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CONTROL OF STRUCTURE-BORNE NOISE TRANSMISSION USING ELECTRO-RHEOLOGICAL FLUID INSERTS

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ABSTRACT

The shear modulus and damping loss factor of electro-rheological (ER) fluids vary when the fluid is exposed to an electric field. This behaviour can be exploited to engineer smart structures with controllable dynamic properties. This paper describes one example of such a system. The intention is to control the transmission of structure-borne sound through a structural member such as a beam. This is achieved by inserting a composite ER beam, comprising two elastic layers between which is sandwiched a layer of ER fluid, into the otherwise uniform beam. The insert behaves as a filter. The frequency characteristics of the energy transmitted through the insert can be controlled by varying the voltage applied to the ER beam. Expressions for the power transmitted through the beam/ER beam/beam combination are given and numerical results are presented.

1. INTRODUCTION

Electro-rheological (ER) fluids are suspensions of very fine dielectric particles (typical diameters are in the 5-50 micron range) in an insulating medium, that exhibit changing rheological properties in the presence of an applied electric field [1]. When an electric field of the order of 0.1 to 5.0 kV/mm with a power density of a few mW/cm^2 is applied to the fluid, long, ordered chains of these dielectric particles, that are resistant to shear deformation, form between the electrodes. The change in rheology of the fluid occurs within a few milliseconds. Physically the fluid changes from a viscous oil to an almost solid gel.

The use of ER fluids in a structure enables the physical properties of the structure to be altered by applying an electric field. This alters the shear modulus and loss factor of the ER fluid, and hence both the stiffness and damping properties of the structure. This paper concerns the control of structure-borne noise through an otherwise uniform beam by the insertion into the beam of a composite ER fluid-filled beam segment. The ER beam insert comprises two elastic outer layers between which is sandwiched a layer of ER fluid.

2. MODEL OF COMPOSITE ER FLUID-FILLED BEAM

Up to a limiting yield stress, an ER fluid can be modelled as a complex, linear viscoelastic solid whose shear modulus G_2 and loss factor β are dependent upon the applied electric field E (kV/mm). For the ER fluid in the numerical examples [2] (Figure 1)

$$G_2^* = G_2 + iG_2';G_2 = 5\times 10^4 E^2; \quad G_2' = 4340E^{0.35}. \tag{1}$$

The composite beam considered here comprises elastic face plates of thickness h_1 with an ER fluid-filled middle layer of thickness h_2 (Figure 2). All layers are of equal width b. The face plates are isotropic with Youngs Modulus E_1. In the Mead and Markus model [3], the equation of motion of the composite beam is given by

$$\frac{\partial^6 w}{\partial x^6} - g[1+Y]\frac{\partial^4 w}{\partial x^4} + \frac{\overline{m}}{\overline{EI}}\left[\frac{\partial^4 w}{\partial x^2 \partial t^2} - g\frac{\partial^2 w}{\partial t^2}\right] + \frac{1}{\overline{EI}}\left[gp(x,t) - \frac{\partial^2 p(x,t)}{\partial x^2}\right] = 0, \tag{2}$$

where $p(x,t)$ is the applied transverse load, $\overline{EI} = 2E_1h_1^3b/12$ is the total bending stiffness of the elastic layers, $\overline{m} = 2\rho_1 h_1 b + \rho_2 h_2 b$ is the mass per unit length of beam, $g = (G_2^*/h_2)(2/E_1h_1)$ is the complex shear parameter and where $Y = (d^2b/\overline{EI})(2/E_1h_1)^{-1}$ is known as Kerwins geometric parameter. Expressions for the axial displacement u_{10} of the mid-surfaces of the face plates, the shear force T, bending moment M, and axial force N_1 in the face plates are given in [3].

In the absence of external excitation, time harmonic waves of the form exp($i\omega t$-ikx) propagate at frequency ω and with wavenumber k, where

$$-k^6 - g[1+Y]k^4 + \frac{\overline{m}}{\overline{EI}}k^2\omega^2 + g\frac{\overline{m}}{\overline{EI}}\omega^2 = 0 \tag{3}$$

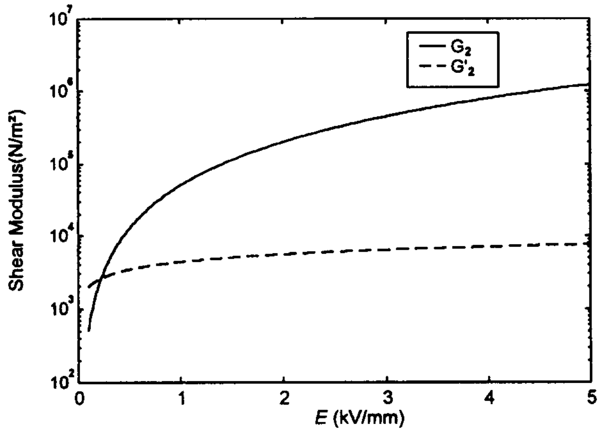

Figure 1. Shear Modulus of ER fluid

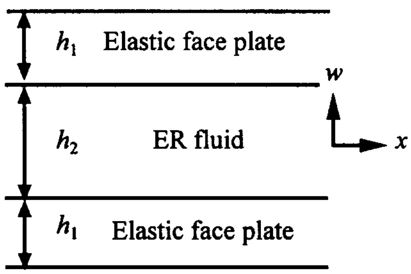

Figure 2. Diagram of ER Beam

There are three different wave modes, for each of which waves can propagate in both the positive and negative x-directions. In general, the motion is a superposition of all these wave components so that the displacement, for example, can be written in terms of the wave amplitudes a as

$$w(x,t) = \sum_{m=1,3}\left[a_m^+ e^{-ik_m x} + a_m^- e^{+ik_m x}\right]e^{i\omega t} \tag{4}$$

These three waves each comprise varying degrees of shear in the ER layer, bending in the outer layers and antisymmetric, axial deformation in the outer layers.

Suppose the amplitudes of the positive and negative travelling waves at some position on the ER beam are given by vectors $\mathbf{a}^+$ and $\mathbf{a}^-$, where $\mathbf{a}^T = \{a_1 \quad a_2 \quad a_3\}$. At this point the generalised internal displacements and internal forces are defined by the vectors $\mathbf{Y}^T = \{w \quad \partial w/\partial x \quad u_{10}\}$ and $\mathbf{F}^T = \{T \quad M \quad N_1\}$. In terms of the wave amplitudes

$$\mathbf{Y} = \Psi^+\mathbf{a}^+ + \Psi^-\mathbf{a}^-; \; \mathbf{F} = \Phi^+\mathbf{a}^+ + \Phi^-\mathbf{a}^-, \tag{5}$$

where Ψ and Φ are matrices that follow from the equations of motion [3,4]. Note that $\Psi^+(k) = \Psi^-(-k)$ and $\Phi^+(k) = \Phi^-(-k)$. The time averaged energy flow, Π, in the beam is given by

$$\Pi = -\tfrac{1}{2}\mathrm{Re}\{i\omega\mathbf{Y}^H\mathbf{F}\} \tag{6}$$

and can therefore be expressed in terms of the wave amplitudes.

2.1 NUMERICAL EXAMPLES

In these numerical examples, the beam is assumed to be of unit width. The elastic layers are taken to be aluminium with $h_1=10^{-3}$, $\rho_1=2.7\mathrm{x}10^3$, $E_1=7.27\mathrm{x}10^{10}$ while for the ER fluid $h_2=3\mathrm{x}10^{-3}$, and $\rho_2=1.06\mathrm{x}10^3$. Figure 3 shows the wavenumbers for positive travelling waves at a frequency f=40 Hz, as functions of the applied electric field. Results at other frequencies (<500 Hz) are qualitatively similar.

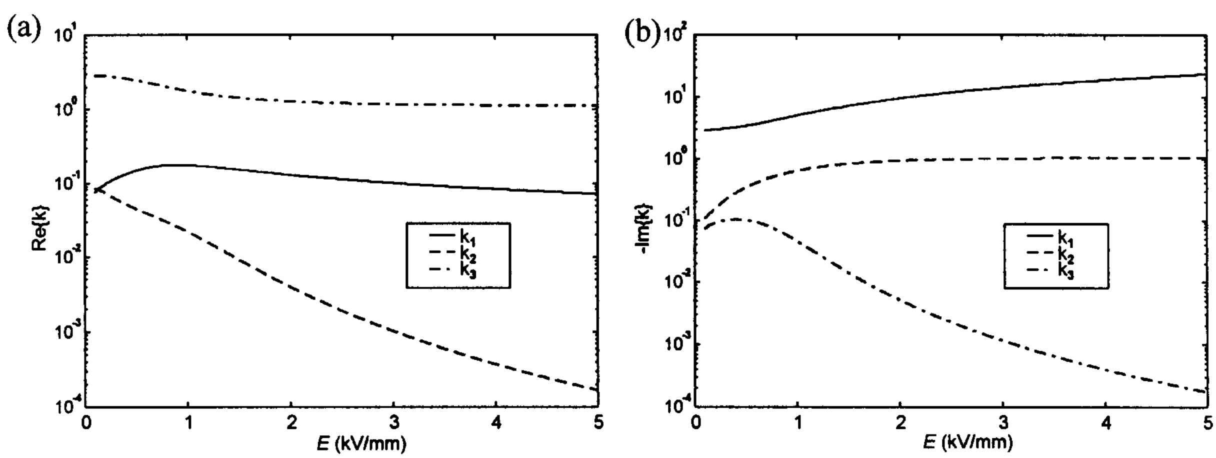

Figure 3. (a) Real and (b) imaginary parts of wavenumbers, f=40 Hz.

The wavenumbers are complex. For small electric fields, wave mode 1 behaves like a bending near field. It attenuates rapidly with distance and generally propagates little energy. Wave mode 3, on the other hand, attenuates relatively slowly. It corresponds to a bending propagating wave and can transport energy relatively freely over large distances. Wave mode 2 propagates with small phase changes and attenuates relatively gradually, the deformation involving, in effect, a "push-pull" axial motion in the outer layers. These characteristics become less distinct as the electric field increases.

3. WAVE REFLECTION AND TRANSMISSION

If a wave is incident upon any discontinuity such as a boundary, an attachment to the beam and so on, then it will be scattered, being reflected and transmitted. In general, wave mode conversion occurs, in that an incident wave of one mode will be scattered into waves of all modes. This is equally true when a wave is incident upon a junction between waveguides of different types. As an example, consider a junction between Euler-Bernoulli (EB) and ER fluid beams (Figure 4). The EB beam a supports two wave modes: a propagating wave of wavenumber $k_P = \sqrt[4]{m/EI}\sqrt{\omega}$ and a near field for which $k_N = ik_P$. This near field decays exponentially with distance and does not propagate significant energy (unless substantial damping is present or if substantial near fields exist in both directions). The ER fluid beam b supports 3 wave modes, as discussed in section 2. The wave amplitudes at this junction are related by

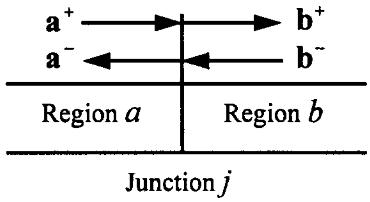

Figure 4. Junction between EB/ER beams

$$\mathbf{a}^- = \mathbf{r}_j^{aa}\mathbf{a}^+ + \mathbf{t}_j^{ba}\mathbf{b}^-;\ \mathbf{b}^+ = \mathbf{t}_j^{ab}\mathbf{a}^+ + \mathbf{r}_j^{bb}\mathbf{b}^-, \tag{7}$$

where $\mathbf{r}$ and $\mathbf{t}$ are matrices of reflection and transmission coefficients.

The generalised displacement and force vectors for each region are given by

$$\mathbf{Y}_a = \begin{Bmatrix} w \\ \partial w/\partial x \end{Bmatrix};\quad \mathbf{Y}_b = \begin{Bmatrix} w \\ \partial w/\partial x \\ u_{10} \end{Bmatrix};\quad \mathbf{F}_a = \begin{Bmatrix} T \\ M \end{Bmatrix};\quad \mathbf{F}_b = \begin{Bmatrix} T \\ M \\ N_1 \end{Bmatrix}. \tag{8}$$

From continuity and equilibrium considerations at the junction it can be shown that

$$\begin{aligned}
\mathbf{r}_j^{aa} &= \left(-\mathbf{E}_a\Phi_a^- + \mathbf{E}_b\Phi_b^+(\mathbf{C}_b\Psi_b^+)^{-1}\mathbf{C}_a\Psi_a^-\right)^{-1}\left(\mathbf{E}_a\Phi_a^+ - \mathbf{E}_b\Phi_b^+(\mathbf{C}_b\Psi_b^+)^{-1}\mathbf{C}_a\Psi_a^+\right);\\
\mathbf{t}_j^{ab} &= (\mathbf{C}_b\Psi_b^+)^{-1}\left(\mathbf{C}_a\Psi_a^+ + \mathbf{C}_a\Psi_a^-\mathbf{r}_j^{aa}\right);\\
\mathbf{t}_j^{ba} &= \left(\mathbf{E}_a\Phi_a^- - \mathbf{E}_b\Phi_b^+(\mathbf{C}_b\Psi_b^+)^{-1}\mathbf{C}_a\Psi_a^-\right)^{-1}\left(\mathbf{E}_b\Phi_b^- - \mathbf{E}_b\Phi_b^+(\mathbf{C}_b\Psi_b^+)^{-1}\mathbf{C}_b\Psi_b^-\right);\\
\mathbf{r}_j^{bb} &= (\mathbf{C}_b\Psi_b^+)^{-1}\left(-\mathbf{C}_b\Psi_b^- + \mathbf{C}_a\Psi_a^-\mathbf{t}_j^{ba}\right),
\end{aligned} \tag{9}$$

where

$$\mathbf{C}_a = \begin{bmatrix} \mathbf{I} & 0 \\ 0 & \mathbf{I} \\ 0 & \frac{1}{2} d_{tot} \mathbf{I} \end{bmatrix}; \qquad \mathbf{C}_b = \mathbf{I}; \qquad \mathbf{E}_a = \mathbf{I}; \qquad \mathbf{E}_b = \begin{bmatrix} \mathbf{I} & 0 & 0 \\ 0 & \mathbf{I} & 0 \\ 0 & 0 & 0 \end{bmatrix}. \tag{10}$$

3.1 NUMERICAL EXAMPLE

As an example, consider a EB beam made of aluminium (ρ=2.7x10^3, E=7.27x10^{10}), of unit width and of thickness h=5x10^{-3}, coupled to the ER beam of section 2.1. The reflection and transmission coefficients for an incident propagating wave in the EB beam are shown in Figure 5. Note that the incident wave is scattered into all three modes in the ER beam (although the amplitude of the mode 2 wave is small), that about half of the incident energy is reflected and that a relatively small reflected near field is produced.

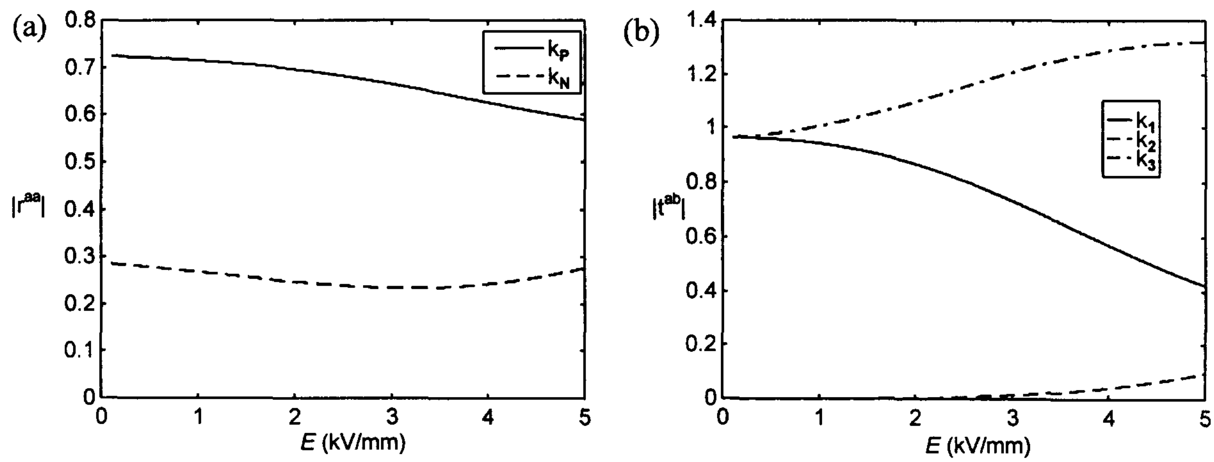

Figure 5. Magnitude of (a) reflection and (b) transmission coefficients for a propagating wave incident upon a junction between an Euler-Bernoulli and an ER beam, f=40 Hz.

4. ER FLUID-FILLED BEAM INSERTS

Consider an otherwise uniform EB beam into which is inserted a length of ER fluid-filled beam. The wave amplitudes at specific locations are shown in Figure 6. They are related by

$$\begin{aligned} & \mathbf{b}_1^+ = \mathbf{t}_1^{ab}\mathbf{a}_1^+ + \mathbf{r}_1^{bb}\mathbf{b}_1^-; \quad \mathbf{b}_2^+ = \mathbf{f}_b\mathbf{b}_1^+; \quad \mathbf{c}_2^+ = \mathbf{t}_2^{bc}\mathbf{b}_2^+; \\ \mathbf{a}_1^- = \mathbf{r}_1^{aa}\mathbf{a}_1^+ + \mathbf{t}_1^{ba}\mathbf{b}_1^-; \quad & \mathbf{b}_1^- = \mathbf{f}_b\mathbf{b}_2^-; \quad \mathbf{b}_2^- = \mathbf{r}_2^{bb}\mathbf{b}_2^+, \end{aligned} \tag{11}$$

where

$$\mathbf{f}_b = \begin{bmatrix} e^{-ik_1 l_b} & 0 & 0 \\ 0 & e^{-ik_2 l_b} & 0 \\ 0 & 0 & e^{-ik_3 l_b} \end{bmatrix} \tag{12}$$

is a matrix that describes wave propagation from one end of the insert to the other. The transmitted and the reflected waves $\mathbf{c}_2^+$ and $\mathbf{a}_1^-$ are given in terms of the incident wave $\mathbf{a}_1^+$ by

$$\mathbf{c}_2^+ = \mathbf{t}^{ac}\mathbf{a}_1^+ ; \qquad \mathbf{a}_1^- = \mathbf{r}^{aa}\mathbf{a}_1^+ , \tag{13}$$

where the net transmission and reflection matrices $\mathbf{t}^{ac}$ and $\mathbf{r}^{aa}$ are given by

$$\mathbf{t}^{ac} = \mathbf{t}_2^{bc}(\mathbf{I} - \mathbf{f}_b\mathbf{r}_1^{bb}\mathbf{f}_b\mathbf{r}_2^{bb})^{-1}\mathbf{f}_b\mathbf{t}_1^{ab} ; \qquad \mathbf{r}^{aa} = \mathbf{r}_1^{aa} + \mathbf{t}_1^{ba}(\mathbf{I} - \mathbf{f}_b\mathbf{r}_2^{bb}\mathbf{f}_b\mathbf{r}_1^{bb})^{-1}\mathbf{f}_b\mathbf{r}_2^{bb}\mathbf{f}_b\mathbf{t}_1^{ab} . \tag{14}$$

These matrices are the sum of a 'direct' component (e.g., $\mathbf{r}_1^{aa}$) and components arising from multiple reflections at the EB/ER interfaces.

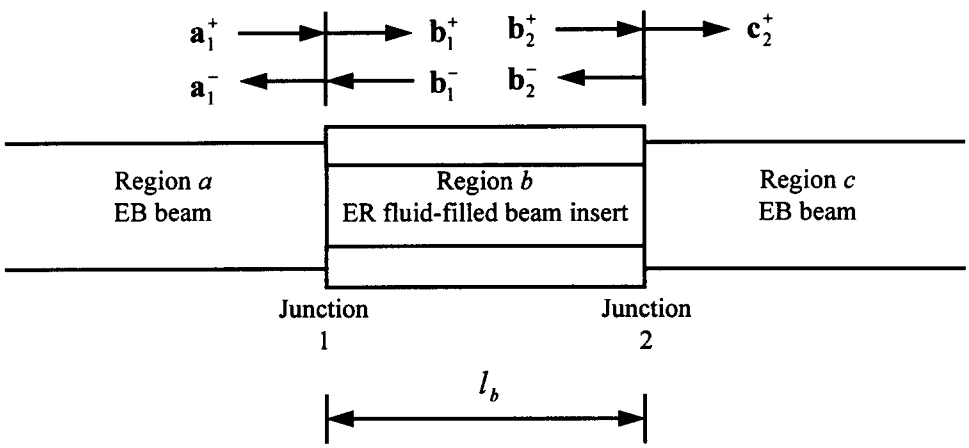

Figure 6. An ER fluid-filled beam insert in a Euler-Bernoulli beam and wave amplitudes.

Since the electric field applied to the ER beam determines the wavenumbers and the reflection and transmission matrices at the EB/ER junctions, it is clear that this field can be used to control the vibrational energy flow (i.e., the net transmission) through the insert. It is also clear from equation (14) that the net transmitted energy is given in terms of the ER beam properties by a rather complicated expression. It is possible to gain some physical insight, however, by considering the much simpler case in which there is only one wave mode. Between the discontinuities, the net wave field consists of the superposition and interference of multiple reflections from the discontinuities. At certain frequencies these waves interfere constructively and the net transmission is large (in the absence of damping $t^{ac} = 1$). At other frequencies destructive interference occurs and the net transmission is small. These frequencies are determined primarily by the wavenumber in the centre section and by the phase of the reflection coefficients r_1^{bb} and r_2^{bb}.

When the insert is an ER fluid-filled beam the situation is complicated by the fact that there are three wave modes in the ER beam and that wave mode conversion occurs at the beam junctions. However the behaviour of the net transmission through the insert is qualitatively similar to the single wave-mode case. Since the transmission depends significantly on the wavenumbers in the ER beam, and since these wavenumbers depend on the applied electric field, controlling this field thus give a mechanism by which the energy flow through the insert can be controlled. In particular the insert can be tuned to provide vibration isolation (i.e., small transmitted energy) over a desired frequency range.

4.1 NUMERICAL EXAMPLE

As an example, consider an ER fluid-filled beam of length l_b=0.17m inserted into a EB beam, the beam properties being those given in sections 2.1 and 3.1. Figure 7 shows the transmitted power per unit incident power τ for various electric fields. Note in Figure 7(a) that there is a distinct minimum at a particular frequency, and that this frequency is tunable by changing the electric field. The bandwidth is typically half the centre frequency or so, thus the insert, which acts as a band-stop filter, can reduce the transmitted power in a desired frequency band. It is therefore especially suited to the isolation of structure-borne vibrations in those cases where the incident power is somewhat narrow band. At higher frequencies (Figure 7(b)) a clear pass/stop band structure, can be seen. This is also tunable, to an extent.

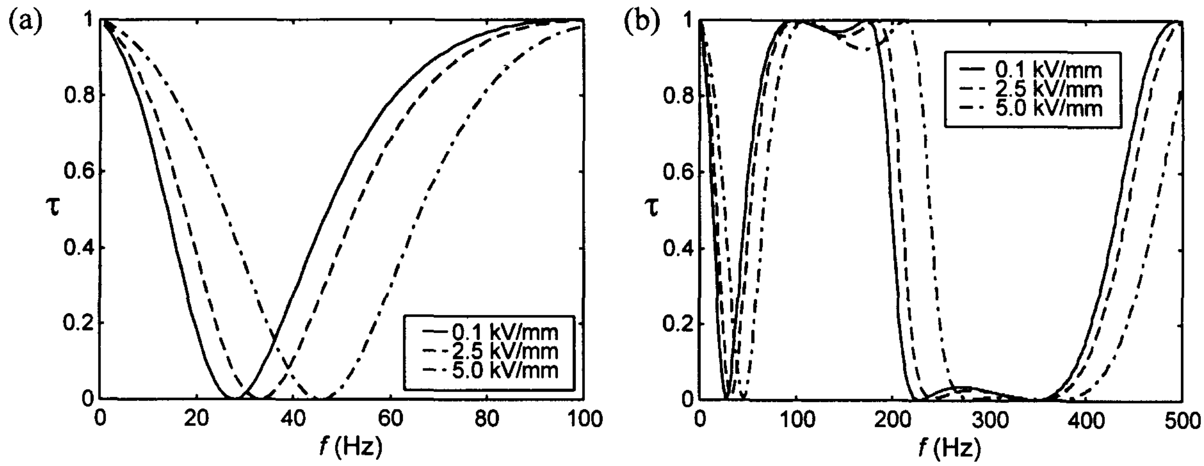

Figure 7. Transmitted power per unit incident power, τ.

Figure 8 shows the transmitted power as a function of frequency for inserts of various lengths. In broad terms, the first minimum occurs at similar values of $k_3 l_b$, so for larger l_b, the first minimum occurs at smaller wavenumbers, and hence lower frequencies. In principle, therefore, for relatively narrow band disturbances, the length of the insert can be designed and the applied electric field then adjusted to provide fine tuning.

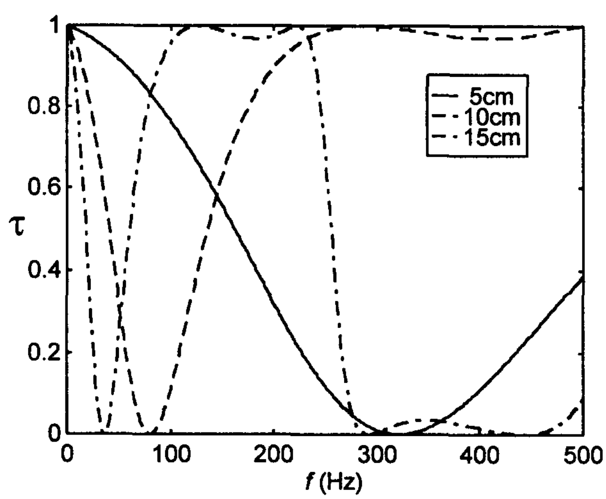

Figure 8. Transmitted power per unit incident power, τ,for various lengths l_b, E= 2.5 kV/mm.

A small amount of energy is absorbed in the ER beam. This is illustrated in Figure 9, which shows the net power in the two EB beams, in the regions *a* and *c* on either side of the insert. The net power becomes small when a strong reflected wave exists, i.e., when the net transmission is small. Only about 1% of the incident energy is absorbed.

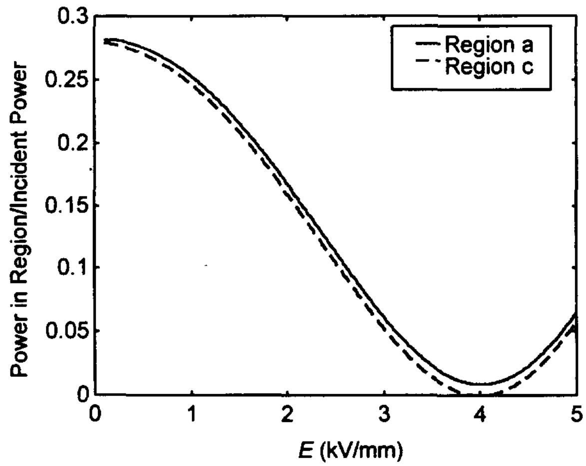

Figure 9. Time averaged power per unit incident power, *f*=40 Hz.

5. CONCLUSION

In this paper, wave propagation, transmission and reflection in ER fluid-filled beams, were described. They depend on, and can be controlled by the applied electric field. The particular case of an ER beam insert in an otherwise uniform EB beam was then considered. The net power transmitted through this insert was seen to depend upon the applied field and to show clear stop bands. The structure is therefore tunable, in that the insert and the applied electric field can be used to control the transmission of structure-borne noise through the structure.

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