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QUANTIFICATION OF INFLOW TURBULENCE FOR PREDICTION OF CASCADE BROADBAND NOISE

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The problem of broadband noise generated by turbulence from a rotor impinging on a downstream stator is examined from a theoretical viewpoint. Equations are derived giving sound power spectra in terms of the 3 dimensional turbulence wavenumber spectrum. Particular attention is given to issues of turbulence inhomogeneity related to separation of individual blade wakes in the near field of the rotor. It is shown that this inhomogeneity can be handled rigorously with no additional complexity in the noise equations. A procedure for measurement of turbulence at a stator inlet with a 2-probe data system is studied. In the process, formulas are derived to compute the 3D turbulence spectrum via transforms of measured cross-spectra and estimates are given for the required probe spacing.

INTRODUCTION

Figure 1 represents the side view of a turbofan with an upstream rotor and downstream stator. Non-uniform flow in the rotor wake that impinges on the stator generates much of the noise radiated by the engine. The wake includes a periodic component, which generates harmonic noise, and a random component, which generates broadband noise. A great deal of effort has been invested in the past on modeling and reducing the harmonic noise. Because of some success in this area, this paper focuses on the broadband component. We consider the random flow to be turbulence (ignoring any random instability effects). Note from the sketch in Figure 2 that the turbulence in the rotor exit flow is highly inhomogeneous because the blade wakes are not mixed and fully merged with each other. This inhomogeneity has inspired various heuristic treatments of stator inflow dating back 25 years (e.g. ref. 1).

In this paper we develop a model for noise generation at the stator treating turbulence via 3 dimensional spectra and covariance functions. The model is used to examine 2 important issues related to noise prediction and related experimental work. The first issue is how to accommodate turbulence inhomogeneity rigorously in a noise theory. The second issue is

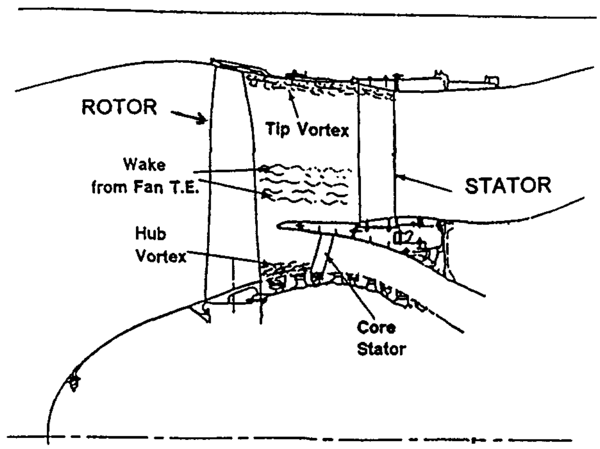

Figure 1. Noise generation in a turbofan by turbulence in rotor wakes impinging on stator.

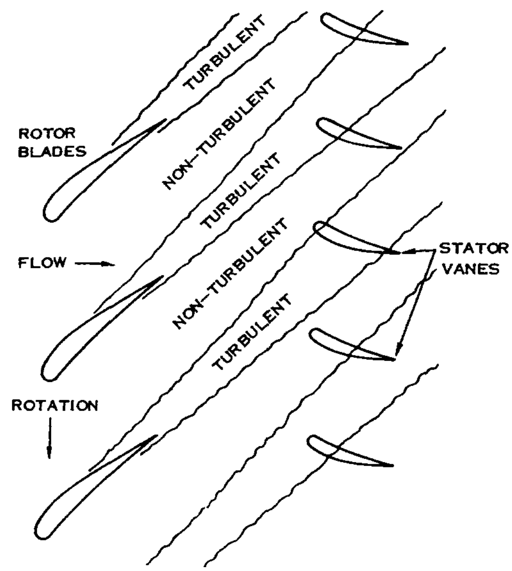

Figure 2. Inhomogeneity of rotor wake at entrance to stator (Hanson, ref.1).

how to measure the wavenumber/frequency spectrum of turbulence at the stator inlet for noise prediction or for theory verification.

BACKGROUND ON GLEGG'S HARMONIC CASCADE THEORY

Most previous noise models for turbulence/stators interaction have been based on some simplifying assumptions that are no longer necessary. Some examples: treatment of the stator vanes as isolated airfoils (as opposed to cascades), 2 dimensional theory, incompressible flow for airfoil loading response, and compact (as opposed to distributed) radiation theory for the acoustic sources. A recent analysis by Glegg[ref 2] overcomes most of these limitations but still is tractable enough for calculations on ordinary computers. As shown in Figure 3, geometry and flow are constant in the z direction. Airfoils are unloaded flat plates. The unsteady flow is harmonic in space and time with upwash given by Equation 1

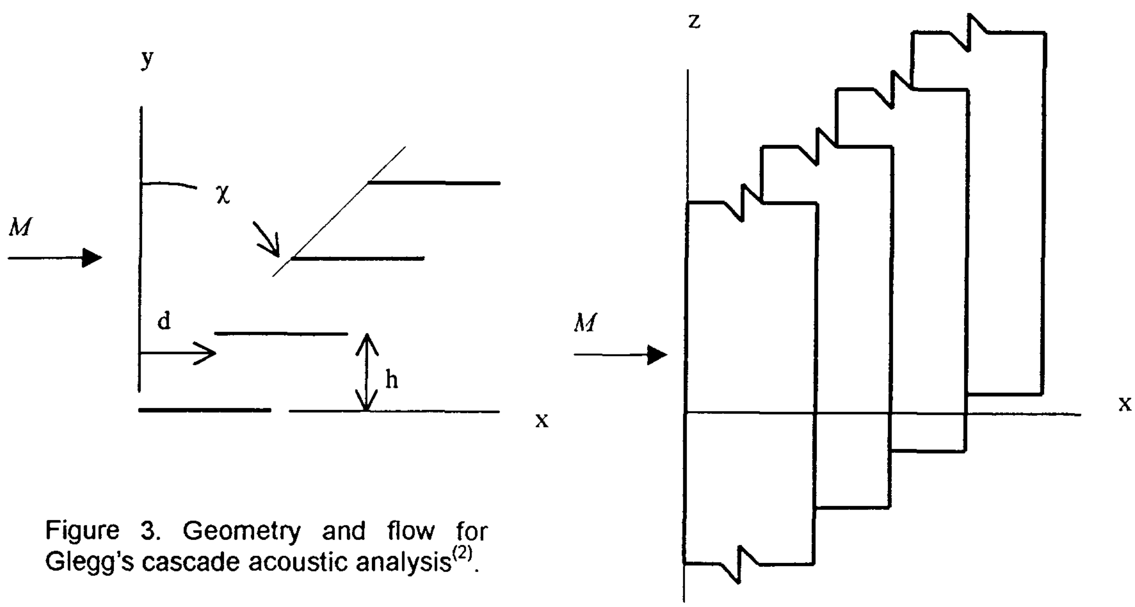

Figure 3. Geometry and flow for Glegg's cascade acoustic analysis[2].

$$w(\mathbf{x},t) = w_o e^{i(\gamma_o x + \alpha y + \nu z - \omega t)} \tag{1}$$

This represents a plane wave that is harmonic in time with complex upwash amplitude w_o. It is also harmonic in space with x, y, and z wavenumbers equal to γ_o, α, and ν. Through an intricate Wiener-Hopf analysis, Glegg derived the following form for the scattered acoustic waves

$$\phi^{\pm}(\mathbf{x},t) = \pm \frac{\pi w_o c^2}{\beta S_e} \sum_{k=-\infty}^{\infty} \frac{\zeta_k^{\pm} D(\lambda_k^{\pm})}{\sqrt{\kappa_e^2 - f_k^2}} e^{i[-\lambda_k^{\pm}(x - yd/h) + (\sigma - 2\pi k) y/h + \nu z]} e^{-i\omega t} \tag{2}$$

where

$$M = U/a \qquad \beta = \sqrt{1 - M^2} \tag{3}$$

$$S_e = \sqrt{d^2 + \beta^2 h^2} \qquad \tan \chi_e = d/\beta h \qquad \lambda_k^{\pm} = \kappa M + \eta_k^{\pm} \tag{4}$$

$$\zeta_k^{\pm} = \beta \sqrt{\kappa_e^2 - (\eta_k^{\pm})^2} \qquad \kappa_e^2 = \kappa^2 - (\nu/\beta)^2 \qquad \kappa = \omega/(a\beta^2) \tag{5}$$

$$\eta_k^{\pm} = -f_k \sin \chi_e \pm \cos \chi_e \sqrt{\kappa_e^2 - f_k^2} \qquad f_k = (\sigma - 2\pi k + \kappa M d)/S_e \tag{6}$$

and $\sigma = \gamma_o d + \alpha h$ is the interblade phase angle. D is the Fourier transform of the discontinuity in potential across the blade and wakes (in the form of an infinite product) and is the major result of Glegg's derivation. The other field variables can be obtained from Equation 2 via

$$\mathbf{u} = \nabla \phi \qquad p = -\rho_o D\phi/Dt \qquad \rho' = p/a^2 \tag{7}$$

where ρ_o and a are the ambient density and speed of sound. The formulation above gives the acoustic waves scattered by a cascade for a single planar wave input. The scattering index k runs over an infinite range but, as usual in this kind of formulation, only a finite number of waves are cut on (propagate undiminished); the remaining waves decay exponentially and, thus, carry no energy. Cuton is governed by the argument of the square root $\sqrt{\kappa_e^2 - f_k^2}$. When the frequency is high enough, the argument is positive and the waves are cut on. The exponential dependence on the space and time variables in Equation 2 permits treatment of any inflow field via Fourier transform methods, as shown in the following section.

GENERALIZATION FOR RANDOM INFLOW

Glegg's formulation above was written for 3D planar, harmonic waves. His theory can be extended to any input waveform via the Fourier transform

$$w(\mathbf{x},t) = \iiiint W(\mathbf{K},\omega) e^{i(\mathbf{K}\cdot\mathbf{x} - \omega t)} d\mathbf{K}\, d\omega \tag{8}$$

where here, and throughout this paper, the integration limits are from $-\infty$ to ∞ unless otherwise specified. Vector wavenumber $\mathbf{K}$ is shorthand for (γ_o, α, v). For later use, the inverse of Equation 8 is

$$W(\mathbf{K},\omega) = \frac{1}{(2\pi)^4} \iiiint w(\mathbf{x},t) e^{-i(\mathbf{K}\cdot\mathbf{x}-\omega t)} d\mathbf{x}\, dt \tag{9}$$

For this kind of analysis, the source function w is considered to be non-zero for a large, but finite, amount of fluid volume and for the associated time, T, required for the flow to pass the cascade at speed U. This insures convergence of all the integrals in the following derivation. With the application of Equation 9 to Equation 2, Glegg's potential generalizes immediately to

$$\phi^{\pm}(\mathbf{x},t) = \int \Phi^{\pm}(\mathbf{x},\omega) e^{-i\omega t} d\omega \tag{10}$$

where

$$\Phi^{\pm}(\mathbf{x},\omega) = \pm \frac{\pi c^2}{\beta S_e} \int W(\mathbf{K},\omega) \sum_{k=-\infty}^{\infty} \frac{\zeta_k^{\pm} D(\lambda_k^{\pm})}{\sqrt{\kappa_e^2 - f_k^2}} \times e^{i[-\lambda_k^{\pm}(x-yd/h)+(\sigma-2\pi k)y/h+vz]} d\mathbf{K} \tag{11}$$

We are interested in computing the spectrum of sound power scattered by the cascade. The starting point for this is the expression for the acoustic energy flux vector applicable to waves in a uniformly moving medium with mean properties given by the density ρ_o, speed of sound a, and velocity $\mathbf{U}$ (ref. 3)

$$\mathbf{I} = \left(\frac{p}{\rho_o} + \mathbf{U}\cdot\mathbf{u}\right)(\rho_o \mathbf{u} + \mathbf{U}\rho') \tag{12}$$

This is the time dependent power per unit area in terms of the acoustic pressure, density, and velocity, p, ρ', and $\mathbf{u}$. The appendix derives the desired form for spectrum of sound power based on velocity potential.

$$\mathbf{I}_\omega = \frac{-i2\pi\rho_o}{T} \omega\, \Phi^* [\nabla\Phi + \frac{\mathbf{U}}{a^2}(i\omega\,\Phi - \mathbf{U}\cdot\nabla\Phi)] \tag{13}$$

This is the local (dependent on position variables) power flux vector. To find the acoustic power leaving the cascade, we must compute

$$I^{\pm} = \mathbf{I}_\omega \cdot \hat{\mathbf{n}}^{\pm} \tag{14}$$

where $\hat{\mathbf{n}}^{\pm}$ is the unit vector for a plane parallel to the cascade leading edge, $\pm(-h/s, d/s, 0)$, where s is the blade gap. By performing the indicated dot product, we find

$$I_\omega = \frac{-i2\pi\rho_o}{T} \frac{\pi^2 c^4 \omega}{\beta^2 S_e^2} \iint W^*(\mathbf{K},\omega) W(\mathbf{K}',\omega) \sum_{k=-\infty}^{\infty} \sum_{k'=-\infty}^{\infty} \frac{\zeta_k^{\pm} \zeta_{k'}^{\pm} D^*(\lambda_k^{\pm}) D(\lambda_{k'}^{\pm})}{\sqrt{\kappa_e^2 - f_k^2}\sqrt{\kappa_e'^2 - f_{k'}^2}}$$
$$\times \left\{ \pm i \left[\left(\frac{h}{s} + \frac{d^2}{hs}\right) \lambda_{k'}^{\pm} + \frac{d}{sh}(\sigma' - 2\pi k') - \frac{h}{s}\frac{U}{a^2}(\omega + U\lambda_{k'}^{\pm}) \right] \right\} \tag{15}$$
$$\times e^{i[(\lambda_k^{\pm} - \lambda_{k'}^{\pm})(x-yd/h) - [(\sigma-2\pi k)-(\sigma'-2\pi k')y/h-(v-v')z]} d\mathbf{K}\, d\mathbf{K}'$$

As Glegg pointed out, it can be shown that the term in curly brackets { } is equal to $\frac{iS_e}{S}\sqrt{\kappa_e'^2 - f_{k'}^2}$. Hence, Equation 15 reduces to

$$I_\omega^\pm = \frac{\rho_o \pi^2 c^4 \omega}{\beta S S_e} \int \int S_{ww}(\mathbf{K}, \mathbf{K}', \omega) \sum_{k=-\infty}^{\infty} \sum_{k'=-\infty}^{\infty} \frac{\zeta_k^\pm \zeta_{k'}'^\pm D^*(\lambda_k^\pm) D(\lambda_{k'}^\pm)}{\sqrt{\kappa_e^2 - f_k^2}} \times e^{i\{(\lambda_k^\pm - \lambda_{k'}^\pm)(x - yd/h) - [(\sigma - 2\pi k) - (\sigma' - 2\pi k')]y/h - (\nu - \nu')z\}} d\mathbf{K}\, d\mathbf{K}' \quad (16)$$

where S_{ww} is the upwash spectrum

$$S_{ww}(\mathbf{K}, \mathbf{K}', \omega) = \frac{2\pi}{T} W^*(\mathbf{K}, \omega) W(\mathbf{K}', \omega) \quad (17)$$

Equations 16 and 17 are the most general form of the sound power spectrum. They are generally too unwieldy for numerical work, considering the 6-fold integral and double sum. The next section addresses the upwash spectrum and develops simplifications that lead to a tractable form of these equations.

RELATION OF UPWASH SPECTRUM TO TURBULENCE SPECTRUM

The first objective of this paper is to develop an understanding of the upwash spectrum S_{ww} from a theoretical point of view. We accomplish this by relating it to the turbulence spectrum and covariance functions and then by establishing the role of inhomogeneity of turbulence in noise calculations. The second objective, gaining an understanding of measurement issues, is addressed in the final sections.

For further analysis of S_{ww} we substitute the inverse transform of the velocity field from Equation 9 into Equation 17. However, before doing this, we make the simplifying assumption that the turbulence can be treated locally as a "frozen gust pattern". This is reasonable because turbulence velocities are generally much smaller than convection velocities; hence, gust patterns change only slightly as they are convected past any point of interest. Also, there are convenient algebraic expressions for restricted forms of convected turbulence that can be used for noise calculation studies. The convected gust assumption allows us to reduce the 4-dimensional transform in Equation 9 to a 3D transform. We use the notation $w(\mathbf{x},t)$ to describe the flow (upwash) pattern in the stator-fixed coordinate system and $\widetilde{w}$ for the same pattern in fluid-fixed coordinates $\widetilde{x}, y, z$. The relationship is

$$w(\mathbf{x}, t) = \widetilde{w}(\widetilde{x}, y, z) = \widetilde{w}(x - Ut, y, z) \quad (18)$$

Substitution of this into Equation 9 and changing the chordwise integration variables via $\widetilde{x} = x - Ut$ lead to

$$W(\mathbf{K}, \omega) = \frac{1}{(2\pi)^3} \iiint \widetilde{w}(\widetilde{x}, y, z) e^{-i(\gamma_o \widetilde{x} + \alpha y + \nu z)} \left[\frac{1}{2\pi} \int e^{i(\omega - \gamma_o U)t} dt \right] d\widetilde{x}\, dy\, dz \quad (19)$$

The integral in square brackets is the delta function $\delta(\omega - \gamma_o U)$. This is equivalent to the form $\frac{1}{U}\delta(\gamma_o - \omega / U)$ needed later to facilitate the chordwise wavenumber integral. We now have

$$W(\mathbf{K},\omega) = \frac{\delta(\gamma_o - \omega/U)}{U}\frac{1}{(2\pi)^3}\iiint \tilde{w}(\tilde{x},y,z)e^{-i(\gamma_o\tilde{x}+\alpha y+\nu z)}\,d\tilde{x}\,dy\,dz \qquad (20)$$

the form to be substituted into Equation 22. The result is

$$S_{ww} = \frac{2\pi}{T}\frac{\delta(\gamma_o - \omega/U)\delta(\gamma'_o - \omega/U)}{U^2}\frac{1}{(2\pi)^6}\iiiint\!\!\iint \tilde{w}(\tilde{x},y,z)\tilde{w}(\tilde{x}',y',z') \times e^{i(\gamma_o\tilde{x}+\alpha y+\nu z)}e^{-i(\gamma'_o\tilde{x}'+\alpha y'+\nu z')}\,d\tilde{x}\,dy\,dz\,d\tilde{x}'\,dy'\,dz' \qquad (21)$$

Next, we make the changes of variables: $\tilde{x}' = \tilde{x} + \Delta x$, $y' = y + \Delta y$, $z' = z + \Delta z$. Through the arguments of the delta functions, we note that effectively $\gamma'_o = \gamma_o$ so that a term in the exponential can be eliminated

$$S_{ww} = \frac{2\pi}{T}\frac{\delta(\gamma_o - \omega/U)\delta(\gamma'_o - \omega/U)}{U^2}\frac{1}{(2\pi)^6}\iiiint\!\!\iint \tilde{w}(\tilde{x},y,z)\tilde{w}(\tilde{x}+\Delta x,y+\Delta y,z+\Delta z) \times e^{-i(\gamma'_o\Delta x+\alpha'\Delta y+\nu'\Delta z)}e^{i(\alpha-\alpha')y}e^{i(\nu-\nu')z}\,d\tilde{x}\,dy\,dz\,d(\Delta x)d(\Delta y)d(\Delta z) \qquad (22)$$

Since it is not possible to know the entire velocity field, we proceed from here with standard stochastic methods. We take the expectation value (statistical average) of both sides of Equation 22

$$\langle S_{ww}\rangle = \frac{2\pi}{T}\frac{\delta(\gamma_o - \omega/U)\delta(\gamma'_o - \omega/U)}{U^2}\frac{1}{(2\pi)^6}\iiiint\!\!\iint \langle\tilde{w}(\tilde{x},y,z)\tilde{w}(\tilde{x}+\Delta x,y+\Delta y,z+\Delta z)\rangle \times e^{-i(\gamma'_o\Delta x+\alpha'\Delta y+\nu'\Delta z)}e^{i(\alpha-\alpha')y}e^{i(\nu-\nu')z}\,d\tilde{x}\,dy\,dz\,d(\Delta x)d(\Delta y)d(\Delta z) \qquad (23)$$

where the angular brackets $\langle\ \rangle$ are read "expectation value of" and can be moved under the integral signs since the velocity field is the only random quantity on the right hand side. We proceed with the analysis using 2 approaches. In the first, we assume the turbulence field to be homogeneous, permitting a quick simplification of Equation 23 and immediate connection to standard formulas for turbulence spectra. Later, we examine the homogeneity assumption and show that it can be relaxed.

The expectation value of the velocity product in Equation 23 is the upwash spatial auto-covariance function, interpreted as follows. Consider the velocity at a base field point given by the first argument $\tilde{x}, y, z$ and multiply it by the velocity at a point removed from the base point by the deltas in the second argument. The expectation value of the product is the turbulence auto-covariance function. For homogeneous flow, the statistical average is the same for any base point and thus depends only on the separation. Thus, we define the auto-covariance function

$$R(\mathbf{S}) = \langle\tilde{w}(\tilde{x},y,z)\tilde{w}(\tilde{x}+\Delta x,y+\Delta y,z+\Delta z)\rangle \qquad (24)$$

where $\mathbf{S} = (\Delta x, \Delta y, \Delta z)$. With this definition, Equation 23 becomes

$$\langle S_{ww}\rangle = \frac{\delta(\gamma_o-\omega/U)\delta(\gamma'_o-\omega/U)}{U} \quad \frac{1}{(2\pi)^3}\iiint R(\mathbf{S})e^{-i\mathbf{K}'\cdot\mathbf{S}}d\mathbf{S}$$
$$\times\frac{1}{UT}\int d\tilde{x} \quad \frac{1}{2\pi}\int e^{i(\alpha-\alpha')y}dy \quad \frac{1}{2\pi}\int e^{i(v-v')z}\,dz \tag{25}$$

Since we have assumed R independent of $(\tilde{x}, y, z)$, integrals over those variables separate for easy treatment. Recall in an earlier section that we assumed, for convergence of some integrals, that the flow was "on" for only a period of time T. Hence, the x integration is over the range UT and the first item on the second line is just unity. The second and third integrals on that line are delta functions. Finally, the triple integral is the standard definition of the 3 dimensional turbulence wavenumber spectrum (Landahl, ref. 4)

$$\phi_{33}(\mathbf{K}) = \frac{1}{(2\pi)^3}\iiint R(\mathbf{S})e^{-i\mathbf{K}\cdot\mathbf{S}}d\mathbf{S} \tag{26}$$

where the 33 subscript is used to denote w_3w_3 correlation, which is our upwash. Thus, our upwash spectrum has been reduced to

$$\langle S_{ww}\rangle = \delta(\gamma_o - \omega / U)\delta(\gamma'_o - \omega / U)\delta(\alpha-\alpha')\delta(v-v')\tfrac{1}{U}\phi_{33}(\mathbf{K}) \tag{27}$$

To apply this, we take the expectation value of both sides of Equation 16, substitute Equation 27, and perform 4 integrations via the delta functions to find

$$\langle I_\omega^\pm\rangle = \frac{\rho_o\pi^2c^4\omega}{\beta SS_eU}\int\int\phi_{33}(\mathbf{K})\sum_{k=-\infty}^{\infty}\sum_{k'=-\infty}^{\infty}\frac{\zeta_k^\pm\zeta_{k'}^\pm D^*(\lambda_k^\pm)D(\lambda_{k'}^\pm)}{\sqrt{\kappa_e^2-f_k^2}}$$
$$\times e^{i(\lambda_k^\pm-\lambda_{k'}^\pm)(x-yd/h)} \quad e^{i2\pi(k-k')y/h}\,d\alpha\,dv \tag{28}$$

This is the sound intensity spectrum which is still a function of position with respect to the cascade via the position variables x and y. If we average this quantity along a line parallel to the cascade leading edges $(x=yd/h)$, there is no variation in the first exponential. The second exponential averages to zero unless $k=k'$, in which case it averages to unity. Thus, in performing the k' summation, the 2 λ's are the same and the first exponential disappears to yield a form suitable for computation.

$$\overline{\langle I_\omega^\pm\rangle} = \frac{\rho_o\pi^2c^4\omega}{\beta SS_eU}\int\int\phi_{33}(\mathbf{K})\sum_{k=-\infty}^{\infty}\frac{\left|\zeta_k^\pm D(\lambda_k^\pm)\right|^2}{\sqrt{\kappa_e^2-f_k^2}}\,d\alpha\,dv \tag{29}$$

This is the sound power/unit frequency/unit area averaged over the area. For initial programming, the Liepmann spectrum(ref. 5) was used for ϕ_{33}

$$\phi_{33}(\mathbf{K}) = \frac{2\overline{w^2}\Lambda^5}{\pi^2}\frac{k_1^2+k_2^2}{[1+\Lambda^2(k_1^2+k_2^2+k_3^2)]^3} \tag{30}$$

because of its simple algebraic form. Here, $\overline{w^2}$ is the upwash intensity and Λ is the turbulence integral scale.

In the wavenumber vector $\mathbf{K} = (k_1, k_2, k_3)$, we identify the streamwise wavenumber $k_1 = \omega/U$ from Equation 20. The index 3 is tied to upwash, which requires the transverse wavenumber $k_3 = \alpha$. By default, the remaining transverse wavenumber $k_2 = \nu$.

Figure 4 shows comparisons with scaled sound power spectrum data from a model test in a NASA-Lewis wind tunnel. The stator was modeled with a single geometry and mean flow appropriate for the tip stator station. Turbulence scale and intensity were back-figured to provide a good match between the noise prediction and test data. The scale, Λ, was taken at 3% of the fan radius and the intensity was 3% of the mean stator inflow velocity. The purposes of this matching were to establish credibility for the acoustic theory and to establish a turbulence scale for a later section on measurement issues.

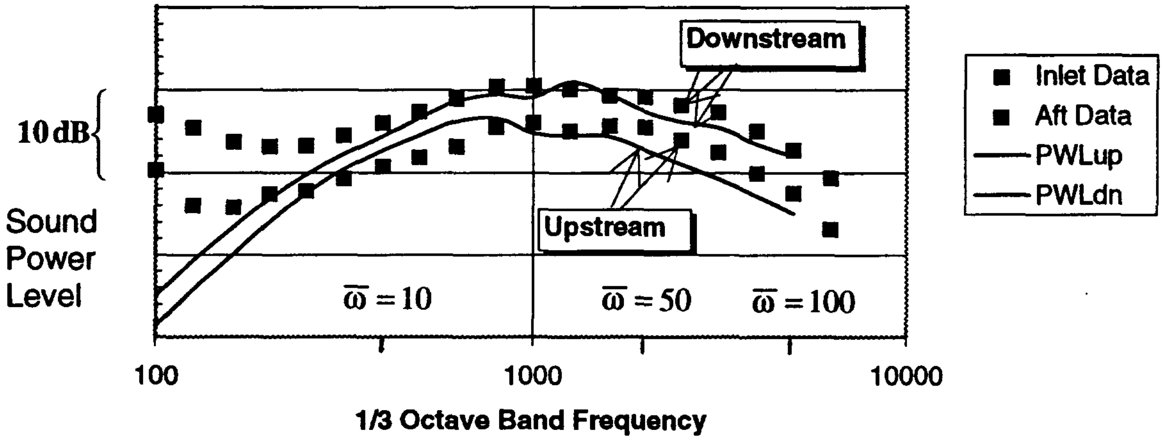

Figure 4. Comparison of theoretical and test noise spectra for scaled model data. Turbulence intensity and scale were chosen for a match to the noise data.

TREATMENT OF INHOMOGENEOUS TURBULENCE

The analysis leading to the relatively simple formula of Equation 29 depended on the assumption of homogeneous turbulence for evaluation of the integrals in Equation 23. We now re-examine this assumption for the type of wake flow shown in Figure 5 to show that valid results can be achieved for inhomogeneous turbulence. The most general form of the correlation function in Equation 24 would be

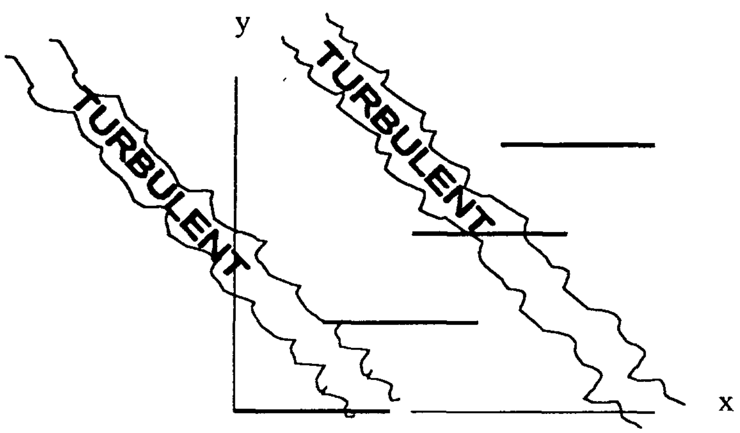

Figure 5. Coordinates for averaging the auto-covariance function

$$\tilde{R}(\tilde{x}, y, z; \Delta x, \Delta y, \Delta z) = \langle \tilde{w}(\tilde{x}, y, z)\tilde{w}(\tilde{x} + \Delta x, y + \Delta y, z + \Delta z)\rangle \tag{31}$$

where the notation now shows the dependence on the position variables $\tilde{x}$, y, and z explicitly. It is reasonable to assume the covariance function to be periodic in $\tilde{x}$ and y; this amounts to ignoring the decay of wakes over a distance equal to the cascade chord. With the above definition and with the justification given below, Equation 23 can be written

$$\langle S_{ww}\rangle = \frac{\delta(\gamma_o - \omega/U)\delta(\gamma'_o - \omega/U)}{U} \quad \frac{1}{(2\pi)^5}\iiiint\!\!\int \left[\frac{1}{UT}\int \tilde{R}(\tilde{x}, y, z; \Delta x, \Delta y, \Delta z)d\tilde{x}\right]e^{-i\mathbf{K}'\cdot\mathbf{S}}d\mathbf{S} \times e^{i(\alpha-\alpha')y}dy \quad e^{i(\nu-\nu')z}\,dz \tag{32}$$

By examining Figure 5 we can see that the integral in the square brackets is the covariance averaged along the coordinate $\tilde{x}$ fixed in the fluid. We call this

$$R_x(y, z; \Delta x\Delta y\Delta z) = \frac{1}{UT}\int \tilde{R}(\tilde{x}, y, z; \Delta x, \Delta y, \Delta z)\,d\tilde{x} \tag{33}$$

which appears still to be a function of y and z. However, in a fan with uniform rotor geometry, the statistics of one wake will be the same as all the others. Hence, the average need only be taken over one wake passage. Once this average is performed, it is independent of origin and thus no longer a function of y. The covariance is, of course, a function of z, the radial coordinate in a fan. Treatment of this effect is beyond the scope of this paper and we ignore variations of statistics in this direction. With this understanding, we can replace R_x by the average correlation $\overline{R}(\mathbf{S})$. This is formally *averaged* over x and then assumed independent of y and z. More generally, we can consider $\overline{R}(\mathbf{S})$ the correlation function averaged over the fluid. With the covariance function independent of y and z, integrals over those variables yield delta functions and the steps leading from Equation 23 to Equation 29 are exactly the same as before. The acoustic equation remains the same but with the understanding that $\overline{R}(\mathbf{S})$ replaces $R(\mathbf{S})$ in Equation 26. Thus, we have shown that the *acoustic* analysis leading to the sound power equation is equally valid for homogeneous and inhomogeneous turbulence. Obviously, determining the average turbulence spectrum function would be a major effort in *aerodynamic* analysis

The analysis above provides the recipe for computing the spectrum of inhomogeneous turbulence as needed for a noise calculation: average the covariance function over the spatial variables and then perform the triple transform indicated by Equation 26 to find the required spectrum. The significance of performing the proper average can be illustrated by asking how to evaluate the average turbulence level in an inhomogeneous flow field. The point on the covariance function for zero separation between evaluation points, i.e. for $\mathbf{S} = 0$, is just $\overline{w^2}$, the mean square upwash velocity. Thus, the above analysis teaches us that the correct value for average turbulence level comes from averaging $\overline{w^2}$, and *not* from averaging the RMS turbulence. For example, if half of the flow contains wakes at 10% turbulence and the other half has zero turbulence, the correct average RMS turbulence would not be 5% but $\sqrt{[(0^2+10^2)/2]}$ or 7.07%.

ESTIMATION OF TURBULENCE SPECTRUM FROM STATOR INFLOW MEASUREMENTS

In the preceding derivations we developed formulas for computation of the sound power spectrum from a specified turbulence spectrum. Much can be learned about noise generation by applying one of the standard algebraic forms for isotropic, homogeneous turbulence. However, the stator inflow in a fan is unlikely to fit this description. Thus, in this section, we address the question of what we would measure at a stator inlet to estimate noise from the spectrum equations above. This will also give us means to evaluate the effects of inhomogeneity and anisotropy on the measurement procedure.

We propose to make measurements with a pair of probes in a plane parallel to the stator face as suggested by Figure 6. Probes at locations a and b would measure upwash signals $w_a(y,z,t)$ and $w_b(y,z,t)$. The first probe would be fixed and the second would be indexed to a series of spacings from probe a for data acquisition. Thus, data would be recorded as a function of Δs and Δz.

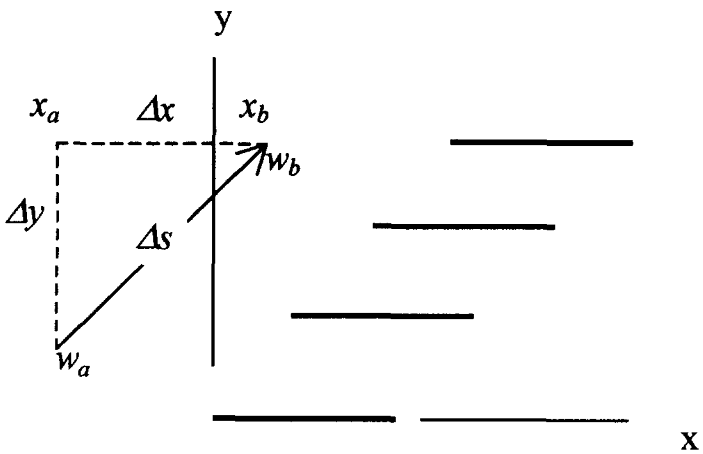

Figure 6. Traversing scheme for measurement probes

Again, we assume that the turbulence can be described as a frozen gust field since it is impractical to measure the time and x correlations independently. From the frozen gust assumption and the measurement in y, z, t, we construct the gust field from

$$w(\mathbf{x},t) = w_a(y,z,t - \tfrac{x-x_a}{U}) = w_b(y,z,t - \tfrac{x-x_b}{U}) \qquad (34)$$

By inserting the first of these into the expression for $W(\mathbf{K},\omega)$ in Equation 9, changing variables for the time integration, and identifying $\frac{1}{2\pi}\int \exp[-i(\gamma_o - \omega/_U)x]dx$ as a delta function, we find

$$W_a(\mathbf{K},\omega) = \delta(\gamma_o - \omega/_U)e^{-i\omega x_a/U}\frac{1}{(2\pi)^3}\iiint w_a(y,z,t)e^{-i(\alpha y+\nu z-\omega t)}dy\,dz\,dt \qquad (35)$$

Substitution into Equation 17 and shifting integration variables on the second triple integral via $y' = y + \Delta y,\quad z' = z + \Delta z,\quad t' = t + \tau$ lead to

$$S_{ww} = \delta(\gamma_o - \omega/U)\delta(\gamma'_o - \omega/U)\frac{1}{(2\pi)^5}\int\int\int\int\int\left[\frac{1}{T}\int w_a(y,z,t)w_b(y+\Delta y, z+\Delta z, t+\tau)dt\right]$$
$$\times e^{i\frac{\omega}{U}(x_a - x_b)} e^{i[(\alpha-\alpha')y+(\nu-\nu')z-\alpha'\Delta y-\nu'\Delta z+\omega\tau]}d\tau\, dy\, dz\, d(\Delta y)\, d(\Delta z) \tag{36}$$

Since the measuring line is parallel to the cascade, we can write $x_b - x_a = \Delta s \sin\chi$ and $\Delta y = \Delta s\cos\chi$ and then change the transverse integration variable from Δy to Δs. Consider the integral in square brackets in Equation 36:

$$R_{ab}(\Delta s, \Delta z, \tau) = \frac{1}{T}\int w_a(y,z,t)w_b(y+\Delta s\cos\chi, z+\Delta z, t+\tau)dt \tag{37}$$

This is the time cross-correlation function of the 2 velocity signals, a standard function of signal analyzers. As the notation indicates, it is independent of y because the inhomogeneous flow is swept past the probes continuously; in averaging over time, any evidence of inhomogeneity in the y direction is lost. Thus, time averaging in the stator system gives us the equivalent of averaging over the fluid that would have to be done by indexing the base probe (probe a) across the wake in either a fluid-fixed or rotor-fixed measurement system. As in the previous section, treatment of inhomogeneity in the z direction is beyond the scope of this paper and is ignored here. Given these simplifications, the α and ν delta functions appear as before and Equation 36 reduces to

$$S_{ww} = \delta(\gamma_o - \omega/U)\delta(\gamma'_o - \omega/U)\delta(\alpha-\alpha')\delta(\nu-\nu')$$
$$\times\frac{\cos\chi}{(2\pi)^2}\int\int\left[\frac{1}{2\pi}\int R_{ab}(\Delta s,\Delta z,\tau)e^{i\omega\tau}d\tau\right]e^{-i[(\frac{\omega}{U}\sin\chi+\alpha'\cos\chi)\Delta s+\nu'\Delta z]}d(\Delta s)\,d(\Delta z) \tag{38}$$

The inner integral

$$S_{ab}(\Delta s,\Delta z,\omega) = \frac{1}{2\pi}\int R_{ab}(\Delta s,\Delta z,\tau)e^{i\omega\tau}d\tau \tag{39}$$

is the cross spectrum of the 2 upwash signals and, also, may be measured by standard wave analyzers. Hence, we can write Equation 38 as

$$S_{ww} = \delta(\gamma_o - \omega/U)\delta(\gamma'_o - \omega/U)\delta(\alpha-\alpha')\delta(\nu-\nu')\cos\chi\,\hat{\phi}_{ab}(\alpha',\nu',\omega) \tag{40}$$

where we have defined

$$\hat{\phi}_{33}(\alpha,\nu,\omega) = \frac{1}{4\pi^2}\int\int S_{ab}(\Delta s,\Delta z,\omega)e^{-i[(\frac{\omega}{U}\sin\chi+\alpha\cos\chi)\Delta s+\nu\Delta z]}d(\Delta s)\,d(\Delta z) \tag{41}$$

This is the desired turbulence spectrum to be used in place of ϕ_{33} in Equation 29 for the sound power calculation. Equations 39 and 41 provide the recipe for data acquisition: measure the cross-spectrum between the signals from probes a and b as a function of spacings Δs and Δz and then perform the transform indicated by Equation 41 to convert the spatial correlation into the desired wavenumber representation.

Substitution of Equation 40 into Equation 16 gives the formula to calculate sound power spectra from turbulence measurements

$$\overline{\left\langle I_\omega^\pm \right\rangle} = \frac{\rho_o \pi^2 c^4 \omega \cos\chi}{\beta S S_e} \iint \hat{\phi}_{33}(\alpha, \nu, \omega) \sum_{k=-\infty}^{\infty} \frac{\left|\zeta_k^\pm D(\lambda_k^\pm)\right|^2}{\sqrt{\kappa_e^2 - f_k^2}} \, d\alpha \, d\nu \tag{42}$$

MEASUREMENT RESOLUTION ISSUES

The previous section provided a recipe to compute the turbulence spectrum from experimental data. Here, we extend the analysis further to estimate the traverse distances required for the measurements. We do this by approximating $\hat{\phi}_{33}$ in Equation 41 by the Liepmann(ref.5) spectrum and then Fourier transforming to estimate the dependence of S_{ab} on the probe spacing Δs and Δz.

By comparing Equations 29 and 42, we can see that equivalence of the 2 spectra is obtained from

$$\frac{1}{U} \phi(\mathbf{K}) = \frac{\cos\chi}{4\pi^2} \iint S_{ab}(\Delta s, \Delta z, \omega) e^{-i[(\frac{\omega}{U}\sin\chi + \alpha\cos\chi)\Delta s + \nu\Delta z]} \, d(\Delta s) \, d(\Delta z) \tag{43}$$

Inverse transformation of this equation is straightforward and yields

$$S_{ab}(\Delta s, \Delta z, \omega) = \frac{1}{U} e^{i\frac{\omega}{U}\Delta s \sin\chi} \iint \phi_{33}(\mathbf{K}) e^{i(\alpha \Delta s \cos\chi + \nu\Delta z)} d\alpha \, d\nu \tag{44}$$

In applying Liepmann's turbulence spectrum,

$$\phi_{33}(\mathbf{K}) = \frac{2\overline{w^2}\Lambda^5}{\pi^2} \frac{k_1^2 + k_2^2}{[1 + \Lambda^2(k_1^2 + k_2^2 + k_3^2)]^3} \tag{45}$$

we non-dimensionalize the wavenumbers k_1, k_2, and k_3, the length scale Λ, and the separation distances Δs and Δz by a nominal duct radius R and substitute into Equation 44 to find

$$S_{ab}(\Delta s, \Delta z, \omega) = \frac{8\overline{w^2}\Lambda^5}{\pi^2 U R^4} e^{i\frac{\omega}{U} S \sin\chi} \int_0^\infty \int_0^\infty \frac{K_1^2 + K_2^2}{[1 + L^2(K_1^2 + K_2^2 + K_3^2)]^3} \cos(K_3 S \cos\chi) \cos(K_2 Z) dK_3 \, dK_2 \tag{46}$$

where $L = \Lambda/R$. S_{ab} is normalized for plotting versus $S = \Delta s/R$ and $Z = \Delta z/R$ so as to give unit value at zero separation according to

$$\hat{S}_{ab}(S, Z, \omega) = \frac{S_{ab}(\Delta s, \Delta z, \omega)}{S_{ab}(0, 0, \omega)} \tag{47}$$

Equations 46 and 47 are used below to anticipate the shape of correlation function (versus probe spacing Δs and Δz) to be measured with the proposed 2 probe system. The only free parameter is the turbulence length scale. However, recall that we already made a comparison of the noise theory with test data in Figure 4 and found a good fit by choosing $L = \Lambda/R = 3\%$. Also, we need to pick some frequencies of interest. Note on the abscissa in Figure 4 that 3

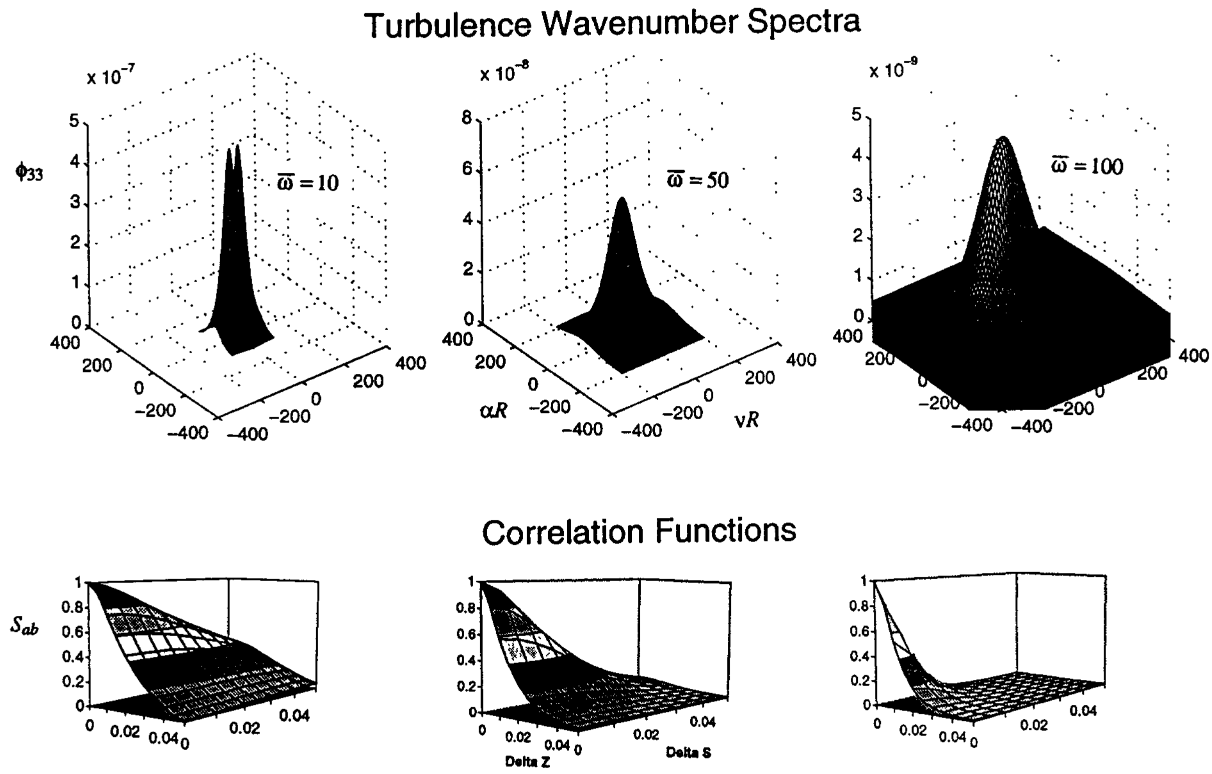

Figure 7. (Top): Turbulence spectra versus transverse wavenumber at the "low", "mid", and "high" acoustic frequencies from Figure 4. (Bottom) Transverse correlation functions indicating required probe traverse distances for the same frequencies.

frequencies are marked indicating "low", "mid", and "high" values for non-dimensional frequency, $\overline{\omega} = \omega R / a$ = 10, 50, and 100. Plots of ϕ_{33} versus $K_2 = \nu R$ and $K_3 = \alpha R$ for these 3 frequencies are given in Figure 7. Note that they broaden with increasing frequency, i.e. the wavenumber range needed for a noise calculation increases with increasing noise frequency. Plots of the frequency dependent correlation function $\hat{S}_{ab}$ are also given in the figure; as the noise frequency increases the correlation curves become narrower. The smaller eddies that produce higher frequencies (at convection speed U) are also smaller in the transverse direction. Figure 7 is a quantification of this obvious concept.

Figure 7 shows that at low frequency, the curve to be resolved extends over about 4% of radius, which should be resolveable even in model scale. At high frequencies, however the narrowness of the correlation curve may pose a measurement problem in fan model scale because of possible probe interference. In a full scale engine, probe separation should not be a problem even at high frequency.

CONCLUDING REMARKS

1. Glegg's harmonic cascade acoustic theory includes dependence on inflow with wavenumbers in all 3 coordinate directions. This permitted us to develop broadband noise formulas driven by turbulence as represented by a 3-dimensional wavenumber spectrum. In general, the acoustic intensity spectrum varies periodically in the cascade pitch direction. However, if we average in this direction (or integrate to compute sound power) the formulas simplify and become tractable for computation.
2. The noise formulas were first derived under the assumption of homogeneous turbulence. However, upon examining this assumption in detail, we found that the noise equations apply equally well to turbulence that is inhomogeneous in the pitch direction provided that the proper average of the turbulence spectrum is used in the equations. The recipe for this, given in the paper, simply requires averaging the turbulence covariance function (or the wavenumber spectrum) across the wakes in fluid-fixed coordinates.
3. It may be desirable to measure turbulence properties at a stator inlet (in stator-fixed coordinates) for use in the noise equations. The algorithms for this were developed showing how to compute the turbulence wavenumber spectrum from a measurable quantity (namely, the cross spectrum of 2 probe signals as a function of probe transverse separation). It was shown that turbulence inhomogeneity does not need to be considered either in the data acquisition or in the data reduction. Sweeping of the turbulence past the probes in the stator-fixed system (caused by rotor rotation) accomplishes the spatial averaging that would be required for measurements in the fluid-fixed system.
4. Requirements for spatial resolution in measurement of stator inflow were investigated. It was shown for frequencies in the upper end of the acoustic spectrum that probe separation must be very small. This may present problems with probe interference.

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REFERENCES

1. Hanson, D. B., "A Unified Analysis of Fan Stator Noise", Journal of the Acoustical Society of America, Vol. 54, No. 6, 1973, pp. 1571, 1591.
2. Glegg, S. A. L., "The Response of a Blade Row to a Three Dimensional Gust", Florida Atlantic University Report, September 1996.
3. Goldstein, M. E., *Aeroacoustics*, McGraw-Hill, New York, 1976.
4. Landahl, M. T. and Mollo-Christensen, E., *Turbulence and Random Processes in Fluid Mechanics,* Cambridge University Press, 1992.
5. Liepmann H. W., "Extension of the Statistical Approach to Buffeting and Gust Response of Wings of Finite Span", Journal of the Aeronautical Sciences, March 1955.

APPENDIX - SOUND POWER CALCULATION

This section derives the recipe (Equation 13) used in the main text for calculation of the spectrum of sound power scattered by the stator. The starting point is the acoustic energy flux vector applicable to waves in uniformly moving media (ref. 3)

$$\mathbf{I} = \left(\frac{p}{\rho_o} + \mathbf{U} \cdot \mathbf{u} \right) (\rho_o \mathbf{u} + \mathbf{U} \rho') \tag{48}$$

By use of Equations 7, this can be expressed in terms of velocity potential:

$$\mathbf{I} = -\rho_o \frac{\partial \phi}{\partial t} \left(\nabla \phi - \frac{\mathbf{U}}{a^2} \frac{D\phi}{Dt} \right) \tag{49}$$

We will represent the potential via the Fourier transform in Equations 10 and 11. Since we are using a 2-sided transform, the expression for potential is real. Thus we can conjugate the first term in Equation 49 without change, for convenience in the succeeding manipulations.

$$\mathbf{I} = -\rho_o \frac{\partial \phi^*}{\partial t} \left(\nabla \phi - \frac{\mathbf{U}}{a^2} \frac{D\phi}{Dt} \right) \tag{50}$$

Now, substitution of Equation 10 into Equation 50 leads to

$$\mathbf{I} = -i\rho_o \iint \omega \Phi^* [\nabla \Phi + \frac{\mathbf{U}}{a^2} (i\omega \Phi - \mathbf{U} \cdot \nabla \Phi)] e^{i(\omega - \overline{\omega})t} d\omega \, d\overline{\omega} \tag{51}$$

This is the instantaneous (time dependent) energy flux. We are interested in its time average

$$\overline{\mathbf{I}} = \frac{1}{T} \int_{-T/2}^{T/2} \mathbf{I} \, dt \tag{52}$$

which requires the operation

$$\frac{1}{T} \int_{-T/2}^{T/2} e^{i(\omega - \overline{\omega})t} \, dt = \xrightarrow[\text{Large } T]{} \frac{2\pi}{T} \delta(\omega - \overline{\omega}) \tag{53}$$

This enables the $\overline{\omega}$ integration reducing Equation 51 to

$$\overline{\mathbf{I}} = \frac{-i2\pi\rho_o}{T} \int \omega \Phi^* [\nabla \Phi + \frac{\mathbf{U}}{a^2} (i\omega \Phi - \mathbf{U} \cdot \nabla \Phi)] d\omega \tag{54}$$

This the local (position variable dependent), time average energy flux and clearly its frequency spectrum is

$$\mathbf{I}_\omega = \frac{-i2\pi\rho_o}{T} \omega \Phi^* [\nabla \Phi + \frac{\mathbf{U}}{a^2} (i\omega \Phi - \mathbf{U} \cdot \nabla \Phi)] \tag{55}$$

which is Equation 13 in the main body of the paper.