-DIMENSIONAL PVDF FILM SENSORS

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Abstract
This paper concerns distributed parameter sensors designed to extract a vibration mode of a structure, as well as its active modal control. Compared to conventional point sensors, the distributed parameter sensors are superior to point sensors provided that they are properly designed. Firstly, taking into consideration the applicability of the sensors, a design procedure for modal filtering using one-dimensional sensors is presented, the number, location and shaping functions for the one-dimensional modal sensors being clarified. Furthermore, the modal filtering using the one-dimensional sensors is found to be applicable for two dimensional structures with any classical boundary condition. Experimental results show the capability of modal filtering even for a degenerate mode of a structure. Finally, using the modal sensors, experiments on active modal control are conducted, showing a significant suppression of the targeted mode without causing instability of the control system.

1. Introduction

Regardless of what control theory is employed, the essential conception of active vibration control relies upon modal control [1~3], which is defined as control that changes the modes of a targeted structure to achieve the desired control objectives. Rosenbrock [1] appears to have been the first to propose modal control of a process, the concept being extended by Murray-Lasso [2], Gould [3] and Schlaefer [4] to include linear distributed-parameter systems.

Difficulty in modal control lies in the tremendous number of sensors required to construct a modal filter, which is indispensable for modal control. When reviewing the reports on modal filtering, most papers would be dependent on the utilization of point sensors for implementing a modal filter, mainly due to the availability of only point-type sensors at that time. There are, however, practical difficulties in implementing such a point measurement-based modal sensing system for real problems. These problems arise from the number of measurements which are required to resolve modes with adequate fidelity. The other way to reduce the burden is to exchange point sensors for distributed parameter sensors[5~8] such as PVDF film sensors. The distributed parameter sensor can be viewed as the limit of an infinite number of point sensors, thereby reducing a large number of point sensors. If it is possible to use distributed parameter sensors, the control information of a vibration field can be obtained not from points, but from lines or areas. As a result of the enhancement of observability, global attenuation of the vibration level over the structure can theoretically be achieved with observation spillover[9] being avoided.

The application of PVDF film sensors can be traced back to Lee and Moon[10] for measuring modal amplitudes. The applications of these sensors for modal filtering thereafter have been limited to one-dimensional structures and cantilever plates. Although there have been some attempts [11] to apply the distributed sensors to a two-dimensional structure, the systematic design procedure for modal filtering of a two-dimensional structure using the distributed parameter sensors has yet to be clarified.

It is the purpose of this paper to present the design methodology of a PVDF film sensor in view of filtering out a modal parameter of a two-dimensional planar structure. First, this paper overviews a conventional modal filter method using point sensors, and points out the problems they involve. To overcome the drawbacks of the point sensor-based modal filter, a novel modal filtering technique based upon a distributed parameter PVDF film sensor is proposed. To begin with, a design procedure for one-dimensional sensors for modal filtering is presented, the number, location and shaping function of the one-dimensional modal sensors clarified. Then, modal filtering using the one-dimensional shaped sensors is found to be applicable for two dimensional structures with any classical boundary conditions. Experimental results show the capability of modal filtering even for a degenerate mode. Finally, using one-dimensional modal sensors, experiments on active modal control are conducted, showing a significant reduction of the targeted modal amplitude without causing instability of the control system.

2. One-dimensional distributed parameter modal sensors

A. General case

In the interest of filtering out a particular modal amplitude, consider the one-dimensional PVDF film sensor of constant thickness, which is attached to a rectangular plate at $x = x_0$ along the y direction with its width varying according to a shaping function s(y). Then, the sensor output amplitude becomes

$$\tilde{q} = \int_0^{L_y} \int_{x_0}^{x_0 + s(y)} \left(e_{31} \frac{\partial^2 w(\mathbf{r})}{\partial x^2} + e_{32} \frac{\partial^2 w(\mathbf{r})}{\partial y^2} \right) d\mathbf{r} \quad . \tag{1}$$

where $w(\mathbf{r})$ denotes the deflection of a plate, and is described as

$$w(\mathbf{r}) = \sum_{i=1}^{\infty} w_i \varphi_i(\mathbf{r}) \tag{2}$$

where $\varphi_i(\mathbf{r})$ and w_i are the i-th eigenfunction and modal amplitude, respectively. Inserting Eq.(2) into Eq.(1) and using the following assumptions on the eigenfunctions,

$$\frac{\partial^2 \varphi_i(\mathbf{r})}{\partial x^2} = -\alpha_i^2 \varphi_i(\mathbf{r}) \tag{3}$$

and

$$\frac{\partial^2 \varphi_i(\mathbf{r})}{\partial y^2} = -\beta_i^2 \varphi_i(\mathbf{r}) \ , \tag{4}$$

the sensor output becomes

$$\tilde{q} = \int_0^{L_y} \int_{x_0}^{x_0 + s(y)} \sum_{i=1}^{\infty} b_i w_i \varphi_i(\mathbf{r}) d\mathbf{r} \quad . \tag{5}$$

Consider the extraction of the s-th modal amplitude, w_s. For this purpose, the shaping function is needed to be defined as follows:

$$s(y) = a_s \varphi_s(x_0, y), \qquad a_s << 1 \ . \tag{6}$$

Equation (5) is further expanded to

$$\begin{aligned} \tilde{q} &= \int_0^{L_y} \int_{x_0}^{x_0 + a_s\varphi_s(x_0, y)} \sum_{i=1}^{\infty} b_i w_i \varphi_i(\mathbf{r}) d\mathbf{r} \\ &\doteq \int_0^{L_y} a_s \varphi_s(x_0, y) \sum_{i=1}^{\infty} b_i w_i \varphi_i(x_0, y) dy \\ &= a_s \sum_{i=1}^{\infty} b_i w_i \int_0^{L_y} \varphi_s(x_0, y)\, \varphi_i(x_0, y) dy \end{aligned} \tag{7}$$

To proceed with the further expansion of Eq.(7), consider the integral

$$\hat{q} = \int_0^{L_y} \varphi_s(x_0, y)\, \varphi_i(x_0, y) dy \ . \tag{8}$$

Multiplying Eq. (8) by $-\beta_i^2$ and applying the properties of the eigenfunctions given by Eq. (4), Eq. (8) may be expanded to

$$\begin{aligned} -\beta_i^2 \int_0^{L_y} \varphi_s(x_0, y)\, \varphi_i(x_0, y) dy &= \int_0^{L_y} \varphi_s(x_0, y) \frac{\partial^2 \varphi_i(x_0, y)}{\partial y^2} dy \\ &= \varphi_s(x_0, y) \frac{\partial \varphi_i(x_0, y)}{\partial y} \bigg|_0^{L_y} - \frac{\partial \varphi_s(x_0, y)}{\partial y} \varphi_i(x_0, y) \bigg|_0^{L_y} \\ &+ \int_0^{L_y} \frac{\partial \varphi_s^2(x_0, y)}{\partial y^2} \varphi_i(x_0, y) dy \end{aligned} \tag{9}$$

Similarly, multiplying Eq. (8) by $-\beta_s^2$ leads to

$$\begin{aligned} -\beta_s^2 \int_0^{L_y} \varphi_s(x_0, y)\, \varphi_i(x_0, y) dy &= \int_0^{L_y} \varphi_i(x_0, y) \frac{\partial^2 \varphi_s(x_0, y)}{\partial y^2} dy \\ &= \varphi_i(x_0, y) \frac{\partial \varphi_s(x_0, y)}{\partial y} \bigg|_0^{L_y} - \frac{\partial \varphi_i(x_0, y)}{\partial y} \varphi_s(x_0, y) \bigg|_0^{L_y} \\ &+ \int_0^{L_y} \frac{\partial \varphi_i^2(x_0, y)}{\partial y^2} \varphi_s(x_0, y) dy \end{aligned} \tag{10}$$

When applying the classical boundary conditions, the boundary terms in Eqs. (9) and (10) vanish at $y = 0$ and L_y. Therefore, Eqs. (9) and (10) may be simplified into

$$-\beta_i^2\int_0^{L_y}\varphi_s(x_0,y)\,\varphi_i(x_0,y)dy=\int_0^{L_y}\varphi_s(x_0,y)\,\frac{\partial^2\varphi_i(x_0,y)}{\partial y^2}dy \tag{11}$$

$$=\int_0^{L_y}\frac{\partial\varphi_s^2(x_0,y)}{\partial y^2}\,\varphi_i(x_0,y)dy$$

and

$$-\beta_s^2\int_0^{L_y}\varphi_s(x_0,y)\,\varphi_i(x_0,y)dy=\int_0^{L_y}\varphi_i(x_0,y)\,\frac{\partial^2\varphi_s(x_0,y)}{\partial y^2}dy$$

$$=\int_0^{L_y}\frac{\partial\varphi_i^2(x_0,y)}{\partial y^2}\,\varphi_s(x_0,y)dy \tag{12}$$

Note that the right hand sides of Eqs. (11) and (12) are equal, so that subtraction of Eq. (12) from Eq. (11) gives

$$\left(\beta_s^2-\beta_i^2\right)\int_0^{L_y}\varphi_s(x_0,y)\,\varphi_i(x_0,y)dy=0\ . \tag{13}$$

For $\beta_s^2 \neq \beta_i^2$,

$$\int_0^{L_y}\varphi_s(x_0,y)\,\varphi_i(x_0,y)dy=0\ . \tag{14}$$

For $\beta_s^2 = \beta_i^2$,

$$\tilde{q}=w_s b_s c_s(x_0)\ , \tag{15}$$

where c_s is a constant of proportionality.

B. Numerical example of a simply supported plate

Consider a simply supported, thin, lossless, rectangular plate of the length L_x (= 0.88 m) and L_y (= 1.8 m) in the x and y directions, respectively, and of the thickness h (= 9mm). Note that the design methodology presented can also be applicable to a rectangular plate with other classical boundary conditions.

In viewing Table 1 listing the modal indices and modal indices of the panel, a numerical example of extracting the (1,4) modal amplitude of the plate using the one-dimensional modal sensors is shown.

Mode number	Mode	Modal frequency Hz	Mode number	Mode	Modal frequency Hz
1	(1,1)	35.292	12	(3,2)	283.588
2	(1,2)	55.716	13	(2,5)	284.136
3	(1,3)	89.756	14	(3,3)	317.628
4	(2,1)	120.744	15	(2,6)	359.024
5	(1,4)	137.412	16	(1,7)	362.077
6	(2,2)	141.168	17	(3,4)	365.284
7	(2,3)	175.208	18	(3,5)	426.556
8	(1,5)	198.684	19	(2,7)	447.529
9	(2,4)	222.864	20	(4,1)	462.551
10	(3,1)	263.164	21	(1,8)	464.197
11	(1,6)	273.573	22	(4,2)	482.975

Table 1 Modal frequency and modal number of a simply supported plate of 0.88m x 1.8m x 9mm

I. Direction of the one-dimensional sensor to be placed

As seen in Table 1, there are 22 structural modes involved in the frequency range up to 500 Hz; Of 22 modes, the (1,4) modal amplitude is designed to be extracted. The natural frequency of the (1,4) mode is 137 Hz, with the closest mode, the (2,2) mode with a natural frequency of 141 Hz, the difference between these modal frequencies being only 4 Hz, and hence the (1,4) mode may be identified as an almost degenerate mode.

As the modal index in the x direction of the (1, 4) mode is 1, eight modes are found to have it in the x direction, e.g. (1,1), (1,2), (1,3) etc. On the other hand, there are three structural modes possessing the modal index, 4, in the y direction. As such, it is obviously advantageous to use the y direction for sensor placement, as the necessary number of the sensors is determined by the number of vibration modes extracted by a one-dimensional sensor.

II. Shaping Function

The shaping function described in Eq.(6) may be given by

$$\varphi(y) = a_5 \sin \frac{4\pi}{L_y} y \,. \tag{16}$$

The number of structural modes extracted by the sensor shaped with Eq. (16) is 3; the (1,4), (2,4), and (3,4) mode with the modal number of 5, 9 and 17, respectively. By using these numbers, the sensor output is described by

$$\tilde{q}_1 = w_5 b_5 c_5^1 + w_9 b_9 c_9^1 + w_{17} b_{17} c_{17}^1 \,. \tag{17}$$

III. Number of the necessary sensors

Among the three extracted structural modes given by Eq. (17), the unwanted mode, the (3,4) mode, with the modal number of 17 has two nodal lines in the y direction. Moreover, considering that three modes are filtered out at this stage, the necessary number of the sensors can be reduced to 2 by placing the sensors along the nodal lines of the (3,4) mode.

IV. Extraction of modal amplitudes

The amplitudes of the two sensor outputs are given by

$$\tilde{q}_1 = w_5 b_5 c_5^1 + w_9 b_9 c_9^1 \tag{18}$$

and

$$\tilde{q}_2 = w_5 b_5 c_5^2 + w_9 b_9 c_9^2 \tag{19}$$

Because of the symmetric location of the two sensors with regard to the central line of the plate, the following relations are obtained,

$$c_5^1 = c_5^2 \tag{20}$$

and

$$c_9^1 = -c_9^2 \,. \tag{21}$$

By inserting Eqs.(20) and (21) into Eqs.(18) and (19), and further expanding the equations, the modal amplitude of the (1,4) mode, w_5, is found to be

$$w_5 = \frac{1}{2 b_5 c_5^1} (\tilde{q}_1 + \tilde{q}_2) \tag{22}$$

By adding the two sensor outputs with opposite polarity, the (2,3) modal amplitude, w_7, can also be obtained by

$$w_9 = \frac{1}{2b_5c_5^1}(\tilde{q}_1 - \tilde{q}_2) \qquad (23)$$

3. Modal filtering/control of a rectangular plate

A schematic diagram for extracting the (1,4) modal amplitude using PVDF film sensors is shown in Fig. 1. The polarity of the film sensor is changed by swapping the top and bottom surface of a sensor, each segment of the sensor connected by a conductive foil. Figure 2 shows the spectrums of the impact force used as disturbance, together with the acceleration response of the panel obtained at the driving point; the corresponding waveforms of the impact force as well as the acceleration are depicted in Figs. 3(a) and (b). As is seen from the frequency characteristics of the impact force, it rolls off at 500 Hz, thereby the maximum frequency range of interest determined as 500 Hz.

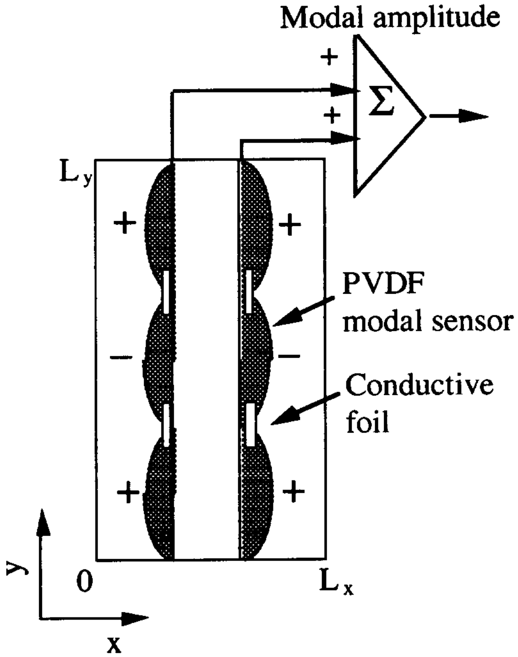

Fig. 1 Schematic diagram for measuring modal amplitude using PVDF modal sensors

As shown in Fig.1, a pair of PVDF film sensors shaped with the function given by Eq.(16) are attached along the two nodal lines of the (3,4) mode in the y direction. Each one-dimensional modal sensor consists of four segments with a half period of the sine function, these segments being connected by conductive foils. When combining these two sensor outputs with the same polarity, the (1,4) modal amplitude may be filtered out, while, with the opposite polarity, the (2,4) modal amplitude extracted. Note the combined sensor output *per se* is the desired modal signal, hence no further signal processing required.

The waveform of an impact response of the extracted (1, 4) modal amplitude is shown in Fig. 3(d), the corresponding frequency characteristics given in Fig. 4. As is seen in Fig. 3(d), a sinusoidal signal with slight damping appears which is filtered out from the turbulent acceleration waveform illustrated in Fig. 3(b). It should be noted that even with a narrow bandpass filter tuned to the targeted natural frequency (the (1,4) modal frequency), a similar waveform may be obtained; however, intrinsic difference between these similar waveforms is whether the dynamic characteristics of the targeted mode are acquired or not. In other words, as is found in Fig. 4, the signal obtained by the modal filter carries the dynamic characteristics of the (1,4) mode in the whole range of frequency (which can be viewed as a single degree of freedom vibratory system), while the signal obtained by the narrow bandpass filter does not.

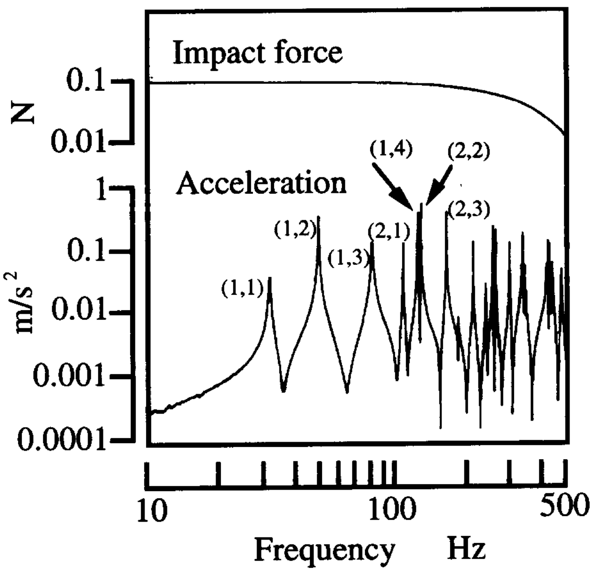

Fig. 2 Spectrum diagram of an impact force and acceleration response at a driving point

If one attempts to extract the (1,4) modal amplitude by using point sensors, at least 50 to 60 sensors, a few times the modal number of interest, should be necessary to ensure the fidelity. In addition, a digital signal processing for calculating an inverse matrix, multiplication and addition of signals are needed, thereby making it almost impossible to use in control systems, because the computation time to produce a control signal cannot be allowed. Thus, a heavy burden is imposed on the use of the conventional modal filtering technique based upon point sensors. In contrast to this, the modal filtering method using distributed sensors can be conducted by merely shaping its width with an appropriate shaping function, which is equivalent to spatial integration, read, addition and multiplication in digital signal processing; hence no calculation time.

To carry out the active modal control, the signal obtained by combining the two modal sensors is needed to be fed back to drive a control actuator. To dampen a resonance peak, velocity feedback is needed; however, the sensor output is proportional to the modal amplitude of displacement, and therefore an approximate differential circuit required.

Figure 3 (e) shows the waveforms of the (1,4) mode suppressed by the active modal control using the distributed modal sensors. The optimal velocity feedback gain was determined experimentally by monitoring the waveforms being suppressed. Because of the control system comprising distributed sensors and a point actuator, the observation spillover leading to a destabilization of the control system may be avoided. Throughout the experiment, it was possible to increase the velocity feedback gain to a significantly high level; the excess amount of feedback gain caused instability of the system due to the saturation of a power amplifier, though. Regarding the waveforms of the acceleration response before and after the active modal control as shown in Figs. 3 (b) and (c), it is hard to recognize the difference in those waveforms, although the (1,4) modal signal was thoroughly eliminated in the uncontrolled acceleration waveform. With this, it is seen that the active modal control was achieved without causing any effect on other structural modes.

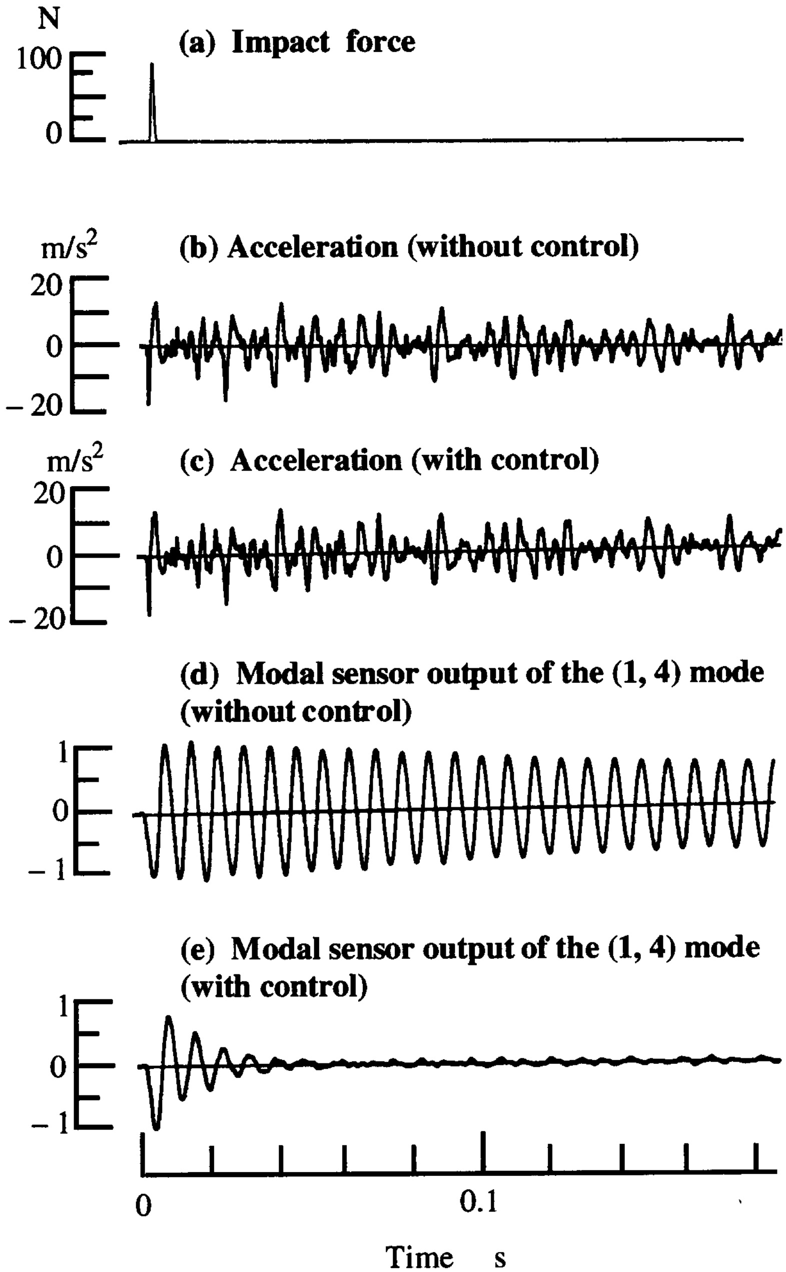

Fig. 3 Time history of an impact force, acceleration waves and modal filter outputf the (1,4) mode

4. Conclusions

The design of one-dimensional shaped PVDF film sensors to provide the modal amplitude of interest has been considered. It was shown that the modal orthogonality of one-dimensional distributed sensors can be applicable for a two-dimensional planar structure with any classical boundary condition. The design procedure for one-dimensional modal sensor has been presented, the number, location and shaping function of the one-dimensional sensor being clarified. Experimental results using one-dimensional modal sensors show that significant accuracy for modal filtering can be attained. The (1,4) mode has been picked up, which is regarded as an almost degenerate mode. It was found that the modal filtering works even on a degenerate mode. The active modal control was performed, showing the significant control effect for suppressing the targeted modal resonance without causing instability of the control system.

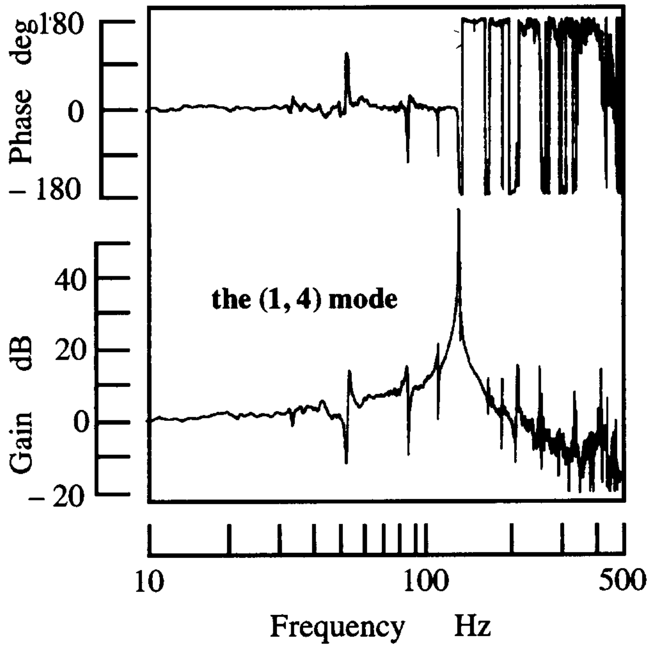

Fig. 4 Modal amplitude characteristics of the (1,4) mode

5. References

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